Designing Lessons to Help Turkish Middle School Students Learn about Orthogonal and Isometric Drawings of Three-dimensional Shapes

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ABSTRACT

This thesis describes a design-based research project to develop, implement and evaluate lessons intended to help middle school students learn about three-dimensional shapes and in particular orthogonal and isometric drawings of polycubes.

In an initial study, four classes from two schools in Turkey were observed during the regular teaching of three-dimensional shapes, and then students were asked to complete a worksheet to examine the outcomes of this teaching. The study found that students' performance on orthogonal and isometric questions of the types asked in national exams were lower than desired. It also analysed the types of mistakes students made and noted the difficulties which may have led to these. Informed by these findings and the wider literature, a model was developed which suggests that teaching of three-dimensional shapes can be: **r**ealistic, **e**xploratory, **t**echnology-enhanced and **a**ctive, hence the RETA principles. These principles informed the design of four lessons on orthogonal and isometric drawings of polycubes, which were researched in the remaining studies of this thesis.

The second, third and fourth study aimed to see whether the RETA-based lessons were engaging and effective and improve them if they were found not to be. Each cycle reported how the RETA-based lessons were experienced by participants and the outcomes they achieved.

Specifically, the second study explored eight students' experiences of the RETA-based lessons in an after-school mathematics course. In general, the results showed that students mostly experienced the lessons positively, and the lessons had the potential to improve their drawings. The third study focused on a teacher's experiences of teaching with the RETA-based lessons in a class of 30 students and its outcomes. The teacher was a typical Turkish maths teacher, having a very different pedagogical approach and background to that of the researcher. This study explored her experiences in teaching in this way and found statistically significant improvement in students' orthogonal and isometric drawing performance with the RETA-based lessons. The final study was a quasi-experiment with 205 students and four teachers where the RETA classrooms were compared to business as usual classes. The results

showed that RETA-based lessons were significantly more effective than traditional methods.

This thesis offers insights and contributions into both the theory and the practice as expected from a design-based research project. The first of these is the RETA principles, which provide a basis for designing lessons on how three-dimensional shapes can be taught. The second contribution is the designed lessons on orthogonal and isometric drawings of polycubes, which are complete and detailed lesson plans that can be reused and adapted by mathematic teachers and researchers. These lesson plans were iteratively improved through three cycles, and by providing accounts of the design changes after each study together with the process involved, the outcomes of the lessons, and what worked and what did not, they are intended to offer detailed information to inform future research and practice.

PUBLICATIONS ARISING FROM THIS THESIS

- Full papers

Saralar, İ., Ainsworth, S., & Wake, G. (2019). A design study on improving spatial thinking of middle school children. *British Society for Research into Learning Mathematics, 39*(1), 1-6.

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Saralar, İ., Ainsworth, S., & Wake, G. (2018). Middle school students' errors in two-dimensional representations of three-dimensional shapes. *British Society for Research into Learning Mathematics, 38*(1), 1-6.

- Abstracts

Saralar, İ., Ainsworth, S., & Wake, G. (2019, August). *Working with a mathematics teacher to teach with technology*. Paper presented at the European Association for Research in Learning and Instruction Conference 2019. RWTH Aachen University, Aachen, Germany: EARLI.

Saralar, İ., & Ainsworth, S. (2018, September). *Turkish middle school students' understanding of polycubical shapes.* Paper presented at European Conference on Educational Research 2018: Inclusion and Exclusion, Resources for Educational Research. Free University of Bozen, Bolzano, Italy: EERA ECER.

Saralar, İ., Ainsworth, S., Foster, C., & Hodgen, J. (2018, June). *How do seventh graders experience two-dimensional representations?* Poster session at the Third Annual Symposium of the Mathematics Education Center. Loughborough University, Loughborough, UK: MEC.

Saralar, İ. (2018, April). *Spatial reasoning in middle school children: Two-dimensional representations of three-dimensional shapes.* Poster session at the Mathematical Cognition and Learning Society Conference 2018. The University of Oxford, Oxford, UK: MCLS.

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1. INTRODUCTION

1.1. Personal Background

When I was 15 years old, I started taking pedagogy courses in addition to core curriculum courses in Bandırma Teacher Training College for four years. After that, I entered the Elementary Mathematics Education programme of the Middle East Technical University. This programme trained mathematics teachers to work with middle school children (aged 11- to 15-year-old), as well as prepare students for graduate programs with research courses and projects. The last year of this programme was mostly devoted to teaching experience in classroom settings. As a trainee mathematics teacher, I taught different mathematical topics in middle school classes and effectively developed my pedagogical skills under the guidance of the headteacher. I obtained my teaching certificate by successfully completing the placement.

Following my teacher training, I studied for a master's degree in the Elementary Science and Mathematics Education programme of the same university. My thesis was a case study on a trainee mathematics teacher's use of technology in mathematics classrooms, which aimed at developing a deep understanding of the change in her technological pedagogical content knowledge throughout the placement (Saralar, 2016a; Saralar, Işıksal-Bostan & Akyüz, 2018). Meanwhile, I started working in a middle school as a mathematics teacher where I had the opportunity to observe students' learning processes. During my teaching practice in middle schools over three years, I experienced difficulties in teaching units requiring spatial thinking in addition to noting the difficulties students face while learning three-dimensional geometry. I attempted to solve this problem by integrating different software packages (e.g., *Cabri*, 2016; *GeoGebra*, 2016) into my classroom. GeoGebra was the main software I used, as it is suggested by the Ministry of Turkish National Education as key to integrate into mathematics classes, and a number of my colleagues used it too. Hence, I decided to investigate those middle school mathematics teachers' beliefs and goals regarding GeoGebra while I was doing my second masters in Learning, Technology and Education at the University of Nottingham. This resulted in a list of challenges teachers and their students face in the process of integrating GeoGebra (Saralar, 2016b; Saralar & Ainsworth, 2017), leading me to the idea of collaborating with teachers so that students will be given chance to study three-dimensional geometry with the support of effective technology-based interventions. This was how this PhD has started.

Following the successful results in language competency tests (YDS), a written exam (ALES) and interview, I won the scholarship of the Turkish government in the field of educational technologies. My scholarship is called YLSY which stands for Selecting and Appointing the Candidates to Send Abroad for Postgraduate Education. After successful completion of my PhD degree, I will be promoted to work as an education specialist at the Ministry of Turkish National Education's Educational Technology department.

I was aware that bringing technology into the classroom was not enough for deep and meaningful learning hence I looked for further ideas to make most of technology, and these were affected by my stance. At the beginning of the thesis, I feel that it is important to set out my pedagogical orientation. I am interested in students' constructing their own understanding, and it leads to the RETA model that is developed and trialled with sample lesson plans in this thesis. Although there are more than one constructivist theories of learning, many agree on the importance of social interaction in the process of knowledge construction and that learners are active in this construction (Bruning, Schraw & Norby, 2011; Woolfolk, 2016). Taken this into consideration, I agree with Woolfolk (2016) and believe in that "learning comes from the learner" and for this to happen "schools must create effective learning environments" (p.396). This is to say, I support active participation of the students in constructing their own understanding of maths rather than direct teaching methods through which a teacher tries to impose their understanding to students. I believe in the importance of the interaction both between students and between students and a teacher throughout the lesson. I also think that oftentimes it is a teacher's job to provide students effective learning environments where students engage in their own knowledge-building. I have held this constructivist thinking throughout the thesis with the hope that learning as an outcome of this constructivist approach not only helps students answer school maths problems correctly but also aids them in reasoning about real-life issues and it facilitates transferring the information that they learned in the classroom to this reasoning.

1.2. Context of the Thesis

Given this information in Section 1.1, it would not be wrong to say that at the beginning of this PhD, the context of the research was ready to investigate.

The Ministry of Turkish National Education has a large budget for bringing the latest technology of time into classrooms since the beginning of the FATIH Project (Movement of Enhancing Opportunities and Improving Technology Project). The project first aimed at providing interactive whiteboards and infrastructure which enable schools to access Moodle (called EBA). It also suggested making use of various dynamic tools and programmes and offered a-day in-sessional training sessions to in-service teachers from all disciplines. The sessions were on effective ways of technology integration. Some of these sessions were given by the technology experts and did not go any further than how to use Moodle. Only a limited number of the sessions were given by prospective teachers who are familiar with teaching in real classrooms. Moreover, the training sessions, unfortunately, were not subject-specific and therefore might be argued to have a limited practical implications.

It is positive that the Turkish government is spending a big budget for education and particularly for technology integration. On the other hand, bringing technology together only with limited and superficial training on effective ways to use it as suggested by the government may neither be enough for effective technology integration and so better learning outcomes (than the current national and international exam results). Therefore, the government followed a new strategy and started recruiting teacher-researchers to study abroad and then to come back to Turkey to work in the ministry from 2013. Some of these researchers started to design sample technology-enhanced lessons for teachers to be used in their teaching. The researcher of this thesis was one of the seven researchers who came to the UK in 2015 for this purpose. Specifically, the present study would like to explore possibilities in geometry education –in relation to redesigning practice of teaching spatial geometry to explore possibilities of using technology to improve middle school students' learning of two-dimensional drawings of three-dimensional shapes.

1.3. Two- and Three-dimensional Shapes

As two-dimensional drawing and three-dimensional shapes are general terms, this section describes their use in Turkish middle school programme and what they refer to in this thesis.

A three-dimensional shape is defined typically as *any shape or object that takes up air space.* Mathematically, 3D shapes or solids can be defined as shapes having height, width and depth. A cube, cylinder, cone, pyramid, sphere or prism are all examples of 3D shapes. A 3D shape, in this study, is a shape constructed from unit cubes and having a non-empty base and no hidden blogs (see red solid in Figure 1.1).

A two-dimensional drawing is any planar shape with height and width such as a triangle or a square. In this thesis, a 2D drawing refers to an orthogonal drawing or an isometric drawing. Before describing the terms (orthogonal drawing and isometric drawing) for their use in this thesis, it is important to understand what orthogonal and isometric are. The term *orthogonal* originated in the "late 16th century from French, based on Greek orthogōnios (right-angled)", it means "of or involving right angles" ('Orthogonal', 2019). As the meaning indicates an orthogonal drawing is a type of drawing which involves right angles to draw separate two-dimensional views (from the front, top, left, right and back). An orthogonal drawing of a view (a single view of a 3D shape constructed from unit cubes, e.g., front view), in this context, can be thought of as a combination of squares. For example, Figure 1.1 illustrates orthogonal drawings of views (from the front, top, left, right and back) of a 3D shape constructed from unit cubes.

Isometric comes from the "mid-19th century from Greek isometria (equality of measure) from isos (equal) and -metria (measuring)", it means "of or having equal dimensions" ('Isometric', 2019). An isometric drawing is a type of drawing in which all dimensions (length, height and width) are drawn in full scale or equally foreshortened instead of foreshortening them to the true projection. In true projection, a 3D shape's dimensions along the line of sight are drawn shorter than those across the line of sight. A key feature of an isometric drawing is that horizontal edges are drawn with 30 degrees angle from the horizontal axis while vertical edges stay vertical in the drawing. An isometric drawing, in this thesis, refers to the isometric projection of cube constructions. An isometric drawing of a cube has three visible faces as equilateral parallelograms that allows drawing all parallel edges as parallel lines (as shown in the blue isometric drawing in Figure 1.1).

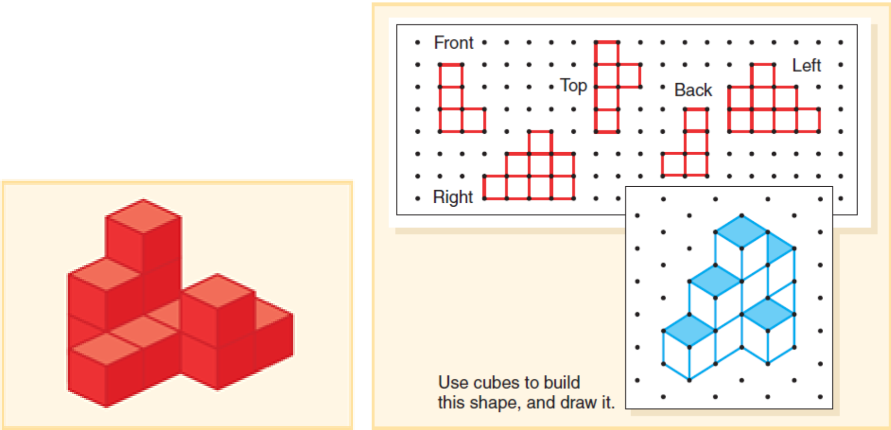

Figure 1.1. Orthogonal and isometric drawings of a shape (Van De Walle, Karp & Bay-Williams, 2010, pp. 431)

1.4. Organization of the Thesis

This thesis has been organised in nine chapters. Following this introductory chapter, Chapter 2 reviews and critiques the literature relevant to the current research. It has two main sections on spatial thinking and geometry education. While the first section introduces spatial thinking and its place in teaching geometry, the second reviews the literature on geometry education, particularly spatial (3D) geometry. The literature review ends with the outline of the research questions.

Chapter 3 explains the methodology underpinning this thesis. This methodology is used to explore the research questions first described in the literature review. It outlines design-based research and how it is used throughout the thesis and discusses the ethical procedures for the studies.

Chapter 4 pertains to the first study which is a case study of current mathematics teaching on 3D shapes in middle schools in Turkey. The current literature and results of this first study fed into a model for teaching 3D shapes, called RETA (realistic, exploratory, technology-enhanced and active) teaching model. Chapter 5 proposes this model for 3D shapes teaching and presents the initial RETA-based lesson plans on orthogonal and isometric drawings of polycubes.

Chapter 6, 7 and 8 are all studies that trial the RETA-based lessons in classroom settings. While Chapter 6 and Chapter 7 further focus on students' and teachers' experiences with the RETA-based lessons, respectively Chapter 8 presents the results of an experimental study on teaching 3D shapes with and without RETA-based lessons.

Finally, Chapter 9 summarizes and discusses the findings of the thesis around the research questions. Additionally, it considers the limitations and implications and proposes future directions for further research on 3D shapes with the RETA model.

2. LITERATURE REVIEW

This chapter consists of two sections. Section 2.1 analyses current discourse about spatial thinking including the domain specificity of spatial thinking and approaches to measure spatial thinking. The immediate proximal aim of this doctoral research is to provide lesson plans to help teachers support their students to do better in the Turkish government geometry exam but the ambition of this thesis goes beyond this exam. This section (2.1) is important because a longer-term aim of this research is to improve students' spatial awareness[1] by giving them an opportunity to work with two- and three-dimensional representations of 3D shapes, so that they can better deal with the spatial problems in their future lives. Thus, it needs to set out if this goal is, in principle, achievable. Section 2.2 presents a review of the literature which seeks to provide an overview of current discourse and understandings about student performance in 3D geometry. It describes students' performance in geometry and main factors that it has been argued to affect this including spatial abilities, drawing abilities, and 3D geometry thinking. Section 2.3 describes spatial thinking and its relation to geometry education. The chapter ends by drawing these literatures together to propose research aims and questions.

2.1. Spatial Thinking

Spatial thinking is an inseparable part of our lives. It starts when the infant first experiences the world, and it never ends. Whether you are a child playing blind man's bluff or an adult packing a suitcase and putting it into a car truck, you always need spatial thinking. From understanding floor plans of a shopping mall to reading complex maps, from deciding to places of the furniture in your house to actually doing the design drawings of furniture and plans of buildings, it is necessary to think spatially. But what is spatial thinking?

Spatial thinking is an overarching and generalizable term that refers to numerous aspects related to space. Before describing the term, it is important to understand *the*

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[1] Spatial awareness is a cognitive skill which requires an organisation of object understanding in reference to another object and in reference to oneself, and it also includes understanding objects' relationships when they alter position in order to use this information in a systematic way for planning movement (Jenkinson et al., 2008; Yarmohammadian, 2014).

spatial of spatial thinking. Although the word spatial seem to be understood as universal and absolute, the literature offers three possible interpretations of it (Witelson & Swallow, 1988). The first of these interpretations uses the term spatial to describe perception as it relates to visual and physiological sense modalities (e.g., Landau, Spelke and Gleitman's (1984) study with blind and blindfolded children for finding new routes using motor control). Secondly, researchers might use the term spatial to refer to mental or physical manipulation of objects in Euclidean space (e.g., two-dimensional and three-dimensional mental rotation tasks). Finally, a third way that researchers use spatial is locative purposes and familiarity with one's environment (Uttal, 2000). Moreover, some researchers have not defined what spatial is, they have had to concentrate on defining what spatial is not; here the contrast is primarily linguistics (Linn & Petersen, 1985). All of these are widely accepted descriptions for the term spatial found in the literature.

To understand the term spatial thinking, it is, of course, important to understand both the term spatial and the term thinking (Ness & Farenga, 2007). Probably, therefore, many ideas arrived from psychology to describe spatial thinking and many psychology-based definitions were offered for spatial thinking. One of them describes the construct of spatial thinking as a subset of mental imagery (one's thinking using mental images[2]) (Gleitman, Gross & Reisberg, 1995). Gleitman and colleagues (1995) claim that spatial thinking is about people's "referring to their mental images as mental pictures and comment that they inspect these pictures with the mind's eye" (p.343). The other considers the term as cognitive processes associated with spatial entities where these entities are events or objects which happen and/or take place in space (Casati & Varzi, 1999). Casati and Varzi (1999) argue that "Spatial thinking, whether actual or hypothetical, is typically thinking about spatial entities of some sort. … For instance, we can imagine a decomposition of objects and events into their parts. The table has four legs (actual); the take-off was the most exciting part of the flight (hypothetical)." (pp.1-3). Although both definitions are commonly cited in the reviewed literature, one could see that while Gleitman et al.'s (1995) definition

[2] Mental images are "mental representations that resemble the objects they represent by directly reflecting the perceptual qualities of the thing represented" (Gleitman, Gross, & Reisberg, 1995, p.343).

excludes the process and steps of spatial events, Casati and Varzi's (1999) definition includes them.

This thesis follows a relatively recent and more comprehensive definition by a committee of researchers working on spatial thinking to describe domain-independent spatial thinking. The committee defines spatial thinking as a combination of three elements: concepts of space, the process of reasoning and tools of representation (Committee on Support for Thinking Spatially, 2006a). *Concepts of space* are considered as the main element which distinguishes spatial thinking from other forms of thinking. The concepts involve an understanding of space so that one could use its properties, such as continuity, dimensionality and proximity, in order to understand and set problems. There are many characteristics of an object in space that are spatial: parts of an object (the tail of a cat), orientation of an object (relative place of a vase, it might be on the table and next to the pencil case), and size of an object (a bird might be smaller than a cat, and bigger than a mouse). *The process of spatial reasoning* needs to be considered and becomes important during problem-solving. It involves the ability to reason by comparing, manipulating and transforming mental pictures in order to suit the problem-solving process; for example, when visualising the shortest distance between two points (Hegarty & Waller, 2005; Newcombe & Shipley, 2015). Spatial transformations including rotation and scaling and using these transformations to figure out, infer and find solutions to problems are also described as being a part of the process of reasoning. Finally, *tools of representation* play a role both in understanding space and in the process of reasoning, for example, when working on the relationships among different views (plans versus elevations of buildings). By tools of representation, the committee basically talks about the Vygotskian representation tool. Hence, representations refer to a wide range of things which can be auditory, graphic (e.g., text, image and video), kinaesthetic and tactile and they are used to describe, clarify and communicate about objects' structure, operation and function and their relationships (Committee on Support for Thinking Spatially, 2006b). These representations are needed because of the existence of objects in different spatial scales. For example, a chemist may treat a molecule as an object and an astrophysicist may treat a planet as an object. Different tools and programs are used to form and understand objects through their representations. Such tools are also utilised in many fields in comparing various representations, such as comparing orthogonal and perspective maps in geography and comparing plans and elevations of buildings in architecture.

The terms visual-spatial thinking (Wickens et al., 2005) and visuospatial thinking (Shah & Miyake, 2005; Wu & Shah, 2004) are sometimes interchangeably used for spatial thinking. However, in this thesis, the term spatial thinking was chosen following the suggestions of Ness and Farenga (2007) because it is the most commonly accepted term in geometry literature. Finally, despite this general description of spatial thinking, it should still be noted that the term has been described by many researchers in different ways (e.g., Lohman, 1988) and the majority of these descriptions are specific to the researchers' disciplines.

2.1.1. Domain-specificity of Spatial Thinking (Spatial Thinking in Disciplinary Contexts)

Research on spatial thinking is distributed across many different disciplines. The term spatial thinking can be and has been found in many disciplines including architecture (March & Stiny, 1985), chemistry (Small & Morton, 1983), engineering (Hsi et al., 1997), geography (Kastens & Ishikawa, 2006; Lee & Bednarz, 2012), medicine (Hegarty et al., 2007), physics (Kozhevnikov et al., 2007), science (Sanchez, 2012) and, most related to the current PhD research, in mathematics education (Cheng & Mix, 2014; Newcombe, 2018). While some of these disciplines are somewhat similar and so one could find a basis for the comparison such as chemistry (e.g., drawings of organic molecules and representing them as 3d structures) and mathematics (e.g., drawings of polycubes and representing the elevations of them), some are rather disparate fields such as architecture and medicine. Because of this distributed nature of relevant research, it is challenging for researchers to uncover all the relevant work and to keep up with the available research. Perhaps, as a result, spatial thinking literature(s) has many definitions and descriptions of the term spatial thinking. In most of the cases, spatial thinking is defined in a disciplinary context and used to mean domain-specific spatial reasoning and skills. For example, while spatial thinking may refer to the relative geographical locations of social phenomena in social sciences or particularly in geography (Lee & Bednarz, 2009; Logan, 2012), it may refer to mental visualisations of the molecules in chemistry.

In this thesis, spatial thinking within a discipline will be considered to include the spatial aspects of a discipline hence it is a part of the discipline. This is, a discipline (or domain) and spatial thinking will be thought of as two different sets that overlap where the overlap of the sets represents common parts in the discipline and spatial thinking (disciplinary spatial thinking, Section 2.3.1 specifies this for geometry education: geometric spatial thinking) (as suggested by Battista, Frazee, & Winer, 2018; Pittalis & Christou, 2010; Widder & Gorsky, 2013). Hence, when reporting students' performance on a task, what is improved or not improved could be spatial thinking, disciplinary knowledge and the intersection between these two things so both of them. In line with this, Section 2.1.2 is devoted to describing the measurements of spatial thinking.

2.1.2. Measuring Spatial Thinking

Given the broadness of the definition of spatial thinking that we can see in Section 2.1, it is unsurprising that studies attempting to measure spatial thinking have actually suggested that it is composed of multiple factors (Cornoldi & Vecchi, 2003; Hegarty & Waller, 2005). Spatial thinking has been measured in the literature in three ways: through spatial tests independent of an academic subject, disciplinary tests and their combination.

Spatial tests mainly focus on items to assess three factors corresponding to most demanding types of processes of spatial reasoning, namely: spatial visualisation, spatial relations and perceptual speed. The first and the most studied factor is spatial visualisation, which is recently described as "piecing together objects into more complex configurations or visualising and mentally transforming objects often from 2D to 3D or vice versa" (Newcombe & Shipley, 2015, p.185). Spatial visualisation test items assess multistep mental manipulations of spatially presented information, for example, determining which combination of small objects would fill a larger one or determining which of the images corresponds to the places of the holes when one opens a folded then drilled image (Carroll, 1993; Linn & Petersen, 1985). Example spatial tests assessing spatial visualisation include Paper Folding Test, Minnesota Paper Form Board Test, Block Design Test, Mental Cutting Test, Space Relations Test, Surface Development Test and Guilford-Zimmerman Spatial Orientation Test

(Hegarty & Waller, 2005; Uttal, Meadow, Tipton, Hand, Allen, Warren & Newcombe, 2013).

The second factor is spatial relation (sometimes called speeded rotation) which implies recognising the relationships among various visual elements of an object (Bosnyák & Nagy-Kondor, 2008; Turgut, 2015). It is often conceived of as the mental rotation of 2D or 3D objects, and it describes how these objects are located in space in relation to a reference object. Mental rotation requires a cognitive process to mentally transform or rotate 2D or 3D objects in any direction indicated through spatial visualisation (Carroll, 1993). Similar to spatial visualization test items, spatial relations test items come with the requirement of mental transformations. Their difference is that they assess single step mental manipulations of two-dimensional objects (usually, rotations on a plane) and they tend to emphasize speed (Carroll, 1993; Miyake, Friedman, Rettinger, Shah, & Hegarty, 2001). Examples of spatial relations measures include Card Rotations Task, Flags Test, Vandenberg Mental Rotation Test (VMRT), Cube Comparison Test, Primary Mental Abilities Space Test (PMA), Thurstone Spatial Relations Test (TSRT), Children’s Mental Transformation Task (CMTT) and Purdue Spatial Visualization Test: Visualization of Rotations (PSVT:R) (Hegarty & Waller, 2005; Uttal et al., 2013). This factor is sometimes seen together with the spatial orientation (i.e., imagining the appearance of objects from different perspectives; see Kozhevnikov and Hegarty’s 2001 study) and named as spatial relations and orientation in the early literature (e.g., Michael, Gilford, Fructer, & Zimmerman, 1957 as cited in Hegarty & Waller, 2005). Alternatively, spatial orientation may be found as a separate factor in the literature (e.g., Lohman, 1988; McGee, 1979). This thesis accepted spatial orientation as a part of spatial visualisation following the suggestions of Carroll (1993) who does not include it as a factor to her factor-analysis considering relatively smaller number of tests to measure it compared to spatial visualization.

The third factor is the perceptual speed which is described as perceiving objects, routes or spatial configurations in the presence of distracting information, sometimes even without all the information present (Newcombe & Shipley, 2015). This includes navigation which refers to coordinated and goal-directed moves in an environment, and they can be either physical movements or metaphorical movements, for example, navigating through a detective story or a mathematics problem (Montello, 2005; Wang & Carr, 2014). Perceptual speed test items assess individual differences in speed and efficiency by looking at relatively easy perceptual judgements can be made them, for example identifying which of the given pictures is the same as the model picture. Examples of perceptual speed measures are Embedded Figures Task, Flexibility of Closure Test, Identical Pictures Test, Morris Water Maze and Radial Arm Maze (Hegarty & Waller, 2005; Uttal et al., 2013). There are also other factors such as closure speed (i.e., spotting figures in a more complex environment, measured by Snowy Pictures Test), visual memory (i.e., remembering the configurations, locations and orientations of figures, measured by Silverman-Eals visual memory task) and spatial perception (i.e., being aware of one's relationship with the environment, measured by Water-level task, Rod and Frame Test), some of which were regarded as minor in the literature (Lohman, 1988). To note, although the debate is still raging as to the number of factors that spatial thinking might be composed of, some studies distinguished spatial orientation (Lohman, 1988; McGee, 1979), but other studies (e.g., Carroll, 1993) have not been able to separate it.

Disciplinary tests are the tests which include domain-specific tasks to measure spatial thinking. These tests measure the change in the participants' spatial thinking through the disciplinary test items. In other words, disciplinary tests measure both spatial and disciplinary knowledge hence the intersection (e.g., Bednarz & Lee, 2019; Huynh & Sharpe, 2013; Lee & Bednarz, 2012). They assess spatial aspects of the discipline by asking disciplinary-specific spatial questions, for example, making orthogonal drawings and cross-sections of a prism in geometry or creating map cross-sections of a landform in geography. In the literature, it is common to see these tests named by the researchers as spatial tests, spatial skill tests, spatial ability tests etc. In this thesis, they are consistently called disciplinary tests (similar to Battista et al., 2018; Uttal et al., 2013).

Finally, there are some studies which use a combination of spatial tests and disciplinary tests and relate the outcomes to each other (e.g., Casey, Nuttall, & Pezaris, 2001; Delgado & Prieto, 2004; Kyttälä & Lehto, 2008). Researchers who use both spatial tests and disciplinary tests mostly aim to find a relationship between disciplinary-independent and disciplinary-specific spatial (disciplinary spatial) performance. Moreover, both spatial and disciplinary tests were observed to be used after disciplinary-specific and disciplinary independent training sessions. While some of the training interventions were in respect of disciplinary-independent spatial outcomes (e.g., improving performance in the computer game Tetris), there were many studies in respect of disciplinary-specific spatial outcomes. Lastly, the same test can be both spatial and disciplinary depend upon the expertise of the participant. Studies show that dentists and chemists initially solve mental rotation problems of teeth/chemical structure in ways that are drawn upon their general spatial skills but with experience, these people solve them using (at least in part) disciplinary skills (Hegarty et al., 2009, 2013; Stieff & Raje, 2010, 2008). In other words, both Hegarty and colleagues (2009) and Stieff and Raje (2008) found that at the beginning students are only able to draw on domain-general spatial skills but by the end of an intervention, they now not only improve their domain-general spatial skills but they may have developed disciplinary spatial skills.

2.1.3. Malleability of Spatial Thinking

Recent research has provided overwhelming evidence that spatial skills can be trained through spatial interventions (Newcombe & Stieff, 2012; Uttal, 2009; Uttal et al., 2013). That is, research has found that spatial test performance can be improved when participants engage in activities that require spatial thinking.

Training sessions involving these activities are often either called *video game training* or *spatial task training* (Baenninger & Newcombe, 1989). This thesis does not discount these influences but is particularly concerned with whether spatial reasoning can be improved through educational interventions in classroom environments. Such interventions are called *course training*, which is a spatially relevant course that used to improve spatial reasoning or rather spatial aspects of disciplinary reasoning (Baenninger & Newcombe, 1989).

Uttal et al.'s (2013) meta-analysis with 206 studies is the first spatial training meta-analysis available in the literature, which included all types of training (video games, courses and spatial task training) described by Baenninger and Newcombe (1989). The meta-analysis found solid evidence to conclude that spatial skills are malleable (overall: g=0.47, SE=0.04). This outcome involves all studies irrespective of the design of the study and indicates that spatial skills are generally moderately malleable. Their study also discovered that students in the training group showed an observable improvement (within design: g=0.62, SE=0.04) even when in comparison to a control group (mixed design: g=0.45, SE=0.04). On average, spatial training increased spatial test performance by nearly half of a standard deviation.

Uttal and colleagues (2013) already showed that training may change spatial skills. Their meta-analysis also showed that the spatial skills gained through training are durable. Considering tests administered immediately after the training as post-tests, and those administered after a couple of days, weeks or over a month as delayed post-test (they define delayed post-test as broadly), no significant differences were reported either between the results of delayed post-tests having varying time delays (p>.67) or between post-tests and delayed post-tests (p>.19). The meta-analysis showed that the effects of training are enduring (post-test: g=0.48, SE=0.05; delayed post-tests: g=0.44, SE=0.08).

Finally, research has found that spatial skills are transferable across tasks if sufficient training or experience is provided and if the tasks share some common underlying psychological spatial skill (Uttal et al., 2013; Wright et al., 2008). Transferability was observable even when studies involved small samples. For example, Wright and colleagues (2008) invited with 31 participants (17 female, 14 male) to daily mental rotation and paper folding activities to practice their spatial skills for three weeks. They divided participants into two conditions; while one group practised mental rotation tasks, the other practised paper folding tasks. Both conditions were asked to complete a mental rotation test and a paper folding test before and after the intervention. The results showed that spatial skills transfer across mental paper folding task and mental rotation task. This is, although there is a greater gain for practised task, significant gains were also revealed for the unpractised task. For example, mental rotation group significantly improved not only in mental rotation (p<.0001, η_p^2=.87) but also in paper folding (p<.0001, η_p^2=.60) by only practising mental rotation tasks, and a similar case was observed for the paper folding group.

Similarly, Uttal and colleagues' (2013) meta-analysis found that specific spatial skills are transferable with a moderate effect size (g=0.48, SE=0.04). They further assessed the degree of the transfer to see how much training in one task transfer to other types of tasks. In order to do the analysis, they classified the spatial tasks in two dimensions: intrinsic vs extrinsic and static vs dynamic. Authors define intrinsic information as "the specification of the parts, and the relation between the parts, that defines a particular object" and extrinsic information as "the relation among objects in a group, relative to each other or to an overall framework" (pp.358-59). This process involved one by one consideration of each spatial task in order to put them into four categories. For instance, they classified a) recognition of an object as a rake as *intrinsic and static*, b) mental rotation of the same object as *intrinsic and dynamic,* c) reading maps as *extrinsic and static*, and d) one's thinking about an object's relations to oneself from a changed position in the same environment involving *extrinsic and static information*. The analysis was conducted with a 2x2 classification of spatial skills, for example, intrinsic and dynamic, and extrinsic and static. The amount of transfer within cells of the 2x2 (g=0.51, SE=0.05) and across cells of the 2x2 (g=0.55, SE=0.10) were more than a half of a standard deviation, indicating that spatial skills are transferable to not only within cells in which training and transfer tasks require similar skills but also across cells which may be anticipated to involve distinct skills and representations.

To sum up, research has shown that spatial skills respond to training and that the improvement or benefit gained from training is long-lasting and transferable.

2.1.3.1. Training to Improve Disciplinary Spatial Performance

There are many disciplinary-specific spatial (disciplinary spatial) training studies which aimed to improve disciplinary-specific spatial performance, hence academic achievement in science and mathematics (Hsi, Linn, & Bell, 1997; Onyancha, Derov, & Kinsey, 2009; Sorby, Casey, Veurink, & Dulaney, 2013; Uttal, 2009). Researchers who reviewed disciplinary spatial training studies to date argued that such training seems promising for increasing students' success in STEM (science, technology, engineering and mathematics) domains (Stieff & Uttal, 2015) and more recent studies have confirmed their results (Sorby, Veurink, & Streiner, 2018).

The effectiveness of disciplinary spatial training is exemplified in many studies aiming to improve academic success in technology courses (e.g., Hsi et al., 1997; Onyancha et al., 2009). For example, Hsi et al. (1997) examined the effects of spatial technological-design training on students' performance in a technological design course. Their training included hands-on technological design activities, computer courseware and problem-solving assessments on orthogonal and isometric drawings. They found that the disciplinary spatial training significantly improved overall course grades (p=.003; no effect size reported), and that there was a significant relationship between spatial skills measured by a disciplinary test and overall course performance (pre: r=.28, p<.0001; post: r=.35, p=.0004), hence they started to design their course curriculum building on the skills that gathered through this training. Similarly, Onyancha et al. (2009) investigated the effects of a spatially based computer-assisted design course on students' success in object geometries and rotation measured by a subset of PSVT:R questions (a test which requires single step mental manipulations of two-dimensional objects in a limited time, described in section 2.1.2 as spatial relation measure (Guay, 1976)) for engineering. They worked with engineering students and measured their spatial skills by using PSVT:R, and divided them into three groups: low group (those who got 60% of the maximum possible PSVT:R score), intermediate group (those who scored between 60% and 80%) and high group (those who scored above 80%). They only invited students with limited spatial skills to the course (approximately 60%, experimental group) and the remaining students were in the control group. The course included work with engineering software packages Physical Model Rotator and Alternative View Screen. While students in the experimental group attended the spatially based computer-assisted design course, the control group did not receive any training. The results showed a significant improvement in the PSVT:R subset questions for engineering scores of experimental group after the four-week course (p<.001, d=1.94) while no difference was observed in the control group (p=.009, d=0.69). Experimental group which is low group not only outperformed control group (pre: p=.79, post: p=.013) but also closed the gap between them and intermediate group (pre: p<.001, post: p=.22). However, it should be noted that this is a study which is pre-screened to include only low scorers. This is, the control group and intervention group did not start from similar levels of knowledge prior to the training which makes it hard to compare groups and interpret the results of the study.

Similar to those of improving achievement in technology courses, studies encouraging disciplinary spatial training to improve academic achievements in mathematics and mathematics-based science courses report an important increase in the students' performance after the training (e.g., Miller & Halpern, 2013; Sorby, 2009; Sorby et al., 2013). Sorby et al.'s (2013) study with almost 700 students is a good illustration of how improvements in spatial skills, measured by PSVT:R, resulted in improved grades in a calculus course. They provided spatially based engineering course to 675

(133 female, 542 male) first-year engineering students and gave lessons on isometric sketching, orthographic projection (orthogonal drawing), transformations of objects and cross-sections during the course. Students were also required to attend a calculus course offered by the same instructor and they studied trigonometry, functions, differentiation and integration as a part of the calculus course. They used a previously-designed workbook for their spatial engineering-maths training (Sorby, 2009). The workbook included problems requiring 2D isometric sketching, orthogonal drawing, transformations of objects and drawing cross-sections. Sorby and colleagues' (2013) findings showed a significant improvement in calculus scores (measured by a disciplinary test, $p<.05$, $d=.20$) for those students who attended the spatially based computer-assisted design course compared to the control group who did not attend the design course. This illustrated a case to how a calculus course that supported with a spatial disciplinary training resulted in improvements in spatial disciplinary performance; however, it is not clear how spatially based engineering course and calculus course were linked to each other, how spatial disciplinary training would help calculus performance and vice versa and how the improvement in spatial disciplinary performance happened as there were many activities in the spatial training course from isometric sketching to study of cross-sections.

Another example of this type of research is Miller and Halpern's (2013) study which supported the findings of Sorby et al. (2013). They used 12 hours of Sorby's (2009) spatial engineering-maths training to improve gifted students' performance in a calculus-based physics course. This was known as a challenging course and students who took the course were initially tested and found to have high spatial abilities (spatial visualization measured by paper folding and mental cutting test and spatial relation measured by mental rotation test). Miller and Halpern's (2013) study showed that the training not only improved students' exam scores approximately 0.4 standard deviations in this physics course but also improved their mental rotation and spatial visualization skills (measured by various spatial tests including the Mental Rotation Test, Mental Cutting Test and Novel Cross Sections Test), and these lasted for a few months after the training. This study is important because it shows that disciplinary spatial training might also help students with already well-developed spatial skills in improving their academic performance.

It should also be noted that the above-explained studies were not the only studies reporting similar results, there were many other researchers who reported similar findings in STEM domains since years (in mathematics: Brinkmann, 1966; Cheng & Mix, 2014; in engineering: Hsi et al., 1997; in science: Hegarty, 2014; Lord, 1985; Sanchez, 2012; in chemistry: Small & Morton, 1983; in medicine: Stransky, Wilcox, & Dubrowski, 2010). Moreover, examples are not only limited to STEM courses, have included samples from social sciences disciplines from archaeology and sociology to economics and criminology (e.g., Hespanha, Goodchild, & Janelle, 2009; Jimenez & Chapman, 2002). This doctoral research focused particularly on spatial geometric training in respect of spatial geometric academic achievement that is further described in Section 2.3.

2.1.4. Gender Differences in Spatial Thinking

That there is a gender difference is almost certainly the most widespread assumption in the popular media about spatial thinking and it is tended to be reported in favour of male participants. The literature does suggests that this may be partially true but there is not any consensus as yet. The majority of the studies report that males outperform females in mental rotation tests but it is not consistent for all types of tests (Newcombe & Stieff, 2012; Vandenberg & Kuse, 1978). There are a few meta-analyses on gender differences in spatial thinking, and the following paragraphs provide a review of them.

The first available meta-analysis on gender differences in spatial thinking was conducted in the early 1970s and looked for spatial perception and spatial visualisation (Maccoby & Jacklin, 1974 as cited in Linn & Petersen, 1985). For spatial perception, 16 of 21 studies included in the analysis were reported statistically significant differences in favour of males. For spatial visualisation, of 32 studies included in the analysis, only eight of them reported statistically significant differences; five were in favour of males and three were in favour of females. Thus, they reported no consistent gender differences in spatial visualisation. A decade later, Linn and Petersen (1985) published a meta-analysis of 172 studies dated after Maccoby and Jacklin's (1974) review until 1982. They showed that there were some gender differences in some of the factors of spatial thinking but not in all of them. That is, there were large gender differences in mental rotation (d=0.73, p<.05) and medium in spatial perception (d=0.44, p<.05) both favouring males but there was not any gender difference in spatial visualization (d=0.13, p>.05). These results confirmed the results of the earlier meta-analysis on spatial visualisation and added that gender differences in mental rotation and spatial perception was robust.

Voyer, Voyer and Bryden (1995) conducted a more comprehensive meta-analysis on gender differences in spatial thinking from 1974 to then-date. The analysis included 286 studies, which covered Maccoby and Jacklin's (1974) and Linn and Petersen's (1985) studies. The results indicated that there were significant gender differences in spatial thinking favouring males (d=0.37, p<.01). Specifically, confirming the results of the earlier studies, they found a large effect size gender difference in mental rotation (d=0.53, p<.05) and a medium one in spatial perception (d=0.44, p<.05), but no gender difference in spatial visualisation (d=0.19, p>.05). Voyer and colleagues (1995) reported that they faced difficulties while synthesizing effect sizes coming from various tests. Hence, the next meta-analysis stated this and only focussed on the mental rotation measured by the PSVT:R (Maeda & Yoon, 2013). It was conducted to estimate the magnitude of gender difference in 3D mental rotation and to see whether and how variables linked to the test conditions influence gender difference in spatial thinking. The analysis included 40 studies published between 1976, when the test was developed, and 2011. The results of Maeda and Yoon's (2013) analysis indicated that males outperformed females in 3D mental rotation (g=0.57, p<.05), and the differences were larger when the test was implemented with strict time limits (≤30 seconds per item, g=0.68, p<.05).

Reilly and Neumann (2013) conducted another meta-analysis on gender differences and spatial thinking, measured by various mental rotation tests. The analysis included 12 peer-reviewed empirical studies and reports dated after 1986 to then-date. The studies were from the United Kingdom, the United States of America, Canada, Poland and Croatia. The meta-analysis found statistically significant relationships between masculinity and mental rotation for both females (r=.23, p<.001) and males (r=.30, p<.001). The analysis also showed that these results did not change according to the country of the study. Moreover, the results indicated that type of test can affect gender difference in mental rotation (VMRT: r=.38, p<.001; Generic mental rotation tasks: r=.22, p<.05; Card Rotations Task: r=.22, p=.07; TSRT: r=.21, p=.06).

Finally, there are two recent meta-analyses of gender differences in spatial thinking, published in 2019. The first one is Lauer, Yhang and Lourenco's (2019) meta-analysis with 128 studies to date published in English. It aimed at exploring the age range at which male advantage emerges and the influence of variables linked to the test conditions to gender differences in spatial thinking, measured by mental rotation tests. The results showed a significant developmental change in the magnitude of gender difference. This is, they reported a small male advantage in childhood (3 to 7 years: g=0.20, p<.05) which increased with age (8 to 12 years: g=.40, p<.05; 13 to 17 years: g=0.54, p<.05). Moreover, the results indicated that variables including dimensionality of task (2D: g=0.30, p<.05; 3D: g=0.50, p<.05), administration mode (digital: g=0.28, p<.05; paper and pencil: g=0.41, p<.05) and test setting (individual: g=0.26, p<.05; group: g=0.45, p<.05) moderate the magnitude of gender differences in mental rotation in addition to the type of test (CMTT: g=0.19, p<.05; PMA: g=0.37, p<.05; VMRT: g=0.58, p<.05).

The second one is Yuan and colleagues' (2019) study which aimed at exploring whether gender differences in spatial thinking differ by the spatial skill. They divided the processes of spatial reasoning into two groups: those requiring small-scale spatial skills (being spatial visualisation and spatial relations) and those requiring large-scale spatial skills (being spatial orientation and navigation). They described small scale spatial skills as mentally representing and transforming 2D and 3D images which can be apprehended from a single perspective (Hegarty & Waller, 2004; Höffler, 2010), and large-scale ones as carrying out the processing in a large environment where the viewer's perspective changes whilst spatial relationship between objects remains (Jansen, 2009; Wang et al., 2014). They obtained 98 effect sizes from 44 studies, 14 of which reporting large-scale and 84 of which reporting small-scale spatial skills. Yuan et al.'s (2019) meta-analysis found that males outperformed females in spatial thinking overall (g=0.72, p<.001) as well as in both spatial skill types but with a larger effect size in large-scale spatial skills (g=1.34, p<.001) than small-scale spatial skills (g=0.62, p<.001).

It is noteworthy that most of the meta-analyses include only one of the processes of reasoning described in Section 2.1.2, mostly spatial relations measured by mental rotation measures and even sometimes using only one particular measure. For example, Maeda and Yoon's (2013) meta-analysis includes studies which measure spatial relation by the PSVT:R but do not include studies with other mental rotation tests and studies measuring spatial visualisation or any other factor.

To sum up, consistent gender differences in favour of male participants were have been found in some processes of spatial reasoning, and they were particularly visible in mental rotation. These findings set the basis of gender consideration in all aspects of this PhD thesis.

2.1.4.1. Alternative Explanations for Gender Differences in Spatial Thinking

Even when it is accepted that there are gender differences in spatial thinking, the underlying mechanism can also still be hotly debated. The reviewed literature suggests at least two distinguishable factors that may explain the nature of gender differences in spatial thinking: biological factors (e.g., neural, hormonal, genetic and evolutionary) and environmental factors (e.g., dissimilar experiences) or indeed both.

On the one hand, many researchers attributed gender differences in spatial thinking to *biological differences* between females and males. They argue that females are born different to males, so the differences are in the genes. For example, Jordan, Wüstenberg, Heinze, Peters and Jäncke (2002) and Koscik, O'Leary, Moser, Andreasen and Nopoulos (2009) suggest that functional and morphological differences in the brains of males and females contribute to gender differences in spatial thinking. Jordan and colleagues' (2002) study found that females and males display different cortical activation patterns during mental rotation tasks. Koscik and colleagues (2009) reported that females have smaller parietal lobe surface area and proportionally greater disadvantageous grey matter volume in the parietal lobe, which is thought to be involved in spatial reasoning.

Other researchers further suggested that hormonal differences may lead to the gender differences in spatial thinking (e.g., Kimura & Hampson, 1994). For example, unique biological development mechanisms for each gender were indicated as the cause of gender differences in spatial tasks (Geiser et al., 2008; Linn & Petersen, 1985). Meta-analyses reported that gender differences in spatial thinking favouring males were found to increase by age (Geiser et al., 2008; Lauer et al., 2019). These differences particularly tend to accelerate around the time of puberty and have been related to hormone levels of oestrogen and testosterone (Broverman et al., 1981).

Others have not specified precise mechanisms but argue that there is an evolutionary basis in gender differences in spatial thinking (Gaulin & FitzGerald, 1986; Silverman & Eals, 1992). Their findings with other species supported the idea that males perform better than females in spatial navigation. In an evolutionary context, navigation only refers to the physical movement (e.g., navigating a maze but not navigating through a detective story or a maths problem). As an extension of studies with other species, studies conducted with people proposed that a similar evolutionary process might underlie gender differences in spatial skills of human (Gaulin & Hoffman, 1988). Both Moffat, Hampson and Hatzipantelis's (1998) and Burkitt, Widman and Saucier's (2007) studies in virtual mazes reported a significant main effect of gender in favour of males on the performance in a virtual water maze. These results were considered as an extension of gender differences in diverse species. They reported that spatial skills reflect the division of labour in hunter-gatherer societies where males developed the skills about spatial navigation and females developed skills of memory for objects and their locations.

Nevertheless, this perspective has not escaped significant criticism from academics (Newcombe, 2010a; Newcombe & Stieff, 2012). For example, Newcombe's (2010) chapter with many examples demonstrated "how the zealotry of many evolutionary psychologists has led them to neglect their obligation as scientists to formulate and defend testable chains of hypotheses" especially for gender differences in spatial thinking (p.261). She pointed out that evolutionary effect is not sensible for because of many reasons. According to Newcombe (2010), the evolutionary writing mostly start with the words about differences in the cognitive functioning between females and males however available research (Guiso et al., 2008; Hyde, 2005; Hyde et al., 2008) provides a satisfying data for the gender similarities hypothesis in various cognitive, emotional and social domains, including mathematics. Moreover, despite obvious differences in some spatial tests (e.g., mental rotation tests), not all spatial tests show the gender differences including some of the tasks which are widely claimed by evolutionary perspective to show such differences such as navigation and memory of objects (Voyer et al., 1995).

On the other hand, gender differences in spatial thinking have also been attributed to *environmental factors*. Many researchers support the idea that girls just get different types of experiences as children to boys, so the differences that results are not due to genes (e.g., Baenninger & Newcombe, 1989; Terlecki, Newcombe, & Little, 2008). They report that dissimilar previous experiences may result in gender differences in spatial thinking. For instance, Johnson and Meade (1987) and Waber, Carlson and Mann (1982) explained more pronounced gender differences about the time of puberty in terms of the process of childhood socialisation. Studies found that some spatial activities may prevail more among boys than girls, such as playing with certain types of toys or doing some kinds of sports. Deno (1995) found that playing with building and construction toys such as blocks and Lego improves spatial visualisation. Ginn and Pickens (2005) reported that doing some kinds of sports (e.g., basketball, football and soccer) increase the performance in mental rotation tasks. Both Cherney (2008) and Feng, Spence and Pratt (2007) found that playing video games improves mental rotation skills. All these activities are more common among boys than girls hence give boys more spatial experiences than girls that lead to gender differences in spatial thinking. This could be linked to the malleability of spatial thinking in a sense that experience and training have a potential to change spatial thinking, as discussed in Section 2.1.3.

Others studies have found that socioeconomic status and culture are also related to the differences in spatial thinking. For example, students from higher socioeconomic environments tend to outperform those coming from lower socioeconomic environments in both spatial tests (Levine et al., 2005) and disciplinary tests including spatial activities such as drawing in geography (Levine et al., 2005) and modelling in geometry (Fuson & Murray, 1978). Finally, de la Fuente, Santiago, Román, Dumitrache and Casasanto (2014) illustrated variations in spatial differences across different cultures (Arabic and Spanish cultures) as evidence of how culture shapes spatial thinking.

The third argument is that gender differences in spatial thinking are caused by both biological and environmental factors. A number of authors argue that biological factors such as hormones and genes may interact (in a complex manner) with environmental factors (Halpern & Collaer, 2005; Wallen, 1996). These biological factors may change spatial skills by influencing girls and boys to seek out or avoid certain activities. For example, boys tend to play with building and construction toys that are known to support spatial thinking (Baenninger & Newcombe, 1989; Tracy, 1987; Verdine et al., 2014). Hence, it is argued that genes cause expression of pre-existing differences and people's seeking out opportunities that then modify and impact on environmental features (which, for example, lead to changes in brain structure). For example, those who chose to be taxi drivers as their career practice their navigation skills as a part of this job. In a study conducted with licenced London taxi drivers, it was found that there is a positive correlation between the grey matter volume in their brains (which is thought to be involved in spatial reasoning) and their years of working experience (Maguire et al., 2000).

To sum up, there are three arguments about the cause of any gender differences in spatial thinking: they are caused by either biology alone or environment alone or both interactively. The reviewed literature is complex and shows that there are many possible explanations. The position taking in this thesis is that as spatial thinking is clearly malleable, there must be at least some strong environmental component.

2.2. Learning and Teaching Geometry

This section presents a review of literature which seeks to provide an overview of relevant current discourse and understandings about student performance in maths, particularly in geometry. The section goes from general to the specific; it first describes students' performance in mathematics by discussing research on geometry performance and reviews national and international test results with a particular interest in geometry performance. It then presents studies on 3D shapes and factors argued to be affecting students' performance in geometry, especially the geometry of 3D shapes.

As geometry is a general term, it is important to understand what geometry refers to in this thesis. In general, geometry deals with the study of properties of space, the measurement of forms that can be designed in space, and the relationships of these forms in Euclidean, elliptic, three-dimensional non-Euclidean, and hyperbolic geometries (Karakas, 2011). Specifically, in many curricula, middle school geometry includes study of only Euclidean geometry, which indeed constitutes a relatively small part in the actual field of geometry. That is, middle school geometry includes the study of two- and three-dimensional shapes together with their representations and transformations, and mathematical calculations of the measurement of lengths, areas and volumes in Euclidean geometry (Altun, 2013; Clements, 2003).

2.2.1. Research on Geometry Performance

Research into students' performance in geometry has been seen as an important research subject in mathematics for a long time (Clements, 2003; Clements & Battista, 1992). For more than 30 years, a large number of these studies reported that students usually perform worse than expected in geometry (e.g., Fuys, Geddes, & Tischler, 1988; Usiskin, 1982, 1987). Although there is often prompts the reaction of students are doing poorly, there may of course be many reasons for this including too high expectations of what students should be able to achieve at specific ages, badly designed tests to measure students' performance and insufficient quality or quantity of teaching.

Early examples of research into geometry performance include many studies from the United States of America. For example, Galbraith's (1981) study through clinical interviews with 170 students concluded that more than 67% of the 12 to 15-year-olds achieved lower than was expected in simple geometric proofs. Carpenter and colleagues (1983) reported that only 20% of 13-year-old students (out of 45.000 tested) were able to calculate the length of hypotenuse from the given two (opposite and adjacent) sides. In a similar vein, Fuys (1988) argued that 19% of the sixth-grade middle school students are 'geometry deprived' (which includes their performance in 3d geometry) and 31% of these students are only able to name the shapes and reference them to visual prototypes. Moreover, Usiskin's (1982, 1987) studies found that students' performance when dealing with two- and three-dimensional shapes also did not meet expectations. Usiskin (1982) collected data from 2699 middle school students attending to year seven to twelve, all enrolled in a one-year geometry course in the USA. His study evaluated students' performance at the beginning and end of the course, without affecting the ongoing geometry programme. In the beginning, students were asked to complete a geometry test (Entering Geometry Test, EGT) as a pre-test which measured their general geometry knowledge. At the end of the project, students' performance measured by using two geometry tests, one measuring the objectives they learnt throughout the term (Comprehensive Assessment Program Geometry Test, CAP in short) and other measuring their geometry proof performance (Proof Test). Students were also asked to complete van Hiele Level Test at the beginning and at the end of the project in order to see whether students' geometry performance was related to their van Hiele levels of knowledge (described in Section 2.3.1.1). Usiskin (1982) concluded that on the average, students answered 54% of the pre-test questions correctly, and no item was correctly answered by more than 80% of the students in the pre-test. Unfortunately, at the end of the course, there was not much change. Usiskin (1982) reported the results for each of the test item one by one for both geometry tests, rather than a comprehensive result for the tests. For both CAP and Proof Test, students' performance, in general, was reported as low. Regarding this, Usiskin (1982) said that

> *"It is hard to believe that, after a year of geometry, 18% to 20% of students cannot identify vertical angles. 44% to 47% cannot find the perimeter of a square from its area, and 65% to 68% cannot calculate and subtract the areas of two circles to find the area of the space between then. (Only about half the students can do any more than simple proofs.) If so little is learned, what is being taught?" (p. 71-72).*

Usiskin's (1982) study also found that van Hiele levels (described in Section 2.3.1.1) that are assigned to students are good descriptors of performance both in pre-test and post-tests. This is, students' poorer geometry performance in the tests were strongly associated with being at the lower van Hiele levels. However, Usiskin's study assessed students' knowledge prior to and after the lesson by using different tests, hence it could not report any direct change in the students' performance and only reported the performance on these tests separately.

While most of the early studies reported poor performance, only a few also provided possible solutions to this performance problem after defining what does not work in their context. One of the most influential works from the eighties regarding this is from Usiskin (1987). His report included discussion on 3d geometry, particularly on transformation geometry. The report not only concluded that middle school geometry was facing performance problems but also provided six dimensions to teach to help students perform better at geometry. The problem as stated in this context was that only half of the students encounter the curriculum and only about one-third of this half understands it. "*The lack of success that characterizes so many students' experiences in geometry discourages other students from taking geometry"* (p.19), he reported. Although there could be many other contributing factors to not enrolling to geometry courses such as students' own experiences with geometry and their career choices, the report argued previous students' poor performance to be one of the main contributing factors which led to only half of the middle school students enrolling in geometry classes in high school at the time. It suggested a possible solution to solve this problem by considering geometry not as a separate part of maths but as an integral part of it; composed of six dimensions. Usiskin (1987) claimed that in order to perform better at geometry, students need to learn about these six dimensions:

1) *The measurement-visualization dimension* considers geometry as the visualization, construction and measurement of figures and emphasises that visualization and drawing are generally neglected but (should not be) in the study of geometry. Hence, including questions such as "*count numbers of cubes on which visible cubes lie*" and "*Tell what a figure looks like after being turned*" are suggested.

2) *The physical real-world dimension* considers geometry as the study of the real and physical world. It emphasizes that even though geometry evolved from the real world, connections with the world when teaching geometry are largely ignored (should be included) in teaching school geometry.

3) *The representation dimension* considers geometry as a vehicle for representing not only geometry but also other mathematical concepts. It emphasizes that the geometry of physical objects (e.g., Cuisenaire rods and dienes blocks) is largely used to represent many maths topics from numeracy to algebra.

4) *The mathematical underpinnings dimension* considers geometry as an example of a mathematical system; therefore, it suggests teaching geometry as a branch of maths not as a separate course only including ideas and proofs of geometry basically because these ideas are connected to other branches of maths.

5) *The socio-cultural dimension* considers geometry as a socio-cultural phenomenon. It suggests studying geometry together with its history and development of ideas.

6) *The cognitive dimension* considers geometry together with one's mental images and cognition, which are mostly available in the studies from psychology.

These highly influential studies were all conducted in the United States, however, more recent research has also been conducted outside of the United States as well.

The findings of more recent studies are barely different from those of the 1980s. Students' difficulty in geometry, its causes and possible solutions, is still the subject of much active research (e.g., Battista, Clements, Arnoff, Battista, & Borrow, 1998; Devichi & Munier, 2013; Fuson, Clements, & Beckman Kazez, 2010; Kaleli-Yilmaz, Ertem, & Güven, 2010; Mbugua, Kibet, Muthaa, & Nkonke, 2012; Oksuz, 2014). For example, Battista (2007) reported in his extensive review of geometry and spatial thinking that many students have difficulties in learning 2D and 3D geometry and concluded his review by saying that "*Despite geometry's importance in mathematical theory and application, students continue to have difficulty learning it with genuine depth*" (p.903). In a similar vein, Fuson and colleagues (2010) reported for the U.S. students (K-12) that geometry and measurements are two of their weakest topics in maths. Studies which looked at the geometry performance in particular topics reported similar findings to those of overall geometry performance. Devichi and Munier (2013), for instance, reported that French students (9-10 years old) encounter difficulties in learning about the concept of angle and listed their misconceptions[3] (and offered lessons to overcome these misconceptions through providing concrete manipulatives and real-life examples). Dagli and Peker (2012) conducted a study on Turkish middle school students' understanding of the perimeter and reported that only about half of the students managed to find the circumference of a circle and about 10% of the students did not even attempt to answer questions; the case was not dissimilar for the calculations of perimeters of other shapes such as squares, rectangles, rhombuses and parallelograms. Similarly, Ulusoy and Cakiroglu (2017) concluded that Turkish middle school students (11-12 years old) struggle acquiring the concept of parallelogram and listed a number of misconceptions including over- and under-generalization. It appears therefore that this list goes on and on.

[3] The word misconception is purposefully chosen to refer 'conceptual or reasoning difficulties that hinder mastery of a discipline' as defined by (Crawford, 2001, p.11) rather than the word error which is 'a simple symptom of a misconception' (Luneta, 2008, p.386). A misconception could be a consequence of 'a misapplication of a rule, an over- or under-generalization or an alternative conception of the situation' (Drews, 2005, p.18).

Hence, research the common thread amongst all these studies is that students have lower performance in geometry than expected. Some have therefore suggested to consider some dimensions in the teaching of it to improve students' achievement. The following section discuss therefore whether it is specifically geometry performance which is not meeting the expectations or whether it is a part of a broader pattern of difficulty including other areas of mathematics such as algebra and measurement.

2.2.2. National and International Test Performance

Another way to understand student performance in mathematics is to use different national and international tests. For this purpose, Section 2.2.2.1 reviews the largest well-known national tests, and then Section 2.2.2.2 particularly focuses on the national and international maths test performance of Turkish students.

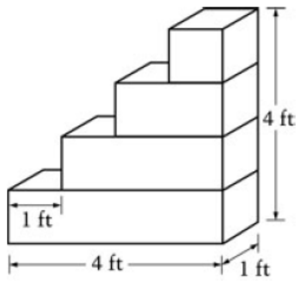
2.2.2.1. Middle School Students' Performance on Well-known National Tests

The United States of America and the United Kingdom are two of the main countries which started to administer the earliest national assessments in mathematics. This section reviews the results of their national tests in mathematics with a particular interest in geometry performance.

The American National Assessment of Educational Progress (NAEP) is one of the largest continuing and well-known national tests. The test has assessed students' performance (most frequently) in mathematics, reading, science and writing since 1969. Many researchers in the field of mathematics report and interpret the results of this national exam every year. One of the first large mathematics assessments within NAEP was conducted in 1986. The test measured students' performance in three main areas: algebra, geometry and fundamentals of mathematics (e.g., proofs). In geometry, the assessment included items evaluating the ability to "*visualise an object or scene from a different perspective than the one given in a diagram*" (Brown et al., 1988, p.341). Sample items included choosing a picture of the top view of a block and choosing a picture that represented the given view of the scene from another perspective, these correspond to orthogonal and isometric drawings respectively. Brown and colleagues (1988) reported the results of this years' exam for middle school students at year seven (12-13 years old). They concluded that American middle school students had low academic achievement in mathematics, especially in geometry, in the study of both two- and three-dimensional shapes and their properties. The students had particular difficulty in answering the items on orthogonal and isometric drawings, which are called *spatial visualisation* tasks by Brown et al. (1988, p.341). More than half of the students who were entered in the exam did not answer these items correctly.

The results of the most recent NAEP assessments were barely different than those of the eighties (National Centre for Education Statistics, 2018b). The last NAEP test was conducted in 2017 and the test included similar items to those of the eighties with an observable difference in the presentation of the questions in realistic contexts, for example, questions on polycubes were asked in the context of building a block tower (see Figure 2.1). In 2017, each question assessed one of the five areas: geometry, measurement, algebra, number properties and operations, and finally data analysis, statistics and probability. Specifically, the assessment of geometry focused on the identification of 2D and 3D shapes and their transformations and combinations. Students in the middle school and beyond were expected to have increased understanding of two- and three-dimensional shapes and to show *adequate* knowledge of symmetry and transformations of these shapes, for example, identifying shapes resulting from rotations (NCES, 2011). Geometry items constituted 16% of the point marks in the most recent exam (target was 20%) (NCES, 2018a). Similar to percentages in the eighties, more than half of the students answered most of the geometry items incorrectly. Particularly, for example, 44% of students answered the item asking students to determine the number of unit cubes used to build a figure incorrectly (Figure 2.1). The item asking for identifying which figures are composites of two given shapes in geometry got only 7% (of the) correct response (NCES, 2018b). Students' performances in the other areas of mathematics were much better than they were in geometry, for example, in number properties and operations, most of the children answered the questions correctly. To illustrate this, one of the questions that measured number properties (a multiplication question which gave the product and asked for finding the factors by arranging the given set of digits; OOOxO=4284, digits that will be used are 1, 2, 6 and 7) got 79% correct response. Hence, it was specifically geometry which the U.S. students found much harder than other areas of mathematics such as number properties and operations.

It should be noted that it is difficult to draw completely firm conclusions about this because of how the results are presented on the NAEP website (the analysis for each of the items). It is not very helpful in evaluating students' overall performance in separate areas of mathematics because students performed differently in different items. Nonetheless, the NAEP’s selected items do appear to show the difference between the percentages of correct responses in geometry and other areas (e.g., 7% correct response for a geometry item, and 79% for a number property item) (NCES, 2018c).



37. Sierra built the block tower with 1-foot cubes. How many cubes did she use?

A. 4

B. 6

C. 8

D. 10

Figure 2.1. Sample item from American national maths assessment (NCES, 2018c)

Standard Attainments Tests (SATs) in England and Wales are another well-known national curriculum assessment, which was introduced between 1991 and 1995 in key stage one (aged between five and seven years) and gradually introduced to key stages two (aged between seven and eleven) and three (aged between eleven and fourteen) as each cohort completed a full key stage. The tests include the assessment of core subjects: mathematics, reading, writing and science, and available grades are above expected, expected standard and below expected. It is compulsory for key stage two children to attend the exam, and maths is a core subject assessed as a part of SATs. The most recent mathematics test (at time of writing) was administered mainly in two main areas: arithmetic and reasoning with a total of three papers, two of them concerning reasoning (Standards and Testing Agency (STA), 2019a). Geometry is a part of reasoning and has a place in both of the reasoning papers, with 4 questions out of 23 in each paper, although only one of these questions were from 3D geometry (see Figure 2.2). Despite the emphasis on geometry in the English national curriculum (see the objectives in section 2.1.6), the same emphasis was not observable in the assessments. Hence, it is hard to draw a firm conclusion concerning 3d geometry performance with only one item.

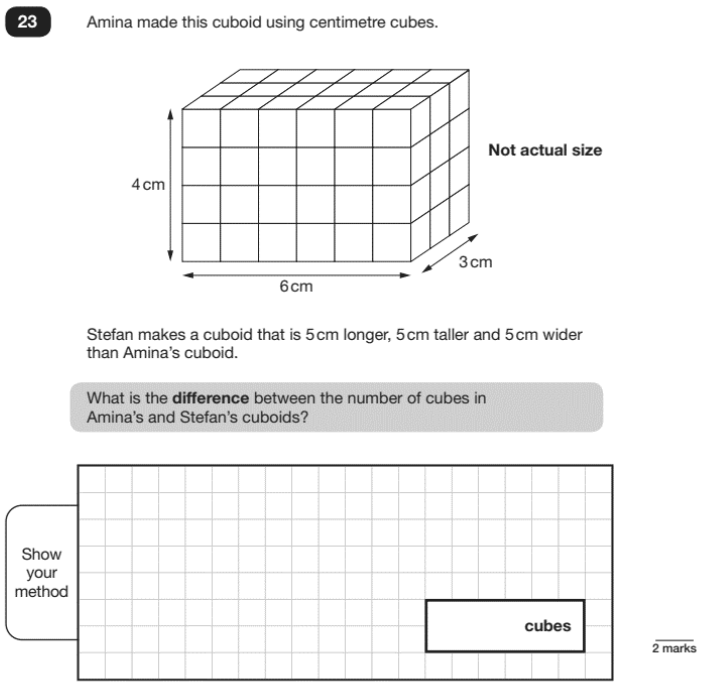


Figure 2.2. Sample item from English national maths assessment (STA, 2019b)

Overall, the results show that each year 20 to 30% of the children in key stage two do not reach the expected standards in mathematics (DfE, 2019). For example, 21% of students did not reach this standard in 2019; 25% in 2018 and 2017, and 30% in 2016 (DfE, 2019). Positively there is an overall increase in the students' mathematics performance over these three years (as seen in the decreasing percentages), nonetheless, the results show that, in 2019, one-fifth of the students did not meet the expected standard in mathematics in England and Wales.

The cases in the United States of America and England and Wales are very similar in respect to the achievement of nationally set goals in mathematics, and both concluded lower performance in mathematics than expected by the governments. American test results were further related to students' low achievement in mathematics to their geometry performance, by particularly presenting the percentage of the correct responses to geometry and number properties and operations items.

2.2.2.2. Turkish Middle School Students' Test Performance

This thesis has a particular interest in improving Turkish middle school students' performance hence it is important to particularly focus on the international and national tests which have been conducted in Turkey.

The PISA (Programme for International Student Assessment) Education Test has assessed 15-year-old students' knowledge of mathematics, science and reading every three years since 2000. Results of the test are calculated by setting the mean of the participating OECD[4] countries at 500 with a standard deviation of 100 in 2003, and these are linked to tests in the following years. A specific focus on geometry, particularly on space and shape can be observed in both PISA 2015 and PISA 2018, and the same strategy is followed in the new PISA mathematics framework for 2021 (OECD, 2018). According to the framework,

> *"Geometry serves as an essential foundation for space and shape, but the category extends beyond traditional geometry in content, meaning and method, drawing on elements of other mathematical areas such as spatial visualisation and measurement. ... The recognition, manipulation and interpretation of shapes in settings that call for tools ranging from dynamic geometry software to machine learning software are included in this content category" (p.25).*

The framework listed area of space and shape as one the main areas to assess in mathematics with the questions looking for the understanding of "transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives and constructing representations of shapes" and gave approximately 25% of score points in overall maths performance to space and shape (OECD, 2018, p.25).

Turkey joined PISA from 2003. As we can see in Table 2.1, Turkey's ranking in the PISA mathematics test results between 2003 and 2018 is always at the bottom end of

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[4] Organisation for Economic Co-operation and Development (OECD) is an international organization with 36 member countries, aiming to shape policies that promote *prosperity, equality, opportunity and well-being* for all.

the distribution (OECD, 2019). Turkey ranked 49th (mean score: 420) in PISA 2015 among 72 countries, with the lowest mean maths score in Turkey's PISA history. Its closest score to OECD is in the last PISA (ranked 41st), still with 35 points lower than the OECD mean score. Hence, international test results do not paint a rosy picture of Turkish mathematics education.

Table 2.1. PISA Mathematics ranking and maths mean scores of Turkey by year

Year	Ranking of Turkey	Mean score of Turkey	Mean score of OECD countries
2003	35th (out of 41 countries)	423	500
2006	43rd (out of 57 countries)	424	494
2009	43rd (out of 74 countries)	445	495
2012	44th (out of 65 countries)	448	494
2015	49th (out of 72 countries)	420	490
2018	41st (out of 79 countries)	454	489

*Adapted from OECD (2019) **At the .05 level of significance

Similar to these international test results, the governmental test scores of Turkey have brought to light that many middle school students do not achieve the goals of national mathematics curriculum (Ministry of Turkish National Education (MoNE), 2013, 2018b). There is a big emphasis on geometry in the Turkish national middle school maths curriculum (Section 2.3.2 will further discuss this); and this was observable in the government exam questions. For example, in 2018 and 2019, almost half of the questions in the mathematics test were from geometry (9 out of 20 in both), and at least three of the geometry questions in each year were from 3D geometry (MoNE, 2018a, 2019).

Over a million students enter the government mathematics test each year in Turkey. The distributions of the number of correct answers in the exam by percentage were more or less the same every year. As displayed in Figure 2.3, which shows the distribution of the number of correct answers in the 2019 Turkish government maths test, the distribution is right-skewed. This means most of the students (given as a percentage, 89%) were clustered around the left (lower) end of the distribution and answered 0 to 10 questions on the test correctly. There were a smaller number of students who answered more than half of the questions correctly (11%).

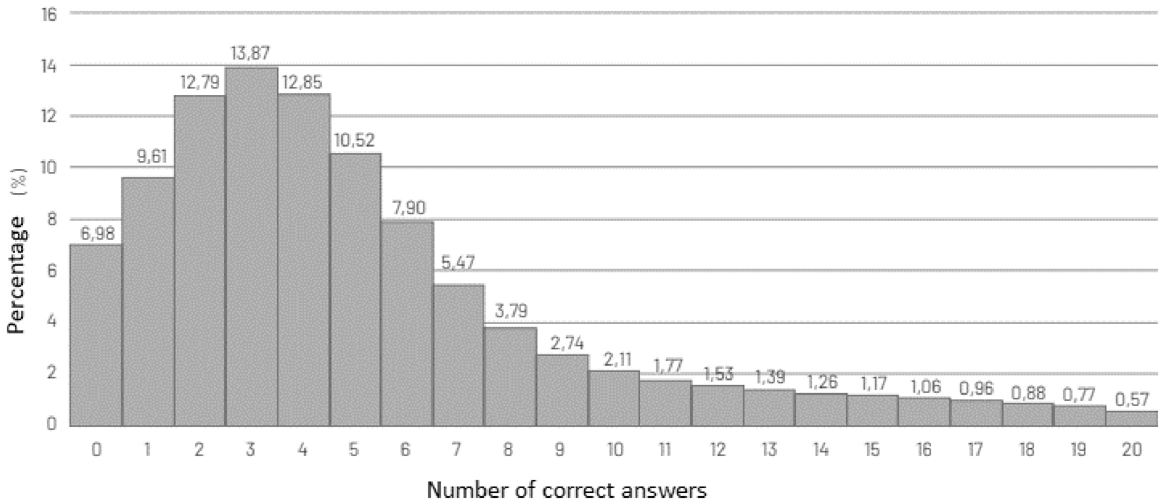


Figure 2.3. Distribution of the number of correct answers in 2019 Turkish government maths test (MoNE, 2019, p.22, used after the permission of the authors)

After the last update in the maths curriculum in 2018, two mathematics exams were conducted in Turkey. Table 2.2 shows Turkish middle school students' maths test performance by gender in the last two years. When we look at students' maths scores, on the average, they answered 6.99 (*SD*=3.99) mathematics questions correctly out of 20 in 2018, with only 83 students answered all of the questions correctly. With this mean, mathematics was the test which students showed the lowest performance in the 2018 exam (other tests being science and technology, history of Turkish revolution and Kemalism, Turkish language, foreign language of choice and religious studies). The average score dropped to 5.09 (*SD*=4.24) in 2019, with 5794 students answered all of the questions correctly. In neither the 2018 nor 2019 tests did the scores of girls and boys differ. Indeed, in 2019, maths test was reported as having the closest mean between girls and boys among all tests. The number of students who answered all maths questions correctly varied between 2000 and 6000 out of over a million students each year in the last ten years (well under one per cent of people taking the test), except for 2018 when the first exam after the update in the curriculum was conducted. Although the number of students at the very high end is not the best interpretation of the performance (means and SDs reported above are a better way of doing this), it is included here to show how small the proportion is (between 0.002 and 0.006).

Table 2.2. Turkish middle school students' performance in maths exam by year and gender

Year	*Mean* (/20)	*SD*	*Girls' mean*	*Boy's mean*
2018	6.99	3.99	6.83	7.15
2019	5.09	4.24	5.07	5.11

**SD*s for girls and boys are not available.

All of these results show that Turkish middle school students' maths performance was poorer than desired. The middle school students' low performance in the mathematics tests could be due to many different reasons, including the validity and reliability of the tests (there are serious reservations about them) to the preparation of the students, and from changes in the curriculum (update in 2018) to parental involvement. Specifically, despite the reliability scores of the maths tests in the last ten years were around *KR-20* = 0.80 (e.g., 0.84 in 2019), the reliability score was 0.65 in 2018 (Kuder & Richardson, 1937). The reason for lower reliability on mathematics test (<0.70) in 2018 is reported as the number of unanswered questions which is arguably because the questions were harder. Moreover, there can be an effect of the calculation of exam points where a student got a full mark for each correct answer and one-third of the mark was deducted for every wrong answer. Hence, students might have tended to leave the questions unanswered if they are not sure about their answer. Finally, this thesis will discuss teachers' contribution to students' performance in Section 2.2.3.3.

This picture means that research which seeks to foster middle school students' improvement in mathematics and to support pedagogical practice is crucial. Given the challenges of delivering a mathematics curriculum and student learning of the entire curriculum, the need for research into innovative classroom practices to improve students' maths performance seems necessary.

Turkish national government mathematics test items set the basis of the questions on the worksheet (described in Section 4.1.2) which is used throughout the thesis.

2.2.3. Teaching and Learning of 3D Geometry

This section is divided into two parts. The first of these presents the current discourse about the study of geometry of 3D shapes with further sub-sections on polycubes and frameworks for teaching and learning 3D geometry, and the second reviews the factors argued to be affecting students' performance in geometry, especially the geometry of 3D shapes and their 2D representations.

2.2.3.1. Studies on 3D Shapes

In this section, studies that looked at student difficulties in learning 2D representations of 3D shapes in geometry will be reviewed. There are not a large number, but this section synthesizes the results from studies that could be identified, which will subsequently be used in the RETA principles proposed in Chapter 5. It is helpful to know about the difficulties and error types reported earlier because they will be used to analyse the difficulties and the errors students made in Study One.

It is argued that 3D geometry is one of the most difficult topics in middle school geometry both for teachers and students (Bakó, 2003). 2D drawings are the most common representations which are used to represent 3D shapes in middle schools (Berthelot & Salin, 1998). *The need to visualize 3D shapes from 2D* (e.g., orthogonal and isometric) *drawings* has often built barriers for both teachers' teaching and students' learning (Christou et al., 2006; Kali & Orion, 1996; McGee, 1979; Parzysz, 1988; Widder & Gorsky, 2013).

Parzysz (1988) reported that in France, the teaching and learning of spatial geometry is reputed to be difficult both among teachers and students. His study found that decoding (reading) and coding (producing) 2D representations of 3D shapes was hard for teachers to teach and middle school students (11- and 12-years old) to learn. It was even harder for students to decode a 3D shape (a square-based regular pyramid in this case) from the visible parts of it in a 2D drawing because of a loss of information when moving the 3D shape to its drawing. Later, Bakó (2003) who reported the French Ministry of Education's survey which showed that only ten per cent of teachers taught spatial geometry. Teachers' most common reason for not teaching spatial geometry was 'not having enough time to teach it'; however, the author suggests the real reason was found to be students' not being able to visualize 3D shapes from teacher's drawing on the board, or that "students cannot see in 3D" (Bakó, 2003, p.1).

Similarly, Duval (1998) who studied teaching 3D shapes from a cognitive perspective, argued that looking at 2D drawings of 3D shapes was not enough to see what the drawings represent, mostly because of the dimensional change in the perceptive organization of the way of seeing. His observation of 13- and 14-years old students' processes of making 2D drawings of 3D shapes showed that the perception/reduction of a 3D shape to its 2D representation was cognitively complex but the dimensional change[5] between 2D and 3D was necessary for processing. However, it is questionable whether everyone processes the given information in the same way and whether there is a common way of looking at 2D representations of 3D shapes. That is to say, some students may not *see* what a teacher *sees* without the teacher having to explain it to them and without the teacher pointing out what the students should have seen.

Another study conducted by the French Institute for Research on Mathematics Education (IREM) was reported by Bayart, Gos, Hindelang and Keyling (2000). Its results suggested that some students consider given 2D representations of 3D shapes as if they were 2D originally, and students do not actually *see* the shape as 3D. For example, when students were asked if four points chosen on a cube (G, N, M and P in Figure 2.4, original draft) were at the same straight line segment (i.e., collinear), the majority of the students were certain that they were collinear and they did not see any other possibility although there were many other equally viable possibilities such as the two possibilities in Figure 2.4.

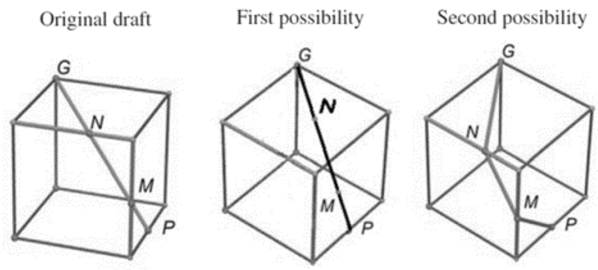


Figure 2.4. Two possibilities of locating the given four points G, N, M and P; adapted version obtained from Widder and Gorsky (2013, p.92)

In a similar vein, Bakó's (2003) experiment with around a hundred 14- and 15-years old students in Hungary showed that the case was not dissimilar there, and students experienced some obstacles in visualizing 2D representations (plane sections) of 3D shapes (cubes). For example, when students were asked to identify as many plane sections of a cube as possible with pen and paper, about a quarter of the students were only able to identify square (32 students), equilateral triangle (25) and rectangle (20),

[5] The dimensional change is “a basic cognitive process in the way of looking at a 2D representation” (Duval, 1998, p.44).

with only a few of them identifying regular hexagon (4), isosceles triangle (2), symmetric trapezoid (2), hexagon (1) and parallelogram (1). Moreover, none of them was able to identify the right triangle, trapezoid, right trapezoid, symmetric trapezoid, rhombus, and pentagon as a plane section of a cube. Presentation of the topic using different representations in dynamic software environments (DOS programs in Pascal) did not change the rate of identifying square, rectangle and equilateral triangle but increased the likelihood of identifying all other plane sections up to 31 students.

More recently, Pittalis and Christou (2010) described and analysed the structure of 3D geometry thinking by identifying different types of reasoning emerging from the literature. They argued that there were four types of 3D geometry reasoning and listed sample tasks which belong to them. The first of these types of reasoning is called *representing 3D shapes*. Activities listed under reasoning are divided into two secondary factors. The first factor included activities such as drawing a 2D representation of a 3D shape (e.g., an orthogonal drawing), constructing a 3D shape from its orthogonal views (e.g., constructing a polycube from its orthogonal views) and translating a 2D representation of a 3D shape to another 2D representation of it (e.g., from orthogonal drawings to isometric drawing). The second factor was about recognizing and constructing nets of 3D shapes (e.g., identifying nets of a square pyramid). The second type of 3D geometry reasoning is called *spatial structuring* as a latent factor. Activities requiring this type of reasoning included manipulating 3D arrays of 3D shapes including cubes, constructing 3D arrays of cubes and giving numbers to cubes that fill a larger 3D shape by spatially structuring the object. The third type they called *conceptualization of mathematical properties*. Activities requiring this type of reasoning loaded on two secondary factors: recognizing properties (e.g., edges, faces and vertices) of 3D shapes, and comparing and contrasting properties and relations of 3D shapes (e.g., a square prism such has six faces and eight vertices but a triangular prism has five faces and six vertices, and the base of cuboids and pyramids can be a square). The fourth, and final, type is named *measurement* and was considered as a latent factor. The tasks measuring this reasoning included estimations of the volumes of 3D shapes without using a formula and calculation of surface areas of 3D shapes. Pittalis and Christou (2010) established the validity of these factors using the data generated from 269 11-14 years old Cypriot students. This empirical work showed that all factor loadings were statistically significant and that each task in their study loaded to one of the six factors described; hence, they concluded that each of these factors could represent different 3D geometry skills, as a part of these four distinct 3D geometry reasoning types.

Moreover, Pittalis and Christou's (2013) study investigated students' skills of interpreting 2D representations of 3D shapes. They administered a geometry test consisting of 18 coding and decoding tasks to 279 11- to 15-year-old Cypriot students and interviewed 40 of them to enrich the profile of coding and decoding skills. The researchers specified coding and decoding skills of Parzysz (1988) for their context. For this particular study, coding was described as producing 2D representations of 3D shapes (e.g., making an isometric drawing of a prism or a polycube), while decoding referred to interpretations of 3D shapes based on their 2D representations and included the process of determining structural elements and geometric properties of 3D shapes from their 2D representations for drawing different parts of them based on the interpretations (e.g., determining visible faces of cubes from an isometric drawing of a polycube for constructing its orthogonal drawings). The results of their mixed-methods analysis identified four categories of student behaviours when interpreting 2D representations of 3D shapes, namely:

- Two-dimensional: Students identified in the two-dimensional category considered 2D representations of 3D shapes as if they were 2D and failed in all coding and decoding tasks because of their lack of conceptualization of the third dimension. This confirms the findings of Bayart et al. (2000) who also reported this type of behaviour.

- Intuitive: Students in the intuitive category managed to correctly answer simple decoding tasks such as identifying structural elements of 3D shapes in plane representations but did not do any of the coding tasks correctly. They were intuitively aware of the third dimension but were not able to manipulate 3D shapes mentally.

- Implicit-conventional: Students categorised as implicit-conventional answered almost all of the decoding tasks correctly and had a satisfactory performance in coding tasks. They did not face any difficulty in tasks asking for interpretations of structural elements of a 3D shape but found it a little harder to interpret geometrical properties and nets.

- Conventional: Students who are in the conventional category answered almost all of the coding and decoding tasks correctly. They were able to interpret 2D representations of 3D shapes by mentally visualizing them and to produce 2D representations of 3D shapes including translations of 2D representations of 3D shapes to each other.

Although the overall percentage in each category is not available, authors reported that 75% of the fifth graders were in the first two categories while almost 50% of the ninth graders were in the last two categories.

Finally, Fujita, Kondo, Kumakura and Kunimune (2017) assessed Japanese students' reasoning in 3D geometry lessons, particularly when they are solving 3D geometry problems of cube representations. They constructed their assessment based on existing literature to date having 12-15 years old students as participants. They administered this geometry assessment to 570 11- to 15-year-old Japanese students. The test included five questions requiring interpretations of 3D shapes (in this case measurements of a cube, such as length of a straight line within a cube). The analysis of the students' tests showed that only 7% of the students answered all five questions correctly. Of the remaining 93% of the students, 15.4% did not attempt to answer questions; 41.8% only answered questions intuitively or by using the visual information; 20% of them judged questions as if they are 2D and 15.8% of them were aware of the 3D representation but did not come up with the correct answer. These categorization of the answers were very similar to four types of students' behaviour to interpret 2D representations of 3D shapes in Pittalis and Christou (2013). Fujita et al. (2017) also coded answers for the nature of the mistakes and reported that the incorrect responses were because of either incorrect reasoning about the properties of a cube or incorrect manipulation of the shape in students' minds (visualization problems) or both.

This thesis will therefore consider the error types/categories reported in these studies when coding the worksheets of students for the nature of errors. However, with its more specific focus it will describe the errors in a much more detailed way for the orthogonal and isometric drawings of polycubes, respectively.

2.2.3.1.1. Studies on Orthogonal and Isometric Drawings of Polycubes

It is not unlikely to see orthogonal and isometric drawings of polycubes in the spatial thinking literature. Such drawings are often reported as a part of the tasks loading on spatial visualization factor (described in section 2.1.2) together with other tasks, rather than being a separate entity. Studies reporting this factor (e.g., Linn & Petersen, 1985; Miller & Halpern, 2013; Voyer et al., 1995) are included throughout spatial thinking sections: 2.1.3.1, 2.1.4, 2.1.4.1 and 2.1.5. This section focuses on studies coming from geometry education literature and synthesizes these studies chronologically. It is helpful to know about the earlier studies on orthogonal and isometric drawings of polycubes because they (and the insights gathered from them) will be considered when designing sample RETA-based lesson plans on orthogonal and isometric drawings of polycubes (see Chapter 5).

Studies on 2D geometry (which includes the study of basic shapes such as parallel lines and angles, study of polygons particularly triangles and quadrilaterals, calculations of perimeter and area, and sample geometric proofs) dominate the geometry education literature. There is a relatively smaller number of studies on 3D geometry (described in Section 2.2.3.1) compared to studies on 2D geometry, and even fewer especially when specified to the geometry of polycubes and their orthogonal and isometric drawings.

As a reminder, the literature on orthogonal and isometric drawings has many names for the terms; for example, orthogonal drawings can be found as orthogonal projections (Jones et al., 2012), orthographic projections/drawings (Moyer-Packenham & Bolyard, 2002), plan/top view and elevations/side views (Yeo et al., 2005), whilst isometric drawings can sometimes be referred isometric projections (Gambari et al., 2014) and perspective drawings (Oldknow & Tetlow, 2008), and sometimes very vaguely as a building or a picture of a building (Ben-Haim et al., 1985). This thesis considered these different names for these 2D representations as synonyms and chose to use orthogonal and isometric drawings which both stand as one of the earliest names for these types of representations in the literature, following the suggestions of Cooper and Sweller (1989). Similarly, polycubes are referred in various names such as polycubical shapes/objects (Cooper & Sweller, 1989), (solid) cube constructions (Ben-Haim et al., 1985) and a solid or an object constructed by unit-sized cubes (Pittalis & Christou, 2010). In this thesis, these names are considered and used as synonyms.

One of the earliest available studies on middle school students' interpretations of 2D representations of polycubes is from Ben-Haim et al. (1985). Ben-Haim and colleagues (1985) conducted a study with 978 years five to eight students (10-13 years old) in the U.S.A. They designed lessons with the activities including matching solid cube constructions to isometric drawings and orthogonal drawings and vice versa and tested students' performance prior to and after these activities. The test was consisted of multiple-choice items asking questions on orthogonal and isometric drawings, such as the number of cubes required to build given isometric drawing, a particular orthogonal view of an isometric drawing (question 2 in Figure 2.5) and symmetric orthogonal views such as the views from the back and front (question 8 in Figure 2.5). The maximum possible score was 32. Lesson observations and interviews with the students showed that students experience difficulty mostly in relating isometric drawings to their constructions from unit cubes. Moreover, descriptive statistics appeared to demonstrate that the mean scores increased as the grade level increases both for the pre-test (year five: *M*=7.39, *SD*=4.89; year eight: *M*=13.23, *SD*=6.00) and post-test (year five: *M*=12.22, *SD*=6.28; year eight: *M*=20.56, *SD*=6.53, no inferential statistics reported) but even in the post-test of year eight, students on the average scored about 12 points less than the maximum possible score (of 32).

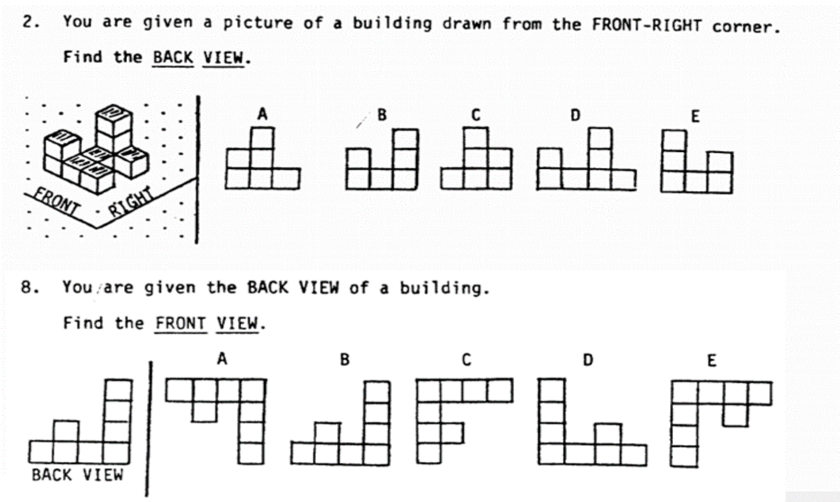


Figure 2.5. Sample questions from Ben-Haim et al.'s (1985) test (p.399)

Another polycubes study was conducted in Australia by Cooper and Sweller (1989). The study examined students' performance to interpret 2D representations of polycubes. For this purpose, students at year seven (ages 11-12), nine (ages 13-14) and eleven (ages 15-16) were provided with various 2D representations of 3D shapes, including orthogonal drawings and isometric drawings (see Figure 2.6). Each student was sequentially presented with different 2D representations and asked to build polycubes corresponding to the 2D representations on a card from the wooden unit cubes provided. The students were also asked to build polycubes based on verbal descriptions and prototypes. Cooper and Sweller (1989) found that building polycubes from the wooden unit cubes when the isometric drawing and the prototype was provided was significantly easier for students than when orthogonal drawings, layer plans, coordinates and verbal descriptions were provided. However, students did not find it any easier to build shapes when the prototype was provided than the isometric drawing was provided.

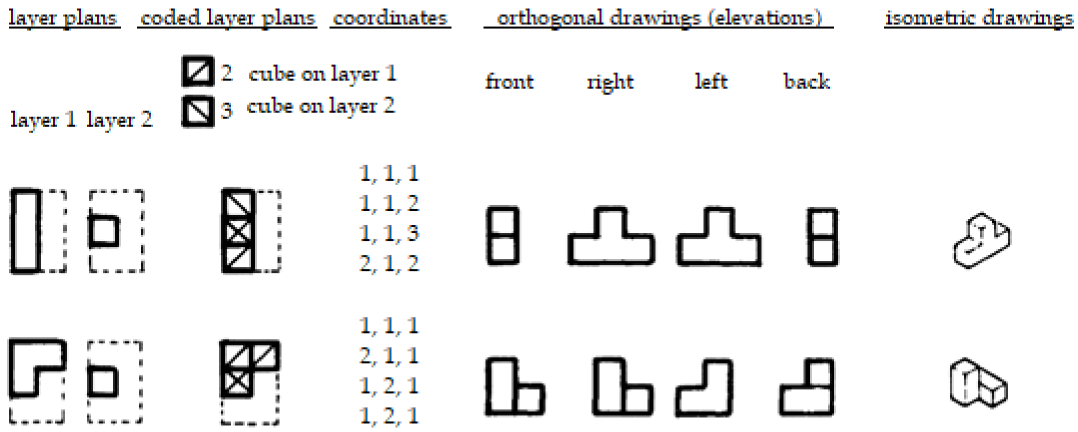


Figure 2.6. 2D representations of polycubes as in Cooper and Sweller's (1989) study (p.205)

Moyer-Packenham and Bolyard (2002) provided an applet to integrate into the teaching of 2D representations of polycubes. Their review explored representations used in the middle grades and highlighted the role of representations in promoting geometric thinking. The authors claimed that students' creation of their own representations can help geometric reasoning and visualization; hence they suggested students' own use of an applet during lessons on orthogonal and isometric drawings of polycubes (see Figure 2.7). The authors also proposed various tasks with the tool. For example, a brief description of one of the tasks they designed is as follows:

Step 1: Pair work or individual work to build random polycubes from snap cubes

Step 2: Practice of orthogonal and isometric drawings of these polycubes in the applet

Step 3: Exchange of the printed drawings from Step 2 with other pairs and individuals to build the drawn shapes from unit cubes

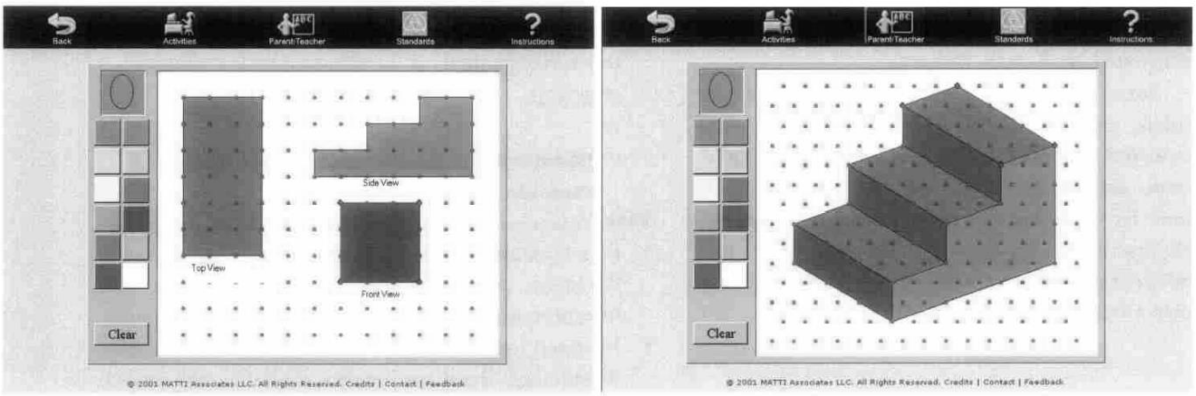


Figure 2.7. Screenshots from matti.usu.edu’s virtual geoboards applet (p.24)

Moyer-Packenham and Bolyard (2002) further argued that students’ engagement with such tasks provides them with various representations for exploration, which hopefully would result in a better understanding of orthogonal and isometric drawings. It should be noted that these claims are based on the literature they reviewed and their own reflections hence they are not supported specifically with their own empirical work.

Similarly, Yeo and colleagues (2005) explored year seven and eight students’ (13- and 14-year-old) experiences of learning orthogonal views of 3D shapes using a dynamic geometry software (ProDesktop). The software in this study used by students to construct and rotate 3D shapes to visualize their orthogonal drawings. The authors included various 3D shapes in their study such as polycubes and prisms and random 3D shapes with slant and inclined surfaces. As a part of the study, students were first taught orthogonal drawings by traditional methods. After a month, the same group of students studied orthogonal drawings with lessons facilitated by ProDesktop. Students were tested prior to and after the lessons with ProDesktop with a worksheet asking for orthogonal drawings of various 3D shapes including polycubes (See Figure 2.8 for a sample test item). They were also asked to complete a survey about their experiences with the tool.

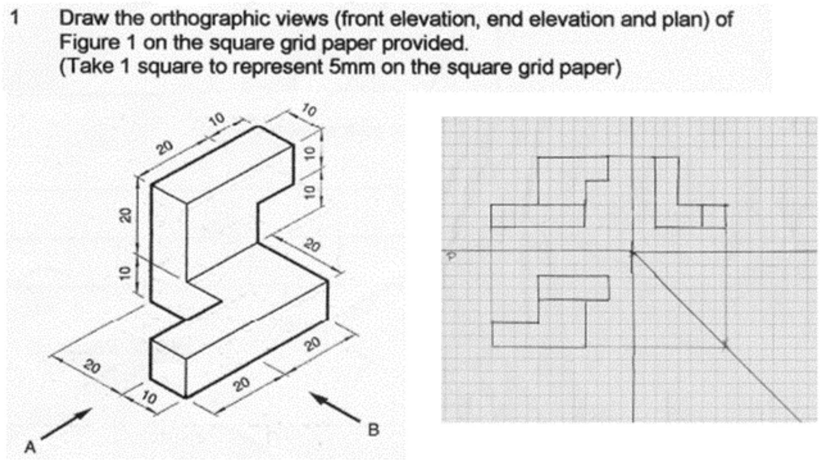


Figure 2.8. Sample test item and a student answer from Yeo and colleagues' (2005) study

Yeo et al. (2005) found that the majority of the students indicated that ProDesktop motivated them to study orthogonal drawings and that the tool facilitated their visualization and their learning of orthogonal drawings. Although the worksheet data was collected, the authors only reported the students' performance vaguely by saying the analysis of the worksheets pointed toward the same conclusion with the survey results that students' answers got better. However, 30% of the eighth-graders and 50% of the seventh graders found the tool difficult to use.

More recently, Jones and colleagues (2012) reported on data they collected from 570 Japanese students aged 12-15. They asked students a question where students were required to draw and interpret isometric drawings and/or oblique parallel perspective drawings of cubes. Oblique parallel perspective drawings are very similar to isometric drawings with a difference that they are set out using 45-degree angles while cubes in isometric drawings are set out using 30-degree angles. The results showed that only about 15% of the students were capable of making the correct drawings, which lead them to a correct solution. Many students' choices of 2D representation to solve the problem was poorer than expected. For example, Jones et al. (2012) illustrated an episode from one of their lesson observations where many of the students chose to draw nets of cubes to solve the problem rather than any of the expected drawings. They also concluded that teachers' prompt of shifting nets of cubes to isometric or oblique parallel perspective drawings increased students' chances to *see* geometric relationships.

Finally, Zilkova and Partova (2019) conducted a series of studies following a design-based research approach. After five design cycles, they developed an applet to help learners visualize orthogonal drawings of cube constructions. Each cycle trialled a version of the applet hence there were five different versions of the applet, all of them were aimed to help visualisation of orthogonal drawings of cubes. All five versions were free and available to use for learners and teachers at the time of writing this thesis.

These studies provide a rich account of students' difficulties in learning 3D geometry with regard to orthogonal and isometric drawings of polycubes (Ben-Haim et al., 1985; M. Cooper & Sweller, 1989; Jones et al., 2012). Some of these studies take a step forward by suggesting the integration of various dynamic geometry software packages into the teaching of these drawings to improve students' performance in these drawings (Moyer-Packenham & Bolyard, 2002; Yeo et al., 2005). This literature fed into the sample lessons on orthogonal and isometric drawings of polycubes with the principles that are evaluated in the empirical work conducted for this thesis.

2.2.3.2. Factors Affecting Student Success in 3D Geometry

The current literature provides an account of a range of factors influencing student success in two-dimensional and three-dimensional geometry. Some of the underlying reasons for low student performance in spatial geometry have been argued to be the following. Rather than presenting the factors as separate entities, this section synthesizes these factors in two categories: those that are related to cognition and individual differences (section 2.2.3.2.1) and those which are more about choices made in the lesson context and the policy context (section 2.2.3.2.2).

2.2.3.2.1. Factors related to human beings' cognition and their individual differences

Firstly, some of the factors affecting student success in spatial geometry are related to human beings' cognition and their individual differences. These factors are spatial skills, difficulties with drawings and working memory limitations.

- **Spatial skills**

Not surprisingly, when we think of 3D geometry performance, one of the first factors that comes to one's mind is spatial skills. The research has established that there is a relationship between spatial skills and geometry performance (Pittalis & Christou, 2010; Winarti, 2018). It will be described in Section 2.3.1 and its subsections 2.3.1.1 and 2.3.1.2 that it is not fully determined whether better spatial skills lead to better geometry or vice versa, but research argued for both (e.g., Buckley, Seery, & Canty, 2019; Lubinski & Benbow, 2006; Widder & Gorsky, 2013). Either way, it is important to note spatial skills is one of the factors that has been argued to contribute to students' performance in 3D geometry.

- **Difficulties with drawings**

Drawing skills are often considered as fundamental sources for understanding spatial geometry. Some researchers believe that drawings are simply tools for representing space but drawings are not related to understanding space (Kosslyn et al., 1977). Others argue that drawings are indicators of children's understanding of space (Goodnow, 1977; Olson, 1970). They suggest that the difficulties with diagrams and drawings could be worth thinking of as a factor which may influence student success in 2D and 3D geometry (Battista, 2007; Kaplan & Ozturk, 2014).

The literature often considers learner-generated drawing as a strategic process for learning similar to summarization and self-questioning (Gobert & Clement, 1999; van Meter, 2001; van Meter & Garner, 2005). 2D drawings of 3D shapes in geometry do not quite fit this consideration. The nature of drawings in spatial geometry is different than free drawing to learn in terms of the aim and the process. This is to say, for example, students do not necessarily construct isometric drawings as a strategy to learn some other concept in geometry. In school geometry, these drawings are mostly constructed '*to learn to represent'* 3D shapes and '*to reason with them'* to come up with a solution to a geometry problem, as they are in science (Ainsworth, Prain, & Tytler, 2011, p.1096). Jones et al.'s (2012) study which is described by the end of the previous section is a typical example of making isometric drawings to reason in geometry.

Although difficulties with drawings are worth taking into consideration, empirical evidence concerning the link between students' difficulties with drawings and their understanding of geometry does not (find) any systematic relationship (Cohen & Jones, 2008; Lehrer, Jenkins, & Osana, 2009; McManus et al., 2011).

- **Working memory limitations**

Another level of factors argued to be affecting student success in spatial geometry is working memory limitations. The majority of the studies on 2D drawings of 3D shapes show that making isometric drawings is hard for students (Jones et al, 2012; Ben-Haim et al., 1985) and some further found evidence to that it is much harder for students to make isometric drawings than to make orthogonal drawings (e.g., Cooper & Sweller, 1989). The literature argues that one's geometry performance on 2D drawings of 3D shapes might be related to working memory limitations. According to what we know from human memory capacity, it is harder to have more elements simultaneously in mind than one when performing a task (Ayres, 2006). Both Ayres (2006) and Paas, Renkl and Sweller (2003) argue that this also applies to 2D geometry drawing (of 3D shapes). They explain the reason for students' difficulty in isometric drawings is the need of having more simultaneous relations in mind to make isometric drawings than of orthogonal drawings. This is to say, one needs to have more elements (orthogonal drawings) simultaneously in mind in order to construct an isometric drawing, while in constructing orthogonal drawings, one could only focus on one element such as only the front view, or only the right view.

Moreover, Halford (1980, 2005) points to particular age groups having difficulties in representing 3D shapes. Within Halford's neo-Piagetian framework, children have specific difficulties because they are younger, and their working memory has not all developed yet. He supported his framework with experimental evidence from his studies with children of varied age ranges. To illustrate, Halford (1980) conducted an experiment on children's construction of 2D and 3D shapes. He had four groups of children between 6.6 and 12.5-year-old (grouped according to their chronological ages). Children were asked to reproduce 2D and 3D shapes presented to them. Halford's (1980) study found significant effect of age and he reported a linear increase in students' performance of 3D shapes with age.

Thus, both cognitive load theory (Ayres, 2006) and Halford's (1980) neo-Piagetian framework argue that working memory has a limited capacity which might eventually affect students' geometry drawing. Hence, working memory limitations are another explanation of students' spatial geometry performance.

2.2.3.2.2. Some choices that are made in the lesson context

Turning now to the other factors, there are some choices that are made in the lesson context by the teachers and policy makers. These include the geometry curricula that students are following, textbooks, teachers' beliefs and their teaching:

- **Changes in the geometry curricula students are following – policy level**

Geometry programmes and curricula have been claimed to be one of the underlying reasons for low student performance (Aksoy & Bayazit, 2012; Battista, 2007; Duru & Korkmaz, 2010; Kutluca & Aydin, 2010; Ural, 2015). Particularly in Turkey, the curriculum radically changed two times in ten years (MoNE, 2009, 2013). The most recent curriculum has been updated in 2018 with further suggestions (MoNE, 2018b). One of the reasons for the ongoing reduction in Turkish middle school students' success, therefore, could be a consequence of these changes because there are not small but radical changes from 2009 curriculum to 2013 one in teaching units, student assignments and portfolios, and suggested teaching methods and technologies (Öksüz, 2015).

The new programme increased the impact of technology on the Turkish education system by suggesting (in fact, telling) the use of educational technologies such as games and educational software packages in classes. Policymakers expected a noticeable improvement in students' academic achievement as a result of this new programme. Thus, it was important for them to examine what the new mathematics programme has brought and how it affected the teacher's geometry teaching. Recent case studies have revealed that Turkish pre-service and in-service teachers believe in the effectiveness of the current maths programme and like its technology-emphasis (Bayrakdar-Çiftçi et al., 2013; Çiftci & Tatar, 2015; Tekalmaz, 2019). However, some case studies also show that teachers are not ready to use these technologies in a student-centred environment and that they would rather prefer to use suggested classroom technologies (e.g., educational software Cabri, EBA, and GeoGebra) themselves in order to teach topics in geometry (Balgalmış, 2013; Balgalmış et al., 2014; Ocak & Çimenci-Ateş, 2015; Saralar, 2016b; Saralar & Ainsworth, 2017). For example, Balgalmış et al. (2014) attempted to understand three teacher candidates' use of GeoGebra within the context of their teaching practices in middle schools. They found that pre-service teachers dominated the use of such technology during class time; they rarely asked students to go to a computer lab to discover a geometrical relationship, and never asked students to use their own tablets in the class time. However, learning with such technology requires interaction with the software in order for the students to explore the topic and relate that with their prior knowledge (Hohenwarter & Jones, 2007; Lavicza & Papp-Varga, 2010). Therefore, these activities may not involve anything more achievable for students than the memorisation of the technique the teacher uses unless students themselves experience and actively engage with the geometry concepts through provided technology.

- **Late and misleading presentation of topics in mathematics textbooks**

Furthermore, students' low performance could be related to the fact that the mathematics textbooks usually do not present geometric problems in the early grades, nor at the middle school level, especially in Turkey (Boz et al., 2016; Küçük & Demir, 2009), therefore, students may not understand geometry concepts as they have received little geometry knowledge from these textbooks. American textbooks, for instance, are not designed to require complex geometrical thinking even by the 6th and 7th grades which correspond to the last year of primary school and the first year of middle school (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). In this context, reasoning about and inferences with 3D shapes in order to solve real-world and mathematical problems count as complex activities. Turkish mathematics textbooks are very similar to American textbooks in their presentation of geometrical content (see Avcu, 2019). Geometrical concepts requiring three-dimensional thinking almost do not take place in the textbooks until the 5th grade, which corresponds to the age of nine and ten years old (MoNE, 2013). Consequently, late introduction of these spatial geometry concepts might influence children's ways of interpreting spatial geometry and/or may result in children facing difficulty when they are asked to solve geometry problems requiring 3D geometrical thinking. It should be noted that teaching topics requiring complex geometrical thinking too early also potentially cause problems with learning 2D and 3D geometry including various misconceptions (Yenilmez & Yasa, 2008); thus, finding the appropriate age range and development level to introduce these concepts could lead to positive results.

Furthermore, textbooks might have potentially misleading 2D drawings which may have caused low student performance. An 2019 study (Widder et al., 2019) used the same question presented in Figure 2.4 together with other similar items. The study exemplified *potentially helpful geometrical information* (PHI, e.g., supplementary verbal explanation of the given 2D drawing that elicits visualization) and *potentially misleading geometrical information* (PMI, i.e., hidden information or altered information). Its aim was to see whether the presented PHI and PMI in geometry textbooks are a-priori measures of visualization difficulty in achieving correct (or desired) comprehension of 2D drawings of cubes (Widder et al., 2019). Hidden information was described as "geometric elements (vertices, edges, surfaces, and intersections of edges) that are occluded by coinciding elements such as one of the two coinciding vertices of a 2D sketch of a cube, is considered hidden." (p.496). Altered information was considered to be consisted "of altered ratios of lengths of edges, altered ratios of sizes of angles, confluent edges that are not confluent in reality, or intersecting edges that do not intersect in reality" (p.496). Its results showed that the interaction between potentially helpful and potentially misleading geometrical information was largely captured by spatial visualization difficulty in geometry for 16-17-year-old students. This is to say, the difficulty of 2D drawings of cubes increases (i.e., students' scores on the test for correct or desired comprehension decreases) when the detail and number of PHIs decreases and PMIs increases (i.e., PHI/PMI decreases) in the textbooks. Hence, the authors found that geometrical information presented (the ratio of PHI/PMI) is an a-priori measure of visualization difficulty in learning 2D drawings of cubes so the information presented in the textbooks might potentially be a reason for poor performance in interpreting 2D representations of 3D shapes.

- **Teachers' teaching**

Insufficient and inappropriate geometry instruction is noted by many researchers as a key factor affecting student success in spatial geometry. Research shows that students' errors in maths occur also because students have difficulties in understanding teachers' instruction methods (Confrey, 1990). Particularly, in geometry education, different levels of the communication of information between the teachers and students can be another cause of misconceptions (Lim, 2011; Pusey, 2003), particularly in transformation geometry (Luneta, 2015). In other words, students may not understand a geometry topic the teacher teaches unless s/he explains the topic at a students' level of geometric reasoning (Pusey, 2003). Hence, Pusey (2003) claims that it is necessary for teachers to be aware of their students' level of geometrical reasoning before attempting to deliver lessons. Some researchers further claim that if teachers design their lessons in higher levels of geometric reasoning than their students have, poor performance in geometry is inevitable (Luneta, 2015; Pusey, 2003).

Moreover, teachers' responsibilities do not end after being aware of their students' level of geometrical reasoning. Teachers' classroom management skills, the supportive climate they provide in the classroom and their choice of activities for cognitive activation are all identified as dimensions of instructional quality which link teachers' teaching with students' geometry outcomes (Klieme et al., 2006; Kunter et al., 2007). While classroom management and supportive climate are likely self-explanatory, cognitive activation requires a little more explanation. "Cognitive activation is an instructional practice that encourages students to engage in higher-level thinking and thus to develop an elaborated knowledge base" (Lipowsky et al., 2009, p.529). Challenging tasks, activation of prior knowledge and a content-related discourse practice are reported as constructs of cognitive activation (Klieme et al., 2006). In cognitively activating geometry lessons, the mathematics teacher encourages students to share and compare their thoughts and solution strategies by giving them challenging tasks, conflicts and contradictory ideas and interpretations (Grouws & Cebulla, 2000). Such challenging tasks were found to be positively correlated with the students' performance in various maths areas, including spatial geometry (Klieme et al., 2001; Wenglinsky, 2002). Moreover, Lipowsky et al.'s (2009) study with 19 Swiss and 19 German maths classes found empirical evidence that both classroom management and cognitive activation have positive effects on geometry outcomes (case of triangles). Hence, if students are not engaged with the cognitive activities and only being invited to solve geometry problems previously demonstrated by the teacher through applying known procedures, this is nothing more than rote learning (i.e., rule and cue following), if at all, and might result in a low geometry performance (Ding & Jones, 2006; Nardi & Steward, 2003; van Hiele-Geldof, 1984).

- **Maths teachers' beliefs and conceptions of students' 3D geometry learning**

Moreover, another reason for low student performance has been offered as being teachers' beliefs and conceptions concerning students' 3D geometry thinking (Barrantes & Blanco, 2006; Even & Tirosh, 2014; López & Nieto, 2006). For example, McKnight, Travers, Crosswhite and Swafford (1985) claimed that teachers believe that students are less likely to learn geometry than other courses in middle school and thus teachers' beliefs could contribute as cause of students' poor performance in geometry. Particularly, Turkish middle school mathematics teachers believe that geometry, especially geometry of 3D shapes, is at the top of the list of mathematics topics where many students have difficulties in understanding and practising (Küçük & Demir, 2009). The majority of research has supported McKnight et al.'s (1985) proposal and reported that teachers' beliefs might influence the way they teach, and students' learning can eventually be affected by this (Hew & Brush, 2007; Sanders et al., 1997; Schoenfeld, 1998; Thompson, 1984). It is of note that in contradiction to the teachers' beliefs, some more recent studies showed that students are open to and equally willing to learn science and mathematics at the very beginning of the academic year and they need a certain amount of time to become disaffected (Aktaş-Arnas, 2009; Aktaş-Arnas et al., 2014).

Finally, teachers' beliefs are also related to the tools and representations they (chose to) use in the classroom. Ainsworth (2006) reports that research provides abundant evidence that external representations support students' learning. Studies (diSessa, 2004; Novick et al., 1999; Zacks & Tversky, 1999) further argue that students can select the representation which fit their needs and learn better with the help of it. On the other hand, particularly in Turkey, teachers of geometry mostly choose the representations they think are effective and do not give students an opportunity to choose the representation which students think it could help them, believing that students do not have necessary competency and skills to choose and use these representations for their learning. This is, for example, many of these teachers do not integrate dynamic geometry tools to represent geometric shapes as these teachers think that their students might not use the tool effectively, students might be distracted from the tool and therefore teacher themselves might not be able to manage the classroom during the activities with these tools (Agyei & Benning, 2015; Saralar & Ainsworth, 2017; Yorgancı, 2018). Teachers' arguments are legitimate that not all students were found to have the skills to choose effective representations for themselves (Chi et al., 1981; Kozma & Russell, 1997), particularly in 3D geometry (Jones et al., 2012). However, teachers' beliefs on the effectiveness of representation and their way to integrate to the class have not escaped from being argued to contribute to students' low performance in 3D geometry.

To note, while the reasons in Section 2.2.3.2 and its subsections constitute the majority of the literature on this topic, some researchers also suggested that children's out-of-school experiences and social forces including parents' attitudes toward maths contribute to students' performance in mathematics, and in geometry as a part of it (Eccles & Jacobs, 1986; Goodall et al., 2017; Hong & Ho, 2005; Soni & Kumari, 2015).

Hence, there is no single factor which could magically be changed the geometry teaching so that the achievement problem in spatial geometry could immediately be cured. One needs to consider all of these factors and others in order to help students get better learning outcomes in spatial geometry.

2.2.3.3. How to Improve Teaching and Learning of 3D Geometry

As described in the two previous sections, students' geometry performance can be seen as problematic because of several reasons. These reasons will be further discussed in 2.2.3.2. This section focuses on how to improve teaching and learning of geometry though designing lessons based on some frameworks.

More recently, some mathematics education researchers who focussed more on the ways to improve mathematics performance as Usiskin (1987) did in the eighties (see Section 2.2.1), intended to provide frameworks for maths teaching. Studies on 2D representations of 3D shapes have focused more on *building frameworks* which describe and analyse '3D geometry thinking'. This thesis inspired from these frameworks when developing the RETA principles for geometry teaching.

As a reminder, while some researchers (e.g., van Nes & van Eerde, 2010) describe 3D geometry thinking as a part of spatial thinking and use the factors of spatial thinking –which are described in Section 2.1.2– to describe 3D geometry thinking, the majority of recent research (e.g., Fujita, Kondo, Kumakura, & Kunimune, 2017; Widder & Gorsky, 2013) describes 3D geometry thinking as a separate domain and uses the definition of Pittalis and Christou (2010). 3D geometry thinking is defined by Pittalis and Christou (2010) as “the conception of thoughts and ideas about 3D geometry concepts by amalgamating various types of reasoning”; and reasoning in this concept refers to “a set of processes and abilities that act as a feasible tool in problem-solving and enable us to go beyond the information given” (p.192).

For example, Yeh and Nason (2004) proposed and examined a framework to teach 3D geometry with technology. They argued that 3D geometry is composed of three inseparable components: communication, objects and spatial thinking. *The communication* referred to (a) spoken and written language to describe 3D geometric entities (including the language such as front-back and up-down) and (b) non-verbal 2D representation of objects in a technological environment. While *objects* were described as any 3D shapes regardless of whether they are a part of maths curricula or not, their *spatial thinking* was geometric spatial thinking, this will be described in section 2.3.1. Taking these three components into consideration, they developed a software package called VRMath, in which realistic representations of 3D geometry problems were presented in various colours together with an available link to a discussion forum. Authors claimed that their “initial *work* with primary school children indicated that *VRMath* is a very effective tool for facilitating construction of knowledge about 3D geometry concepts and processes” (p.6). It is of note that this claim is stronger than the evidence have; the study was with only two primary school children (six and seven graders) in a lab environment.

Recently Goodall, Johnston-Wilder and Russell (2017) suggested a framework to teach mathematics that many pupils can experience at home or at school in the UK (not specified in but including 3D geometry). According to them, the mathematics teaching should be ALIVE (accessible, linked, inclusive, valued and empowering) in contrast to TIRED (tedious, isolated, rote, elitist and depersonalised) mathematics found by Nardi and Steward (2003). In order to understand ALIVE, we first need to understand TIRED framework. Nardi and Steward's (2003) study in the UK with 13-14 years old students found that mathematics teaching can be experienced by students as TIRED:

1) Tedious: Most of the students viewed maths as a boring and irrelevant subject with no transferable skills to real-life. Moreover, they reported that learning maths offers little opportunity for being active.

2) Isolated: Students perceived mathematics as an isolated subject where students mostly needed to work individually to come up with a solution to a maths problem.

3) Rote: Many students viewed maths as a set of rules to follow hence, for them, there were unquestionable and unique solutions to answer maths problems.

4) Elitist: Students experienced maths as a challenging subject and developed the belief that only exceptionally smart or gifted students can excel in maths.

5) Depersonalised: Most of the students in the study believed that their maths learning is not but can be facilitated by somehow making teaching suitable to each student's needs.

As a contrast to this TIRED maths, Goodall and colleagues (2017) introduced five principles to improve performance in mathematics:

1) *Accessible principle* refers to the use of appropriate enactive tasks through which students can establish their own understanding. The principle is suggested with the belief that these activities leave almost no reason for students to be excluded from developing mathematical thinking.

2) *Linked principle* implies the referral of the previous knowledge so that students can link the new information to what is already known and understood.

3) *Inclusive principle* suggests including all students to the process of learning maths through various activities as opposed to the belief that only exceptionally smart students can learn maths.

4) *Valued principle* emphasizes the integration of real-life examples into the teaching of mathematics to understand the value of maths. The researchers further suggested that the examples should be chosen from those which are valued by people as worthwhile both personally and culturally.

5) *Empowering principle* refers to the students' agency that empowers students to take ownership of their learning. The aim is to help students develop an

understanding of lifelong learning while making as much progress with mathematics as possible. This principle further suggests ideas on how students can be empowered in their learning through purposefully designed mathematical tasks whilst developing skills needed for the 21st century, skills such as creativity and technology literacy.

2.3. Spatial Thinking and Geometry

2.3.1. The Relationship between Spatial Thinking and Geometry

This section explains the relationship between spatial thinking and geometry. After the description of the position taking in this thesis, it continues with two sections that review historical and recent studies on this relationship, respectively.

There is no doubt that the relationship between geometry and spatial thinking is contested. On the one hand, some researchers support the idea that spatial thinking and geometry are independent of each other mostly because of the complex nature of spatial thinking (Clements & Battista, 1992; Pittalis & Christou, 2010; Tartre, 1990). They believe that the process of spatial thinking (such as cognitive processes while creating mental representations for mapping and navigation) makes it more complex than what is needed for learning basic geometry. On the other hand, a number of authors have concluded that geometry and spatial thinking are interrelated (Cheng, Huttenlocher, & Newcombe, 2013; Guven, 2012; Lean & Clements, 1981; Wheatley, 1990; Widder & Gorsky, 2013); for example, citing studies that have found a positive correlation between spatial ability and academic performance in maths, and specifically geometry (Cheng et al., 2011; Fennema & Sherman, 1977, 1978; Grüßing, 2011; Guay & McDaniel, 1977; Ishida, 2011). Many researchers have developed geometry activities such as tessellations, isometric dot paper and block building activities to improve children’s spatial thinking, measured by disciplinary tests (Battista, 2007; Clements & Battista, 1992). It is also argued that gaining spatial thinking skills brings advantages to students including expertise in mathematics and specifically in geometry (Clements, 1998; Jones, 2002b; Schoenfeld, 2006; Uttal & Cohen, 2012), and geometrical intuition[6] (Jones, 2002a). The balance of evidence

[6] Geometrical intuition is a mental phenomenon that describes the skill to acquire without reasoning and inference in geometry (Fujita et al., 2004).

suggests that there is a link between geometry and spatial thinking and these two have a mutually beneficial relationship (Battista, 2007; Clements & Battista, 1992; Gergelitsova, 2007; Jones, 2002a).

Geometry and spatial thinking have the study on the properties of and the measurement in space in common although each covers more than that. In this thesis, geometry and spatial thinking will be thought of as two different sets that overlap, where the overlap of the sets represents common parts of geometry and spatial thinking. That is to say, geometry and spatial thinking have shared parts regarding the study of space such as orientation and visualization of two- and three-dimensional shapes but they individually are more than these parts.

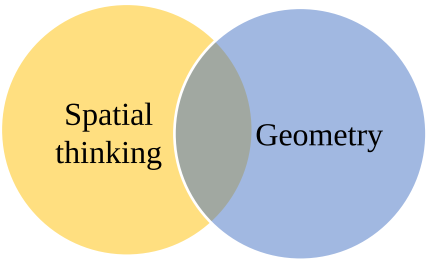


Figure 2.9. Venn diagram of spatial thinking and geometry (size is not meant to be implied)

Moreover, even though an ongoing debate on the nature of spatial thinking exists, this thesis focused on spatial aspects of geometric reasoning which are thought of as a cluster of cognitive processes that are important for constructing mental two-dimensional representations for three-dimensional objects and changing them to suit problem-solving process in geometry (Battista, 2007; Clements & Battista, 1992). Specifically, in the context of this thesis, the process of reasoning involves the skills to visualise three-dimensional shapes constructed from unit cubes and to create and manipulate internal mental images in order to make orthogonal and isometric drawings. Therefore, this PhD research focusses on spatial aspects of geometry reasoning to improve spatial geometric academic achievement (i.e., spatial and geometric thinking in the words of Ness and Farenga (2007)).

2.3.1.1. Spatial Thinking and Geometry: Historical Review

"Geometry literature often seems to have a life of its own outside the broader study of spatial thinking." (Cheng et al., 2013, p. 1050). A lot of what we know about spatial thinking in geometry today classically started with the Van Hiele who sees geometric thinking as a part of spatial thinking. Van Hiele (1959) and Van Hiele-Geldof (1984) asserted that the progress of students' geometric thinking depends on a series of sequential and hierarchical levels ([0. precognition,] 1. visual, 2. descriptive-analytic, 3. abstract-relational, 4. formal deductive, 5. rigour-mathematical) to accomplish desired thought in geometry. They argue that effective learning in middle schools can be achieved only if students reach to descriptive-analytic or abstract-relational levels. In a study based on Van Hieles' theory with children at age eight, Rosser, Lane, and Mazzeo (as cited in Clements & Battista, 1992) asserted that those children's spatial skills dealing with the transformations (such as iterations, and rotations) is at the third level in the hierarchical development order, after matching figures (level 1) and recalling and reconstruction of figures (level 2). However, this position is not without its controversy. As previously discussed, spatial skills are malleable, and age and level and experience of schooling appear to be factors affecting the spatial skills needed in geometry. Similar studies to that of Rosser and others have continued to be conducted and some evidence on different age ranges has been found by researchers (see Clements & Battista, 1992). Even though student ages have been reported, such studies relate students' geometric thinking to the instruction in addition to their out of school experience rather than their ages as Halpern and Collaer (2005) suggests. Moreover, Van Hiele's theory of geometric thinking is solely based on two-dimensional geometry.

Gutierrez (1992) extended Van Hiele's model to 3D geometry by analysing students' behaviour when they compare or move solids (3D shapes) in the ground based on the results of her experiment which aimed at improving spatial visualisation in sixth graders through practising questions from 3D geometry (e.g., orthogonal and isometric drawings, and cube rotations). The following summarizes her claims for students' abilities for each extended van Hiele level. She claimed that students at level 1 compare 3D shapes paying attention to certain elements such as faces and vertices without paying sufficient attention to mathematical properties (e.g., parallelism and sizes of angles) or only using them with a visual role. They are not able to visualise 3D shapes or invisible parts of them, neither their movements in an orientation question. Students at level 2 still focus on the elements of 3D shapes but base their explanations on elements that lead to the differences in mathematical properties. Of note, while some of these properties are known by them from the name of a 3D object (e.g., a rectangular prism has a rectangular base), some are more obvious from their observation of it. Students at this level are able to visualise 3D shapes if their current and final positions are available, but are not able to plan a position in the next movement. Students at level 3 make informal justification based on the mathematical properties which are either previously known by them or found by the analysis of the observation of the 3D representations and their elements. These students are able to visualise hidden parts of 3D shapes in addition to their visualisation of the shape and to plan the next movements. Students at the fourth level examine 3D objects before attempting any manipulation by focussing on elements and properties, some of which are not observable hence are formally deducted from definitions and other properties. They have well-developed spatial visualisation skills which allow them to make accurate decisions on the basis of the formally deducted properties through less number of movements than those at level 3.

Piaget and Inhelder (1967, 1971) proposed an alternative to Van Hieles and Gutierrez that sees spatial thinking as an innate ability. While Van Hieles (1959, 1984) and Gutierrez (1992) did not specify age ranges or possible effects of innate factors, Piaget's four stages correspond with the age of children. The claim is that spatial thinking – or, in this context, organising objects in two- and three-dimensional frames – is an innate ability which develops with children's interaction with 3D objects. Piaget and Inhender believe that improvement in geometrical thinking leads to better spatial thinking. They maintained that children need to be at about nine years old to have wider geometrical reference frames (than young children) and this allows them to make spatial inferences. These were also discussed at an earlier section, which was on nature of gender differences in spatial thinking (Section 2.1.4.1).

Clements and Battista's (1992) review on geometry and spatial thinking suggests building a new model on Piaget's and Van Hiele's which combines their strengths. They noted in their review that some of the students in the studies they reviewed, aged 11 to 14, were less competent than the younger children; and not all of them managed to achieve Piaget's task. Indeed, some of the studies they reviewed showed that only half of the students in this age range mastered such tasks. Nonetheless, for Piaget, this age group (formal operational) is expected to be capable of interpreting information in abstract forms, finding analytic solutions to problems, and also making formal and deductive reasoning.

Thus far one it can be seen there is some inconsistency or discrepancy in the historical literature review. However, these studies are considered as the basis of the growth of studies on spatial reasoning in geometry and should not be discounted when considering new approaches. These also showed that more studies including different age groups (especially those of aged between 11 and 14 as discussed above) are required to measure spatial thinking in geometry. Therefore, it seems important to investigate spatial thinking in middle school curricula, which corresponds to this age group in the Turkish education system. It is crucial to note that teaching Turkish middle school students to help them improve spatial awareness for the achievement of curricular goals in three-dimensional geometry is a goal of the present research.

2.3.1.2. Spatial Thinking and Geometry: Recent Research

Before presenting more recent research, section begins by describing its relationship to sections 2.1.3 and 2.1.3.1. Section 2.1.3 looked at whether spatial skills can be trained through spatial interventions and reported the results of studies including all types of training (video-games, courses and spatial task training). Section 2.1.3.1 reported on disciplinary-specific spatial training studies which aim to improve disciplinary-specific spatial performance with examples from across disciplines from STEM domains (e.g., engineering courses and technology courses). This section specifies the discipline as mathematics with a further focus on geometry where possible and reviews studies that look for a relationship between spatial thinking and mathematics.

The literature has argued that there is a relationship between spatial thinking and mathematics for more than 30 years (see Lubinski & Benbow, 2006); but it has not fully determined whether better spatial reasoning leads to better mathematics or the reverse or whether they most depend upon some other underlying skill.

The resent research has found that an increase in spatial skills may lead to better learning outcomes in mathematics and particularly in geometry (Casey et al., 2001; Cheng & Mix, 2014). For example, Cheng and Mix's (2014) study suggests a causal relationship between spatial reasoning and maths that starts early in the development. Specifically, Cheng and Mix (2014) worked with six- to eight-year-old children and divided them into two groups. While the intervention group received spatial training by practising mental rotation tasks, the control group completed crossword puzzles. Both groups completed a mental rotation test, a spatial relations test and a maths test on calculations. The results showed that students in the intervention group (but not in the control group) improved on the mental rotation test (p<.001, η^2=.23) and maths calculations test (p=.005, η^2=.14). These findings were attributed to the possibility of spatial training's priming children to reorganise the questions spatially. However, this is just one study with 58 children which reports such results; hence, more studies are needed for generalisation. Moreover, as discussed later in sections 3.1.1 and 3.1.4, while such experimental research is crucial to understand the causal relationship between spatial thinking and mathematics, research that bridges these lab-based interventions and classroom practice (such as research following DBR approach and designing course training) is also needed.

Hawes, Moss, Caswell and Poliszczuk (2015) described major branches of mathematics where the research has concluded that spatial reasoning (the second component of the committee's spatial thinking definition in section 2.1) is an important aspect. They argued that while branches such as algebra and mental arithmetic appear to heavily rely on spatial thinking, geometry goes further in that it is an inherently spatial branch of mathematics. Concerning the various skills considered as spatial reasoning, it is argued that mental rotation plays a big role in geometry achievement (Bruce & Hawes, 2015; Casey et al., 1995). Moreover, mental rotation skills were found to be related with students' geometry performance (Battista, 1990; Delgado & Prieto, 2004). Hence, it is not surprising to see studies that relate spatial thinking to geometry performance commonly use performance on mental rotation tasks. For example, Casey and colleagues' (2001) study with 187 eighth-graders found that students' spatial skills measured by mental rotation tests were correlated with their scores on the related parts of the Trends in International Mathematics and Science Study (TIMSS, 1995). Despite the fact that their aim was to explore gender differences in maths performance, their study also showed links between Vandenberg mental rotation test results and maths performance.

Researchers have also looked at how different initial spatial skills might influence problem-solving strategies in STEM domains. Typically, research has found that students with different initial spatial skills use different problem-solving strategies

(Gilligan et al., 2017; Hegarty et al., 2013; Tuvi-Arad & Gorsky, 2007). Particularly in geometry, research has suggested that spatial skills affect the way one interacts with the available sources and change their problem-solving strategies (Buckley et al., 2019; van Garderen, 2006; Widder & Gorsky, 2013). For example, Widder and Gorsky (2013) focused on secondary school students' learning processes when they were asked to use a 3D computerised software in order to visualise three-dimensional geometric objects (a cube, triangular prism, and square-based pyramid). Before observing the school students' learning processes, students were asked to complete a disciplinary test with questions including measurements in cube representations (spatial geometry test). The test included two types of items; a-type items which probe understanding based on verbal information together with formal geometric knowledge (e.g., items expecting one to conclude base of a triangular prism is an isosceles right-angled triangle given a 2D sketch of the prism and complementary verbal description) and b-type items which probe understanding based on visualization (e.g., visualizing a 3D shape from its orthogonal drawings). Widder and Gorsky (2013), first, found that students with high spatial skills used the tool less than those with low spatial skills. Moreover, students with high and low spatial skills had different purposes for using the tool. While students with limited spatial skills used the tool to discover the relationships, to see the structures and to calculate the measurements, students with well-developed spatial skills used the tool only for the reflection of the structures, such as rotations, to see perspective drawings.

Finally, research has further found that "the spatial skills rely on neuronal networks partially shared with mathematics" (Tosto et al., 2014, p. 462). Tosto and colleagues' (2014) study with 4174 pairs of 12 years old twins observed an overlap between spatial reasoning and mathematics (r>.40). They measured spatial reasoning through spatial tests including jigsaw items and hidden shape items, and mathematics performance through a test with items corresponding to English curriculum components, namely (a) non-numerical processes (e.g., rotational and reflective symmetry in geometry), (b) understanding numbers (e.g., 27 + 27 + 27 = 27 x □) and (c) computation and knowledge (e.g., straightforward calculations). While genes explained 60% of the overlap between spatial reasoning and mathematics, environmental factors explained the remaining 40%. These factors equally affected female and male students in spatial thinking and mathematics at the age of 12.

The longer-term aim of the work underpinning this thesis is improving spatial awareness of students by providing them with the opportunity to work with representations of 3D shapes so perhaps in future, they can better cope with spatial problems. Given the relationship between spatial thinking and geometry, in principle, this seems achievable.

2.3.2. What spatial thinking is taught in various countries' mathematics curricula?

In various national mathematics curricula, objectives requiring spatial thinking mostly can be found in three-dimensional geometry. Mathematics curricula include three-dimensional geometry in many countries, such as Canada, England, France, Germany, Japan, the Netherlands, Poland, Singapore, Switzerland, and the Turkish Republic (Department for Education [DfE], 2009; Hoyles, Foxman, & Küchemann, 2002; Ministry of Turkish National Education [MoNE], 2013).

In the Turkish Middle School Mathematics Curriculum (year five to eight), it has been argued that providing spatial thinking education is one of the goals, for the following reasons: (1) spatial thinking is important to understand a multitude of situations in real life since it helps students improve skills of producing and using information; (2) spatial thinking aids students to understand various mathematical concepts and to relate those with each other, and they could use these relationships in everyday life and in other disciplines; (3) spatial thinking helps students express their own thoughts and reasoning in the problem-solving process; (4) spatial thinking guides students to understand the necessary mathematical knowledge and skills – rather than rote learning/ memorization of formulas and concepts – to receive further education in mathematics and/or related fields (MoNE, 2013). The curriculum, which was designed in 2013, is still in use but was updated in 2018.

The Turkish Middle School Mathematics Curriculum integrates spatial thinking in all years of the middle school program and identifies objectives related to spatial thinking by calling them spatial relationship objectives (MoNE, 2018b). It gives specific hours of teaching time to teach each unit. Students study mathematics for four lesson hours (each being 40 minutes), and applications of mathematics for two lesson hours each week. Although there is a set program for the maths lessons, teachers are allowed to choose what they want for their students to practice in the maths application lessons. Spatial relationship objectives from the curriculum for each year are as follows.

In year five (nine and ten years old), students learn about cuboids with three spatial relationship objectives: At the end of the teaching unit on geometrical shapes for eight to ten lesson hours, students should be able to

- *describe a rectangular prism (i.e. cuboid) and properties of it; realise square prism and cube are special forms of a rectangular prism*
- *draw faces of rectangular prisms and decides whether given drawing belongs to a rectangular prism (dynamic software can be used)*
- *calculate surface areas of rectangular prisms (MoNE, 2018b, pp.56-57).*

In year six (ten and eleven years old), students learn about geometric shapes with five main spatial relationship objectives: At the end of the teaching unit on geometrical shapes for fifteen lesson hours, students should be able to

- *understand that if one places unit cubes into a rectangular prism so that there is no space in the prism, the number of unit cubes equals to the volume of that prism*
- *form different rectangular prisms having the same volume by using unit cubes and explain the relationship as the volume of a rectangular prism equals to the multiplication of the base area and the height*
- *derive and apply the volume formula of a rectangular prism (requires the knowledge of area and lengths)*
- *recognise standard volume units, and converts m^3, dm^3, cm^3, and mm^3 to each other*
- *estimate volumes of rectangular prisms (MoNE, 2018b, p.63).*

In year seven (eleven and twelve years old), students study geometrical shapes with two spatial relationship objectives (these two are the objectives of the lesson plans in the current thesis): At the end of the teaching unit on geometrical shapes for four to six hours, students should be able to

- *draw orthogonal views (views from the top, left, right, and front) of the 3D shapes which are constructed from unit cubes and relate them with each other (e.g. left-right views are symmetric)*
- *construct 3D shapes from the given orthogonal views and make an isometric drawing corresponding to given views (the use of isometric paper is suggested) (MoNE, 2018b, pp.69-70).*

In year eight (twelve and thirteen years old), students learn about geometric shapes with six spatial relationship objectives: At the end of the teaching unit on geometrical shapes for eight to ten lesson hours, students should be able to

- *identify right prisms and list their common properties (identical ends, flat faces and the same cross-section all along its length)*
- *determine the main elements (two circular bases and one rectangular side) of a right circular cylinder, construct and draw them*
- *derive and apply the surface area formula of a right circular cylinder*
- *derive and apply the volume formula of a right circular cylinder*
- *identify a right circular pyramid, determine its elements (a base and a face), construct and draw its face*
- *recognise the right cone and determine its main elements (MoNE, 2018b, pp.75-76).*

Moreover, an elective preparatory mathematics course was designed to prepare middle school students for their first year of secondary school (for year eight students to prepare them for year nine). The teacher guidelines for the course further suggested teachers to integrate spatial geometry games into their teaching with the hopes of better learning outcome (Ministry of Turkish National Education, 2016b).

Another example is the English National Curriculum Mathematics Programmes for key stages one (years one and two), two (years three to six) and three (years seven to nine) (Department for Education and Employment [DfEE], 1999; Department for Education [DfE], 2013c, 2013b) in which 2D and 3D shapes, the language to analyse and interpret their properties and strategies to solve problems including these shapes, have had an important role. In a report on the previous curriculum, for instance, spatial intuition was emphasised as "enormously powerful tool and that is why geometry is actually such a powerful part of mathematics" (The Royal Society/ Joint Mathematical Council, 2001, p.7). The report further claimed that students can develop geometrical intuition and extend their spatial thinking through playing with 3D shapes.

In the English National Curriculum Mathematics Programmes, objectives on 3D shapes can be found from Key stages one to three; the objectives are called statutory requirements and they are as follows for key stages one and two: "Pupils should be taught to

- *recognise and name common 3D shapes, for example, cuboids including cubes, pyramids and spheres; describe position, direction and movement, including whole, half, quarter and three-quarter turns (year one, five and six years old)*
- *identify and describe the properties of 3D shapes, including the number of edges, vertices and faces; identify 2D shapes on the surface of 3D shapes, [for example, a circle on a cylinder and a triangle on a pyramid]; compare and sort common 2D and 3D shapes and everyday objects (year two, six and seven years old)*
- *draw 2D shapes and make 3D shapes using modelling materials; recognise 3D shapes in different orientations and describe them (year three, seven and eight years old)*
- *no objective on 3D shapes was listed for year four*
- *identify 3D shapes, including cubes and other cuboids, from 2D representations (year five, nine and ten years old)*
- *recognise, describe and build simple 3D shapes, including making nets (year six, ten and eleven years old)" (DfE, 2013b, pp.10-44).*

In key stage three, statutory requirements are not divided into years but given as a whole for years seven to nine (eleven to fourteen years old) and they are as follows: "Pupils should be taught to

- *derive and apply formulae to calculate and solve problems involving perimeter and area of triangles, parallelograms, trapezia, volumes of cuboids (including cubes) and other prisms (including cylinders)*

- *use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3D*

- *interpret mathematical relationships both algebraically and geometrically (DfE, 2013a, p.6).*

Yet another example is the American National Mathematics Curriculum, which has been amended to integrate spatial reasoning into geometry teaching after the suggestions of the National Council of Teachers of Mathematics, NCTM in short (Fuson, Clements, & Beckmann Kazez, 2010). The council (2010) suggested integrating spatial reasoning into mathematics curriculum starting from the early stages and provided an alternative teaching unit. This unit is named as Geometry, Spatial Reasoning and Measurement and is integrated into mathematics curriculum in the US from pre-kindergarten to year eight (Fuson, Clements, & Beckman Kazez, 2010b, 2010a; Schielack, 2010a, 2010b). In the first book of the series (for pre-kindergarten to year eight), the council noted that "Geometry, spatial reasoning and measurements are topics that connect to each other and the other mathematics, and that connects mathematics to real-world situations" (Fuson, Clements, & Beckman Kazez, 2010b, p.57). The council's (2010b) list of ideas and key skills for geometry and spatial reasoning included:

- *recognize and name common three-dimensional shapes (including real-world objects) including spheres, cylinders, prisms, and pyramids*

- *use the relational language of right and left; identify and create symmetric figures (e.g., mirrors as reflections)*

- *build simple 3D structures from pictured models*

- *compose and decompose solid shapes, thus building an understanding of part-whole relationships" (p.59).*

A final example is the German National Mathematics Programme, whose developers believe that "work with three-dimensional objects strengthens students' spatial ability" while including three-dimensional geometry into their curriculum (Hoyles et al., 2002, p.17). They expect that further work with three-dimensional objects from the counting of unit cubes to rotation, translation and reflection of 3D shapes is important for improving students' spatial thinking, which will be required in their practical life. Although there are at least 13 curricula followed in 16 German states at the primary level, all are informed by national mathematics programme. One example from Germany is North Rhine-Westphalia's primary mathematics curriculum in which two of the content domains are devoted to spatial thinking and 3D shapes. This curriculum aims for students (seven to eleven years old) to reach the following content-based competencies:

"Spatial orientation and spatial visualisation, and solid figures

- *trace lines with a pen (eye-hand coordination), name overlapping figures (figure-ground discrimination), and identify forms (visual consistency)*
- *make orientations in two-dimensional space using a map*
- *describe spatial relations on the basis of pictures, arrangements, plans, etc., as well as from imagination*
- *visualize the movement of shapes and objects and describe the results of movement in advance*
- *identify geometrical objects, sort them according to geometrical characteristics, and describe them using mathematical terminology (e.g., area, edge)*
- *construct wireframe and solid models of objects and build more complex cube constructions*
- *find various nets for cubes*
- *identify two- or three-dimensional views of buildings and construct buildings according to a plan*
- *define and compare volumes of objects with unit cubes"* (Mullis, Martin, Goh, & Cotter, 2016, pp.2-3).

It is possible to see how specific the learning objectives are in the Turkish curriculum which is very similar to how the objectives are described in the NCTM books for each year in America. Curriculums in Turkey and America are different than most of the European curriculums, such as English mathematics curriculum (which specifies statutory requirements for key stages, not for a particular year) and German maths programme (which describes learning objectives for six years in general and is needed to adapted to 13 different curriculums at primary level).

Although spatial thinking is a part of numerous countries' mathematics curricula, students' poor performance in geometry, especially in topics related to spatial thinking objectives, have been reported by many researchers from different nations (Battista, 1999; Battista, Clements, Arnoff, Battista, & Borrow, 1998; Fuson, Clements, & Beckman Kazez, 2010b). The focus of my work, as a design-based research project, is to provide lessons to aid teachers so that they can help students improve their performance in the Turkish government geometry exam.

2.3.3. Summary of Section 2.3

To conclude, there is a complicated relationship between spatial thinking and geometry. There are many descriptions of the same terms because of the distributed nature of spatial thinking across different disciplines (e.g., psychology and maths education), and this makes working on spatial geometry hard. Hence, not surprisingly, this literature review shows that spatial geometry is an under researched area compared to many other areas in mathematics, for example, number sense. The research does show that it is not easy or trivial for students to understand 3D geometry. Students are not doing as well as many people including researchers, practitioners and policy makers think they should and the reasons for that are multifaceted. Attempting to resolve aspects of this problem directly motivated the research questions addressed in this thesis.

2.4. Research Questions

Review of the literature on spatial thinking and geometry resulted in the development of five research questions. These questions are:

1. How do the seventh-grade middle school students learn 3D shapes in Turkey?

   a. What are the students' difficulties in learning about 3D shapes?

   b. What are the students' errors in representing 3D shapes?

2. What principles can inform how 2D representations of 3D shapes are best taught to grade seven students in Turkish middle schools?

   a. What are the important elements of 3D shapes lesson plans?

   b. How can specific lessons be designed to teach 3D shapes?

3. How do seventh grade students experience these lessons?

4. What are the opportunities and challenges for a maths teacher when adopting these lessons?

5. What are the outcomes of these lessons for these students?

   a. How do learning outcomes (orthogonal and isometric drawings) differ between students who participate in the new lessons and those who study traditional lessons?

   b. Are these results influenced by gender?

These questions are further refined and investigated in the relevant chapters (Chapters 4 to 8).

3. METHODOLOGY

In this chapter, the choice of methodology, specific methods that were used in this thesis and ethical issues are addressed. This chapter is divided into three sections. It begins with the description of the nature of the overarching methodological approach: design-based research and explains how the four studies in this thesis fit into a design-based research project. This includes an introduction to design-based research and its phases, a justification for the approach and a discussion concerning issues with design-based research. It then describes the specific methods that were used throughout the thesis, including data generation, analysis and presentation. The final section discusses ethical issues. It should be noted that the specific methods (e.g., interview questions) employed in each study are described in detail in their own chapters.

3.1. Choice of Methodology: Design-based Research

Design-based research is pragmatic and it aims to improve educational practice in innovative ways, often together with technological interventions. This PhD research aims to improve Turkish middle school students' learning of orthogonal and isometric drawings through designing a new model and lesson plans and benefitting from the available technology in this context. As explained in the Introduction, the researcher is a mathematics teacher herself and is funded by the Ministry of Turkish National Education to find effective ways of integrating the available technology (a tablet for each student and a smartboard for each classroom) to improve current maths teaching practices. It was considered important to investigate existing practices and identify any missing components in order to propose suggestions to improve these practices. Hence, DBR fits the goals of the research by giving the researcher the opportunities to not only investigate and note the problems in teaching orthogonal and isometric drawings in the regular lessons, but also design and test interventions for overcoming these problems and propose a possible solution to them by suggesting a design for teachers to use in their practice.

3.1.1. Introduction to Design-based Research

In this thesis, a design-based research (DBR) approach was employed. In some countries (e.g., the United States), this approach was initially named design experiments and was intended to overcome the perceived limitations of experiments that compare intervention and control groups (Collins, 1990). DBR was developed in response to the need to develop methodological solutions to explore interventions in authentic educational contexts rather than laboratories (Brown, 1992; Collins, 1992). In other countries (e.g., the Netherlands), design-based research emerged to develop and improve curriculum materials (Gravemeijer, 1998), and it was initially called developmental research (Freudenthal, 1998; Goffree, 1979). The terms design experiment (Collins, 1992), design research (Edelson, 2002), development research (van den Akker, 1999) and developmental research (Richey & Nelson, 1996) are sometimes interchangeably used for design-based research. In this thesis, the term design-based research was chosen to use, following the suggestions on Bakker (2018) which describes the term as "the research that is possible due to the existence of a new design" (p.29). Design-based research, in this thesis, refers to a family of related research approaches with similarities which are together with differences in aims of characteristics including design experiments, developmental research and others (Phillips, 2006; van den Akker, Gravemeijer, McKenney, & Nieveen, 2006b; Wang & Hannafin, 2005).

There are some characteristics of design-based research that often go together (Bakker, 2018c; Cobb et al., 2003; Collins et al., 2004; Phillips, 2006; Plomp, 2006). Simply, researchers who use this approach to methodology, create, test and refine their interventions based on their findings, often acting as a teacher or collaborating with one in classrooms in order to develop new models, artefacts and practices that can be generalised to other contexts. The following paragraphs described the main characteristics of design-based research.

First of all, DBR has an *interventionist* nature similar to experimental studies. Researchers who use this approach create, test and refine their interventions based on their findings. Hence, DBR differs from ethnographic traditions which do not affect the researched context. It is true to say that DBR is a test-bed for innovation with its characteristic of creating and testing new interventions (Cobb et al., 2003).

Secondly, DBR is *iterative* in the sense that it has cycles (DBR studies) of an intervention. It involves putting the first version of intervention into a real life situation to see how it works. The analysis of this cycle feeds a new cycle, and its analysis informs the next one. Each cycle consists of preparation, design of an intervention or its refinement, implementation of the intervention and retrospective analysis. Moreover, design-based research often has one overarching purpose but different stages and cycles of the research might have other purposes. If one considers a project that aims to provide guidance on how to teach a specific topic in mathematics, it could have multiple phases. It is likely to begin with a description and evaluation of current learning and teaching environments such as students' prior understanding of the topic and existing teaching practices, an innovative design to be tested such as activities and lesson plans, and a comparison and an evaluation of a design through various forms (e.g., performance tests and interviews) from different perspectives (e.g., students' experiences and teachers' experiences).

The third characteristic is that DBR is *process-oriented* so it is both prospective and reflective. Educational ideas (for students or teachers) which are developed in the design can be adapted and tailored throughout the empirical work (e.g., when a design idea does not work as intended). Reflection can be done after each lesson even if there is more than one lesson and lead to changes in the designed activities for the next lesson. Design and testing are not separated, as opposed to many other interventionist approaches to research (Bakker, 2018c), instead, they are intertwined with each other and interwoven together. This is one of the main characteristics of design-based research, which distinguishes it from some experimental approaches that focus on a hypothesis before and after an intervention.

The fourth characteristic is that DBR is *theory-oriented* and grounded in relevant research, theory and practice. It, at least partly, uses theory at the beginning of the research and aims for generating one at the end for learning and means (or design artefacts) to support this learning. The generated theories are domain-specific and related to the design so humble, which reflects its pragmatic roots. A humble theory is not a grand theory of learning, it is, in fact, a theory which is "accountable to the activity of design", explaining and framing both what works and what does not work in a specific domain with accounts for them coming from practice (Cobb et al., 2003, p.10). This characteristic distinguishes DBR from action research which does not as directly aim at informing theory.

Another and a key characteristic is that DBR is *utility-oriented* in the sense that "the theory must do real work" (Cobb et al., 2003, p.10). DBR aims to tide over the gap between educational practice and theory (Bakker & van Eerde, 2014; Cobb et al., 2003; Edelson, 2006; van den Akker, 1999). It proposes and explores a design artefact such as designed activities and lesson plans (Kelly, 2006) but not in a lab rather in a real-world setting such as a classroom and mostly collaborating with real practitioners (van den Akker et al., 2006b). It refines both theory and practice. "The value of a DBR theory lies in its ability to produce changes in the world" (Barab & Squire, 2004, p.6). This again reflects its pragmatic roots. This is, DBR has an advisory purpose which is to provide theoretical insight and understanding into how to support or promote a specific way of learning and teaching *in practice* (van den Akker, Gravemeijer, McKenney, & Nieveen, 2006a). It focuses on the development of a design artefact (e.g., activities and lesson plans) and provides information and knowledge to similar practices.

The final characteristic is that DBR is *integrative* and flexible and thus allows the researchers to use various types of data and different methods to work through research questions. DBR is mostly associated with the use of mixed methods (Sandoval & Reiser, 2004). Most researchers who follow a DBR approach would agree with Maxcy (2003) who asserted that "It is perfectly logical for researchers to select and use differing methods, selecting them as they see the need, applying their findings to a reality that is both plural and unknown" (p. 59). The selection of methods in DBR for investigating issues in authentic real-world settings is once again in line with its pragmatic roots (Anderson & Shattuck, 2012). Furthermore, the combination of qualitative and quantitative methods helps to increase objectivity, validity and applicability. However, it is crucial to mention that DBR does not normally focus on replicability and generalisation to a population, instead it uses these methods for the exploration of the researched context. DBR potentially sets foundations and bases of experiments (e.g., randomised control trials) which could also be a part of DBR in later cycles (Kelly, 2006). This extension may start with controlling some of the variables in a later cycle and may end up with randomised control trials having control and experimental groups if the particular DBR project has the capacity and need.

Given these characteristics, DBR is neither interpretivist nor positivist. It is pragmatic and it comes with the belief that methods of the study should be chosen according to the needs of the research questions in order to answer them better (Reimann, 2011; Walker, 2011). DBR with a pragmatic philosophy is flexible and thus allows the researcher to use various types of data and different methods to work through research questions hence it is mostly associated with the use of mixed methods (Biddle & Schafft, 2015; R. B. Johnson & Onwuegbuzie, 2004). It acknowledges issues in the debate of using mixed methods and settles them by arguing that qualitative and quantitative approaches are compatible (House, 1994; Howe, 1988; Tashakkori & Teddlie, 1998).

Design-based research can be adapted to and used in various contexts from language teaching (Bergroth-Koskinen & Seppala, 2012) to gifted education (Jen et al., 2015), from mathematics education (Evans, 2018) to game-based learning (Squire, 2005) but not specialised in one. Thus, reviewed literature suggests a variety of criteria for DBR (e.g., see Wang & Hannafin, 2005, p.7), which is not possible to follow all at the same time. It can be used in formal and informal education. Accordingly, Collins et al. (2004) suggest not embodying each and every idea they proposed for DBR but to move in the direction of embodying as many of them. The following points, therefore, describes characteristics which are often associated with DBR but were not followed in this thesis.

- Involvement of practitioners as co-investigators (Kelly, 2006; van den Akker, 1999): In this thesis, opportunities were created for students during the classroom discussion at the end of the lessons and during the interviews to share their opinions about the lessons and how they could be improved. In addition to this, teachers as practitioners were given opportunities during both pre-interviews on lesson plans and debrief sessions to share their ideas about how lessons should be. Although this beginnings of involvement, the term co-investigator implies a deeper involvement with the research.
- Involvement of multidisciplinary research teams (Cobb et al., 2003; Collins et al., 2004): Different research teams that have specialists in different roles would probably have deepened the understanding of the phenomenon under investigation. Although there were not separate teams for different roles, the

research involved a mathematics educator, a cognitive scientist and a local practitioner in addition to the PhD researcher.

- Separation of roles as designer, evaluator and implementer (Collins, 1992; Plomp, 2006): It is true that the researcher created opportunities for teachers to adapt and implement the lesson plans in Study 3 and 4 unlike Study 2 where she acted as a teacher in an after-school course. Nonetheless, she played the roles of primary intervention designer and evaluator and all other necessary roles because of the requirements of a doctoral thesis; along with the support of her supervisors.

3.1.2. Phases of Design-based Research

This section outlines the characteristics of the phases of design-based research.

- **Phase One – Exploration and analysis of the problem**

The first phase of design-based research can be thought of as the exploration and analysis of the problem; both in context and though existing literature (Herrington et al., 2007). In this phase, similar to ethnographic studies, the researcher observes the researched context without intending to influence it, mostly by using a set of qualitative methods to generate a rich account of design in practice. The exploration phase may also include quantitative methods such as tests for measuring students' performance in a specific domain in order to better investigate the problem (e.g., the errors made by the students). Moreover, the researcher reviews the literature and seeks advice from the experts in the field. This phase is the time to start establishing the researcher's perspective on learning, design principles, particular lesson objectives to be investigated and the research questions.

- **Phase Two – Design and construction**

After redefining the problem in its setting in detail and reviewing the literature concerning it, DBR then begins to develop "solutions informed by existing design principles and technological innovations" and applying them in the real settings, which can also be thought as the second phase of a design-based research (Herrington et al., 2007, p. 4093). In this phase, DBR differs from ethnographic studies as the researcher interferes with the researched context. With the insights gained in Phase One, the researcher designs the intervention (i.e., lesson plans with necessary materials and activity sheets) and instruments to measure its implementation (e.g., worksheets and interview questions). These instruments may be revised in later iterations. In this phase, the researcher should be careful to consider how s/he will assess the intervention. A range of instruments might be needed to measure whether the intervention is working and it worked in an intended way. As such, in addition to an instrument to measure students' performance, observation protocols, questionnaires in various forms (e.g., evaluation forms), and interviews with participants are suggested to assess participants' experiences and outcomes for them (Collins et al., 2004).

- **Phase Three – Evaluation and reflection**

In the third phase, the researcher collects the data by implementing the proposed intervention in Phase Two. Unavoidably, the intervention will not be enacted exactly as it was anticipated. Hence, necessary refinements are taken into consideration including changing and removing some of the activities in the design or adding some others according to the needs of the participants. The next iterations are implemented after these changes in order to get better results - mostly seen as better learning outcomes in design-based research in education (Cobb et al., 2014; Herrington et al., 2007). These iterations are named differently (e.g., year, cycle, case study) by different design researchers. The aim of these iterations is to use the lessons learnt in each iteration to design a better one. The iterations also allow the researcher to collect more detailed information about the case in order to generate or perhaps propose a local and humble theory (Cobb et al., 2003, 2014) as explained in Section 3.1.1. A humble theory, for this PhD thesis, is a domain-specific instructional theory as of Stephan and Akyuz (2012) – who built a humble theory about teaching integer addition and subtraction in middle schools based on other theories such as constructionism and realistic mathematics education.

3.1.3. The Current Thesis as a Design-based Research

In this section, how the phases of design-based research were performed with this PhD thesis are considered.

As discussed in the literature review, studies show that students struggle to represent three-dimensional shapes in two dimensions. This thesis aimed to improve Turkish middle school students' orthogonal and isometric drawings of 3D shapes by providing insights into how it was already taught in Study 1 (Chapter 4) and what the literature suggests could improve this. This resulted in the RETA (Realistic, Exploratory, Technology-enhanced, and Active) model and associated lesson plans (Chapter 5). It then tested and refined the model and the lesson plans through cycles of DBR focussing on students' experiences of the lessons and outcomes for students in Study 2 (Chapter 6) and a teacher's experiences of teaching with the RETA-based lesson plans and its outcomes in Study 3 (Chapter 7). Finally, results of a quasi-experimental study with more than 200 students are reported in Study 4 (Chapter 8). All cycles helped the researcher to understand the problem from different perspectives to refine the design in support of the overarching aim. The following paragraphs describe the four studies of this thesis and how they fit the phases of DBR.

In line with Phase One of DBR, Study 1 explored the current teaching of 3D shapes and the learning outcomes associated with this teaching in its naturally occurring situation, i.e., Turkish middle school classrooms. It was helpful to explore students' learning experiences and current pedagogy in natural classroom settings in order to better understand the reasons underlying the struggle in representing three-dimensional shapes that was largely reported in the literature (and noted in the public exam performance). One could relate this with the exploratory endeavour of design-based research, which aims to explore the problem in detail in its own setting instead of testing pre-determined variables as in experimental methods. This study explored particular problems including students' orthogonal and isometric drawing errors on the worksheets with a detailed investigation of the nature of the errors (Section 4.2.1), students' perceptions of their performance in these drawings and the challenges they faced (Section 4.2.2) and the current pedagogy which could have caused them (Section 4.2.3). As might be expected from DBR, mixed methods were used throughout the study. A set of qualitative (e.g., observations and interviews) methods were used to explore students' difficulty in representing 3D shapes orthogonally and isometrically. It was also necessary to understand the learning outcomes associated with current practice. Hence, quantitative methods (e.g., worksheets) were used to test orthogonal and isometric drawing performance. These guided the researcher to design the first version of the RETA model (Section 5.2) and the lesson plans (Section 5.3).

In Phase Two, based upon the insights gained in the first phase, the researcher drafted a model -which is called the RETA 3D shapes teaching model - and lesson plans to teach orthogonal and isometric drawings of 3D shapes. Lesson plans were supplemented with documents such as activity sheets and slides. These were revised in later iterations in order to better fit the needs of the students and the teachers. Moreover, the researcher was carefully considered how the intervention would be evaluated in this phase. As such, in addition to preparing worksheets to measure students' performance, observation protocols, lesson evaluation forms and interview questions were drafted to assess participants' experiences of the lessons and outcomes of the lessons for them in accordance with the suggestions of DBR researchers.

In accordance with Phase Three of DBR, the data were generated by implementing the RETA-based lessons. Modifications were made to some activities in the lessons as they were changed, removed and new activities were added as analysis suggested. As expected from DBR, the iterations permitted the collection of more detailed information from different perspectives. Study 2 explored students' experiences of the lessons and outcomes for students and Study 3 focused on a teacher's experiences of teaching with the RETA-based lesson plans and their outcomes. Finally, Study 4 (Chapter 8) employed a quasi-experimental study with 205 students where RETA classrooms were compared to business as usual classes. Implementing DBR that ended with an experiment brought an additional value to research for the Turkish government.

3.1.4. Justification for the Research Approach

Many research methods are criticized for either not having or having only a slight aim of contributing to practice (Levin, 2013; Nuthall, 2004). Levin (2013) who criticised this in his review of the relationship between research findings, policy and practice reported that research rarely shapes practice in education even though there were recent the attempts of the policymakers in the last decade. Researchers have argued that teachers rarely benefit from research findings (Broekkamp & Hout-Wolters, 2007) and it does not help teachers as practitioners much to know whether one intervention is better than the other (Nuthall, 2004). Rather, teachers need to understand the mechanism which makes it effective so that they can adapt it to their classroom contexts. Moreover, they need research findings free from complex research terms, in a straightforward (to apply), familiar and concrete form, which are called design artefacts (e.g., developed interventions such as lesson plans and activities as in this PhD thesis) in DBR (Kelly, 2006).

Unlike many research methods, DBR does not only contribute to the theory or the practise; it is intended to tide over this gap between theory and practice. McKenney and Reeves (2018) who performed an extensive literature review reported that DBR allows for contributions in various forms both to the theory and the practice. Its contributions to the theory to date have included setting design standards for a particular teaching method, developing educational software and making a district-wide reform in curricula. These theories were always together with practical contributions to educational practice. The main practical contribution of DBR is the design artefact, which is intended to solve everyday problems in a classroom context. Similarly, the goal of this research was improving students' performance, theorise what made it difficult and easy and suggest a design for teachers to use in their practice.

In relation to the implementation context, DBR provides evidence about the process of learning and the factors affecting these processes in real classroom contexts rather laboratory settings. In short, DBR mostly works in real classrooms (or other contexts) so as to overcome the criticisms of researching in artificial and fixed lab contexts. Barab and Squire (2004) argue that it is not valid to report laboratory settings as natural environments as the findings might be inadequate. For example, participants might guess questions of the research (in)correctly and behave accordingly rather than doing what they *normally* do (Goodnow, 1976) or they might not relate the lab setting with the real context and again behave different than *normal* (Brown, 1992). Hence, it is not possible to interpret participants' behaviour independent of the researched context. Working in real educational contexts allows DBR to better understand how the designed intervention works or does not work in the complexity of a classroom. Correspondingly, all of the studies of this PhD research are in real middle school classrooms. The research started with the observation of middle school classrooms in two schools to explore the current teaching of orthogonal and isometric drawings in Study 1. It continued with an after-school course in a middle school in Study 2, which was followed by two more cycles in regular middle school classrooms in Study 3 and 4.

Structured principles are needed to clarify the mechanisms by which the results were achieved and DBR provides these for researchers and practitioners (Cobb & Gravemeijer, 2008). These can help to appreciate the degree of transferability of the intervention by applying them to different iterations of the intervention in selected classrooms (Gobo, 2007). This is, researchers can start realising the things in common among contexts by studying how the results were achieved in different contexts for generalisability – to a humble but analytic theory rather than a population. In DBR, these structured principles are called *design principles* and design principles of this thesis are described in Section 5.2.

3.1.5. Issues with Design-based Research

Design-based research intends to bridge the gap between theoretical research and practice but such work also brings challenges. It is important to provide an overview of criticism about DBR which reveals its limitations as a research method. The following paragraphs do not represent a complete account, rather highlights challenges concerning this research and describes some criteria that emerge from concerns to be taken into account in design-based research in mathematics education and their relation to this thesis.

One of the issues is the degree of fidelity of the intervention to the designers' intentions. Design researchers do not expect to have full fidelity to their intention because of the context of the implementation being classrooms rather than labs. However, core aspects of the intervention still need to be implemented as intended to help the researcher answer the research questions. These should be shared with practitioners prior to the implementation so that when they adopt the design into their lessons, the main parts of the design should still be there. In cases which this is not possible, it is unlikely to collect data to answer research questions hence the cycle must be repeated (Brown & Campione, 1996). Researchers have suggested good practice involves a) describing the intervention fully and explicitly to the practitioners and b) having practitioners as co-designers in the research wherever possible in order to implement it with fidelity (Kelly, 2006; Stears, Malcolm, & Kowlas, 2005; van den Akker, 1999). Moreover, c) showing that the intervention is based on research, d) testing the intervention in a friendly environment and e) micro-cycles are all suggested to reduce this risk of losing fidelity. In this thesis, the researcher tested early stages of the intervention with other PhD students in Education prior to Study 1. Furthermore, the intervention was trialled with small number of students in an after-school course context in Study 2. Hence, this study could be thought of as a micro-cycle of the next study. In Study 2 (cycle 1), the researcher acted as a mathematics teacher in an after-school course and she noted her reflections whenever she needed to make certain choices (e.g., the lesson plans suggested students' work in pairs; however, the researcher needed to decide how the pairs were composed). It was the closest version of the implementation of the RETA model therefore the difference between intended model and enacted model was small. Finally, the researcher met with teachers several times before their implementation. These meetings included the discussion of research aims, design principles, lessons plans and the corresponding activities including how to use the tool in the lessons and trying to represent some of the shapes on it. In Study 3 (cycle 2), the researcher collaborated with a mathematics teacher to adapt and adopt the lessons to use in her own classes. She conducted four semi-structured interviews prior to lessons. The aims of these interviews were to get to know the teacher and to discuss her opinions about the lesson plans and underlying pedagogical stands. One of the interviews focused on the discovery of the software which was used in the lesson plans. Moreover, there were debrief discussions after each lesson and the aim of the debrief discussions was to discuss how the lessons went including strengths, problems and issues to consider for the next lesson (and in the future). In Study 4 (cycle 3), the same procedure was repeated with a number of teachers.

Another issue is the complexity of working in a real classroom. Brown (1992) describes a real classroom as "rich, complex and constantly changing" environment in which many things might go wrong (p.144). She discussed how a real classroom consists of many components and these constantly change. Unexpected things and problems might occur in any classroom whilst the interventions are expected to be working in a smoothly functioning learning environment. This makes a real classroom difficult to research and from which to make inferences. Because of the complex nature, working in a real classroom brings the challenge of collecting a big amount of data including recordings of the lessons, observation notes, interviews and students' work. Handling a big amount of data in DBR might be challenging for the researchers. It makes the analysis of the data harder because of the time and resources required (Collins et al., 2004). Lack of analysis may cause misinterpretations and conclusions might be speculative and local (Cobb et al., 2003). The data of this thesis generated from real classrooms as expected in DBR. This data included lesson recordings, observation notes, interviews with both teachers and students and students' work including worksheets and GeoGebra constructions. Report on all the data generated in all four studies has been impossible given the word length constraints of a PhD. Instead, separate studies reported different angles of the data after their detailed analysis and in respect of specific research questions. For example, Study 2 (cycle 1) reported students' experiences of the RETA-based lessons and outcomes for them whilst Study 3 (cycle 2) reported a teacher's experiences of adopting and using these lessons.

The final issue is the time delay during iterations. DBR involves iterative cycles of testing and refinement. As explained in phases of DBR, the intervention is refined by using the results of a previous cycle, and this process continues until having the best version of the intervention which meets the research aims. For projects with time constraints and/or limited funding as of a PhD thesis and programme/course development, compromises might be needed in completing the whole analysis before starting to the next cycle because of the tight deadlines. This thesis included four studies in a three-year time. In order to maximise the time for the analysis, the data generation started in the first year. Moreover, the challenge of time delay during iterations was reduced by focussing on different perspectives in each cycle as explained in the previous paragraph.

3.1.5.1. Argumentative Grammar for DBR in Mathematics Education and this Thesis

Other than the issues above, there is a criticism concerning the existence of argumentative grammar in DBR. Kelly (2004) – who defines an argumentative grammar as "the logic that guides the use of a method and that supports the reasoning about its data" (p.118) – argues that argumentative grammar for DBR is not explicit and agreed-upon like that of more mature methodologies, such as ethnographies and randomised trials. Consequently, some researchers argue that design-based researchers "lack a basis for a warrant for their claims" (e.g., Kelly, 2004, p.119). However, many researchers have now contributed to and proposed argumentative grammars for design-based research in different fields (Bakker, 2018a; Cole et al., 2014; Penuel & Frank, 2015). A decade after Kelly's (2004) argument, Cobb et al. (2014) proposed an argumentative grammar for design-based research in mathematics classrooms.

The first step proposed was that research following a DBR approach should show that "students would not have developed the documented forms of mathematical reasoning but for their participation" in design-based research (p. 21). This step is clear because DBR seeks to explore innovative types of thinking that can be argued to rarely arise in traditional mathematics teaching. Therefore, the first study of this thesis was devoted to understanding current learning practices and its outcomes. It also looked for the errors made by the students in orthogonal and isometric drawings and the nature of these errors and current pedagogy which may have caused them. Students were interviewed on their performances and explained their reasoning when drawing the answers on the worksheet. Building upon the results of Study 1, Study 2 and consequent studies were conducted to explore and trial possible solutions.

Secondly, necessary aspects of the learning environment which have the potential to "support the emergence of successive forms of mathematical reasoning" should be identified (Cobb et al., 2014, p.24). Manipulated aspects of the classroom environment should be highlighted in the research report instead of unchanged/constant aspects. DBR should show the means to support the learning in various forms. This thesis proposes the RETA model with four principles (realistic, exploratory, technology-enhanced and active) for teaching 3D shapes in mathematics. It recommends integrating lesson plans which are supported with these principles for teaching orthogonal and isometric drawings of 3D shapes in the researched context.

Moreover, Cobb and colleagues (2014) describe the overall aim of DBR as improving students' learning by means of innovative methods together with new directions in educational technology. Hence, they put the design of the intervention together with an educational tool and a set of designed activities around it and suggest adding a clear description of the educational technology utilised in the designed mathematics lessons, if any. In accordance with this, the technology-enhanced principle of the RETA model refers to the strategic use of dynamic geometry tools in teaching orthogonal and isometric drawings of 3D shapes to provide multiple representations of them. The potential benefits and disadvantages of integrating dynamic tools are discussed in Section 5.2.3.

Finally, the findings of DBR should be potentially generalizable. This is difficult in a PhD thesis where a single researcher is conducting mostly small-scale research. In order to handle this limitation, Cobb et al. (2014) suggest that the context and procedures should be explained in detail. The intent for this is not preparing an environment for other researchers to replicate the study by following the same procedures but informing them about the procedures so that they can modify and use them according to the needs of their classrooms. Hence, in this thesis, the context of the research, all data collection materials and time spent in each phase of the data generation (e.g., time spent for the observation and interviews) are provided by giving examples in each study chapter.

To sum up, even though there are these issues, they were mitigated in the ways explained in this section leading me to conclude it is a good choice and design-based research fits the needs of my research which was intended to design lessons to improve Turkish middle school students' orthogonal and isometric drawing performance by providing insights into how it was already done/taught and how the practices could or should be (improved).

3.2. Methods

3.2.1. Data Generation

In all studies, the data generated through interviews, observations and students' work. In Study 2, students also completed lesson evaluation forms, however these were found not straightforward for them to complete and hence they were eliminated for the next iterations. The sample is strategically selected to match the research questions. The researcher conducted interviews with a) students in Study 2 to answer the third research question on students' experiences of her teaching and b) a teacher in Study 3 in order to answer the fourth research question about investigating the opportunities for a teacher and challenges of her.

3.2.1.1. Sampling

As explained in the literature, many researchers believe that the optimal age for training spatial ability for academic achievement is around twelve years old which corresponds to middle school age (Ben-Chaim et al., 1989; Piaget & Inhelder, 1967; Rafi et al., 2005), that is grade seven in Turkish schooling system. Considering this, the researcher had access to two public middle schools throughout her PhD and worked with the seventh graders in these schools. These schools in Turkey were the ones the researcher was already in contact with. This comes with both advantages and disadvantages. For example, this limits the possibilities to straightforwardly generalise the findings to other schools as they may not be representative. However, this disadvantage is weighed by the benefits of working in these schools. The researchers' prior relationship with the schools, where she did her teacher training high school internship and conducted the study of her MA dissertation, assisted her in sampling and in data generation as she knew the selected schools had enough technological infrastructure for the future cycles and enough number of students for a quasi-experimental study in the final cycle. There were three to four seventh grade classes in each middle school and about 25 to 30 students in each classroom.

Study 1 was conducted in both schools where two mathematics teachers in each agreed to be a part of the study and invited the researcher to observe their lessons while they were teaching two-dimensional representations of 3D shapes. Study 2 and 3 were conducted in one of these schools. Study 2 was an after-school course where the researcher acted as a teacher to teach the same topic to eight volunteer students. In Study 3, the intention was to collaborate with a teacher to adapt and adopt the lessons to use in her own class hence the number of students was increased to a class of students around 30. Finally, Study 4, which included intervention and control groups, was conducted in both of the schools including a number of teachers who volunteered to be involved. The characteristics of the participants of each study are described in detail in their own chapters.

The spatial ability which affects geometry performance is believed to be influenced by gender (e.g., see da Costa, 2017); therefore, gender was considered in all studies. An equal number of female and male students were interviewed in all studies. Moreover, an equal number of them was selected for a particular study where possible; for example, four female and four male students were chosen as participants from all volunteer students to participate in Study 2.

3.2.2. Data Analysis

3.2.2.1. Analysis of Quantitative Data

Parametric tests were chosen to analyse quantitative data and ANOVA was the main statistical test used in this thesis together with a number of post-hoc tests as mostly questions concerned differences between pre and post-tests. Data were checked for the various assumptions of parametric tests including independence of observations, homogeneity of variances and normality prior to the analysis.

One of the assumptions of ANOVA is data has a normal distribution. Social research, however, often creates non-normal datasets. On the one hand, authors such as Field (2013) suggest using non-parametric tests if the data is not normally distributed. However, using non-parametric tests decreases the power to detect an effect particularly whilst using small datasets (Coolican, 2014b, 2014a). Moreover, Glass, Peckham and Sanders (1972) found that skewed distributions which are analysed using ANOVA have almost no impact on significance levels or on statistical power if the test is two-tailed and the kurtosis and skew are within certain recommended limits. Harwell, Rubinstein, Hayes and Olds's (1992) meta-review on using ANOVA with non-normal distributions found considerable support for such an analysis to be robust to non-normal data given that variances are equal.

The Shapiro-Wilk test is used in this thesis to test normality when using ANOVA (Shapiro & Wilk, 1965). The group variance, skew and kurtosis of the distributions were considered when the variables differed significantly from normality. The analysis proceeded if measurements were within the limits of Glass et al.'s (1972) paper. In addition, although outliers have been shown to influence parametric tests (Zimmerman, 1994), they were not removed unless they were considered extreme and unless there is a significant difference between results with and without the outliers (Kruskal, 1960).

Finally, the agreement between the raters for coding of the worksheets was investigated by Cohen's Kappa after meeting the assumptions of the test (Cohen, 1960). These assumptions were: using nominal scale coding rubrics, drawing data from paired observations (repeated measures design), having the same number of categories for each variable, providing independent scoring of two raters and same two raters' judging all observations in a study (Carletta, 1996).

3.2.2.2. Analysis of Qualitative Data

A thematic coding strategy was used to analyse observation and interview data. Although it is widely used in education, it is the term can be problematic as it has also been applied to other methods (e.g., discourse analysis and content analysis) or it is not identified as a method at all and hidden under the words of qualitative analysis (Braun & Clarke, 2006).

In this thesis, a thematic coding strategy refers to the active role of the researcher's identification of the patterns in data, selection of the ones that are of interest and their reports (as in Taylor & Ussher, 2001). This strategy was chosen as it provides rich and detailed data analysis and it is flexible, in a sense that it could be used in both methods stemming from a particular theory and those independent of a theory or aiming for one (Braun & Clarke, 2006). Through this freedom, it fits design-based research that includes a design which at least partly based on a theory or a theoretical framework and generates or perhaps proposes a humble theory (van den Akker et al., 2006b). Moreover, because a detailed theoretical understanding of approaches like those of grounded theory and discourse analysis is not required for the thematic coding strategy, it provides a more accessible and flexible style of analysis which is suitable for novice researchers in qualitative analysis (Braun & Clarke, 2006). Finally, any philosophical worldview with its assumptions of the nature of data can orient a thematic coding strategy. In other words, this strategy can be shaped to different theoretical orientations which should be made clear in the analysis. In this thesis, it is pragmatic or perhaps pragmatic post-positivist orientation which guided the thematic analysis of the data as expected from a DBR project. All this freedom sometimes brings the critique of *anything goes* (Antaki et al., 2003) Hence, this thesis followed the guidelines of Braun and Clarke (2006) throughout the analysis.

In Study 1, this included the inductive process of open coding to analyse and interpret data. As Marshall and Rossman, (2011) suggest, the analysis procedure followed the sequence of organising and coding the data, generating themes, testing understanding and searching for alternative explanations and writing the report. All interview transcripts were analysed for meaningful statements, significant phrases and sentences directly related to the interview questions, which were then contrasted with lesson observations and significant actions of students and the teacher by the researcher. In Study 2, the researcher deductively analysed the data and looked for the experiences of and outcomes for students, particularly focusing on the RETA principles. That is, she looked for whether and how the RETA principles are working in the lessons through how they impacted students' experiences and outcomes. Individual extracts of data (a sentence or a sentence group) were coded as many different themes as they fit into therefore some extracts coded only once, some more than once and some were not coded. Analyses of Study 3 looked for how these principles are experienced by a teacher and what were the outcomes of them for the students. Analysis scheme was described in detail in the particular study chapters.

The interpretation of data raised two main issues which have also been discussed by early DBR researchers. One of them was the consequence of the researcher's having different roles such as intervention designer and the researcher who evaluates the intervention that is somehow related to the first one (Collins, 1992). The proposed solution to this is having separate researchers for different roles. However, as explained earlier, the researcher carried out all of the roles because it was necessary for a PhD thesis. To minimise this risk arising from this issue, she actively searched for negative and contradictory evidence throughout the analysis.

The second issue was the Bartlett effect, which is the bias that happens when the researcher selects from the data only the parts confirming her earlier proposal (Brown, 1992). In order to minimise this, the researcher followed three strategies suggested by Robson (2011) for improving the quality of the analysis. Firstly, she used triangulation techniques which could be described as looking for evidence from different sources to see whether they confirm or supplement each other. For example, if the researcher took an observation note on a students' struggle on a particular activity during a lesson, evidence about this from the interview with the same student and from the student's lesson evaluation form was checked to see whether they point toward the same conclusion. In addition to using triangulation techniques, she particularly looked for negative evidence and noted the contradictory examples in the data (to what she was concluding) during the analysis. She challenged herself by looking at negative evidence and contradictory examples and included them in the findings. Finally, the technique of weighing the evidence was used. The researcher looked at the frequency of evidence as to the number of participants. For instance, whether eight participants share a similar experience or only one was taken into account. Although the generalisation is not possible with the number of participants available, this technique is still helpful given the study's scope.

3.2.3. Presentation of Findings

There are a number of common features to how the research is presented in each study. Some are outlined below:

Each study chapter includes a brief introduction, methods, results and summary of results. Study 2 was the first empirical study which the intervention was trialled. Therefore, starting from this chapter, chapters on the subsequent studies include a design changes section which describes refinements to the design of the intervention and the measuring instruments. As the nature of Study 1 is different from the other three studies (analysis of existing practice, not intervention), it does not include a design changes section, instead, it is followed by a chapter on design principles and sample lesson plans.

3.3. Ethical Issues

All research in this thesis was conducted within the guidelines of the University of Nottingham's Code of Research Conduct and Research Ethics (The University of Nottingham, 2016) and the School of Education in particular which has adopted Revised Ethical Guidelines for Educational Research (BERA, 2011). All information collected was anonymised, confidential and only available to the researcher and her supervisors. Pseudonyms were used throughout the studies to replace teachers' and students' real names.

Particular ethical issues related to each study and ethical permission numbers from the University of Nottingham are noted in the following sections.

3.3.1. Working with Children and Teachers

There were some ethical issues regarding working with children and teachers. The following describes these issues and how they were handed in this thesis:

- **Working with children:** First of all, the research involves work with children (delivery of lessons, discussion and interviews) which might bring some risks such as students might feel uncomfortable or not safe (Fraenkel et al., 2015). However, being a qualified middle school mathematics teacher who worked in middle schools for a few years, the researcher had experience in working with middle school students so she understands the practices and protocols of working within the school environment and with middle school students. Moreover, the students who were involved in the research worked in a school setting which is familiar to them and they were ensured that they could withdraw from the study anytime without any consequence if they would like to so.

- **Children volunteering to be a part of the lessons:** The researcher only asked volunteers to be a part of all cycles of the research as suggested (Berg, 2001). Yet, not all students who volunteered were able to part of the research in Study 2. Study 2 was the first trial of the intervention. It offered an after-school course run by the researcher. The intent was to test the intervention with a small number of students in detail before collaborating with a teacher to use it in her regular lessons. Hence, this trial of the intervention was with eight students and investigated their experiences of the lessons and outcomes of the lessons for them in detail. This situation was handled by collaborating with the mathematics teacher of the students. The teacher informed the researcher that there were ongoing after-school classes in the school for some of the lessons including mathematics. These classes were open for those who need and/or want to do extra practice. Hence, students who were already registered for the maths classes were not chosen for the study so that they can continue their existing classes, and all remaining students being four girls and four boys were accepted for Study 2 – despite the first intention was to work with six students. Moreover, volunteer students had the right to withdraw themselves from research at any point, as well as schools', teachers' and parents' having the right to withdraw the children.
- **Children having to discuss with the researcher about her teaching:** The researcher acted as the teacher of the after-school course. At the end of the course, she conducted one-to-one interviews with students and some of the interview questions asked students' comments on her own teaching. Researching about the researcher's own teaching might bring the risk that students could feel nervous or uncomfortable (Mercer, 2007). Hence, the researcher stressed to them that they would not be judged for their answers,

this was not a test and it was important for her to understand their experiences to design better lessons to help them.

- **Teachers not being judged or criticized:** The thesis included teachers as participants in Study 1, 3 and 4. Teachers were asked about their teaching experiences, the methods and strategies they use to teach 3D shapes and the reasons for their choices during the interviews. Interviews with the teachers who adapted the RETA-based lessons included questions about their experiences of teaching with these lessons, any problems faced and their actions to overcome these problems. These questions might bring the risk of teachers feeling judged or criticized. Nonetheless, being a mathematics teacher herself, the researcher shared her experiences and talked about her teaching journey including both good and bad experiences at the start to make the teachers feel comfortable as in Mullings's (1999) study. By doing so, she followed the suggestions of early researchers who argue that sharing experiences both helps to develop trust between the interviewer and the interviewee (Logan, 1984; Oakley, 1981) and encourages the interviewee who has the opinion of “I will show you mine, if you show me yours” to share their experiences more openly (Mercer, 2007, p.8). She also ensured the teachers that they would not be judged for their answers and the answers would only be used for research purposes.
- **The formal nature of the system:** The researcher started working for the Ministry of Turkish National Education in 2014 and she was funded by the ministry for her PhD research. After successful completion of her degree, the researcher will be promoted to work as an education expert at the ministry. Knowing that she came from the ministry, headteachers and teachers might feel obligated to be a part of the research because of the status of the researcher as a ministry officer (Merton, 1972). As Drever (1995) says, “people’s willingness to talk to you and what people say to you are influenced by who they think you are” (p.31). This was handled by asking their consent, as well as their own willingness to volunteer. Even if the ministry was the gatekeeper, the participants were not affected by the gatekeeper in any manner. They had the right to say no both at the beginning and during the research. The head teachers and teachers were volunteered to be a part of the research and gave

their consents. All head teachers involved in the research were ensured that they could withdraw their school from it anytime without any consequence if they would like to; the same applied for the teachers involved in the research as well. In Study 1, four teachers out of six volunteered to be in the study and agreed for their lessons to be observed during their regular teaching of 3D shapes. In Study 3, a volunteer teacher adapted the RETA-based lessons and agreed for her lessons to be observed. In Study 4, nine teachers volunteered to be in the study and four of whom also volunteered to adopt the RETA-based lessons whilst other five teachers chose to be in the control group and continue their normal practice.

- **Recording the lessons and interviews:** Lessons and interviews were recorded during all four studies. Considering that some of the teachers who want to be in the study may not want to be recorded, hence in the later cycles, separate tick boxes were added to consent forms for being in the study and for being recorded during the lessons and for being recorded during the interviews. The researcher only recorded the lessons of the teachers who gave their consent for it. She took observation notes in five-minute intervals in cases that she could not get teachers' consent for recording the lessons. Moreover, the teacher in Study 3 gave her consent for the recording during interviews at the beginning but she did not want to be recorded during the interview on the use of GeoGebra. This request was accepted by the researcher and she was not recorded during this particular interview. Similar tick boxes were used in the parent consent forms, too. This procedure was particularly helpful when recruiting participants who might otherwise refuse to take part in the study (Berg, 2001).

3.3.2. Approval from the Ethics Committee

The Ethics Committee of the School of Education at the University of Nottingham approved all of the studies of this thesis. This section summarises the steps to receive approval.

Before the submission, the researcher needed to obtain a Disclosure and Barring Service (DBS) check to have access to schools. She previously had received it with an F0104171462 reference number. After a discussion with the head of the ethics committee, it was decided that the researcher did not need another one for further studies. Instead, she needed its equivalent from Turkey and collected a valid, up-to-date certificate of good conduct from there separately for each study. This showed the committee that the researcher does not have any criminal or problematic records.

After obtaining this, the researcher submitted the ethics form of the University of Nottingham together with the necessary documentation. These included three types of documents: certificate of good conduct, research instruments (interview questions, observation protocol, sample activities and sample worksheet questions) and participant information sheets together with the consent forms. Participant information sheets (Appendix A) and consent forms (Appendix B) were separately prepared for the head of the schools, teachers, parents and guardians and children. Especially, for the children, the language of the forms was simplified and was made easy to understand for them.

Study 1 did not cause any concerns for the committee. It was a case study which observes how teachers teach 3D shapes in regular classrooms and there was no interruption of the researcher to the naturally occurring case. The ethics committee only asked the researcher to remove a marginal comment which was mistakenly forgotten while submitting the ethics documents. Ethics was received on May 23rd, 2017; Ref: 2017/64 (Appendix C).

Study 2 was based on the implementation of the intervention in a class environment, and the researcher as the after-school mathematics teacher. Lesson activities were not harmful and there was not any sensitive topic for the children. The study included completion of worksheets and interviews with students on their experience of learning 3D shapes including questions asking their comments on the researcher's own teaching. Students were ensured that there would not be any consequence of their comments other than the improvement of the lessons. There was not an issue raised by the ethics committee about the submitted documents. The ethics committee commented that this was an exemplary ethics submission. Ethics was received on October 19th, 2017; Ref: 2017/94 (Appendix D).

Study 3 was almost the same as Study 2 except for a collaboration of a teacher to adapt and deliver the lessons rather than the researcher. Therefore, it required only an amendment and no separate ethical permission. The researcher sent the interview questions with the teacher and the amendment in the information sheet and consent form where a teacher was expected to teach the lesson in a regular maths classroom instead of the researcher teaching it in an after-school course. No issue was raised by the ethics committee. Amendments for this study were submitted together with the previous ethics submission hence no further action was needed.

Another amendment was required for Study 4 as it included a number of teachers who volunteered to be in intervention and control groups. It was also the time the university started including the European Union General Data Protection Regulation (the EU GDPR) in the ethics form. The researcher declared in the ethics form that she familiarised herself with the GDPR and carried out the research complying with it. No issue was raised by the ethics committee. Amendments were accepted on September 20th, 2018.

All the ethical procedures described in the documentation were approved by the committee and were followed in practice. In addition to these, all of the instruments were translated into Turkish and got the approval of the Ethics Committee of the Directorate General of Innovation and Educational Technologies of Ministry of Turkish National Education.

4. STUDY 1: AN INVESTIGATION OF STUDENTS' LEARNING OF 3D SHAPES

The current study explored middle school students' progress within the context of learning two-dimensional representations of three-dimensional shapes. The aims of this study were investigating how 3D shapes currently taught in natural classroom settings in Turkish schools in order to better understand the reasons underlying the difficulty in representing 3D shapes (that was largely reported in the literature). Specifically, this chapter tries to answer the following research question.

1. How do the seventh-grade middle school students learn 3D shapes in Turkey?
    a. What are the students' difficulties in learning about 3D shapes?
    b. What are the students' errors in representing 3D shapes?

4.1. Methods

4.1.1. Participants

The study was conducted in two public middle schools (explained in Section 3.2.1.1 Sampling) where two mathematics teachers from each school agreed to be a part of the study and invited the researcher to observe their lessons while they were teaching two-dimensional representations of three-dimensional shapes. These teachers volunteered as they had an interest in developing their understanding and practices within their own school context.

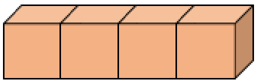
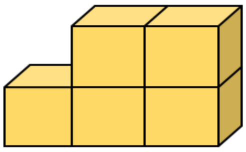
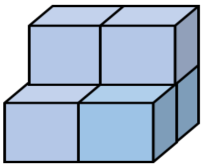
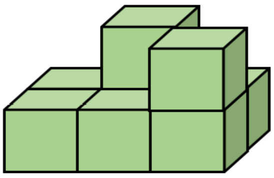
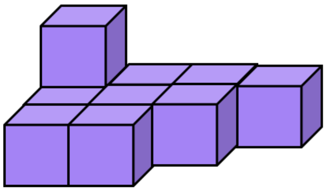
The student sample was drawn from the seventh grade middle school students within these two schools. The sample consisted of 199 students (107 females and 92 males) aged between 12 and 14 years. In-depth understanding of students' reflections on the lessons and their reasoning and solution strategies was generated by inviting 16 students (8 females and 8 males) to an artefact-based interview. The interviewed students were selected randomly from those who answered at least two questions of the worksheet correctly to ensure that they have the basis of the knowledge to answer the questions in the interview.

4.1.2. Materials

Worksheets had ten questions in total with two types of two-dimensional representations of three-dimensional polycubical shapes and five questions from each. The first of these was about constructing orthogonal drawings of the given isometric drawing of a polycubical shape (orthogonal drawing questions) and the second half of the questions were about constructing an isometric drawing corresponding to the given orthogonal drawings of a polycubical shape (isometric drawing questions).

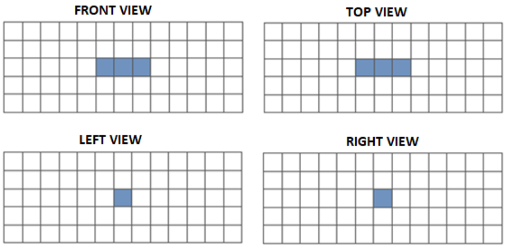
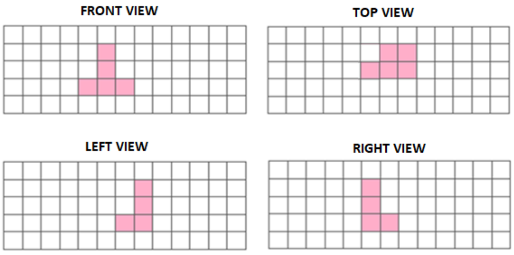
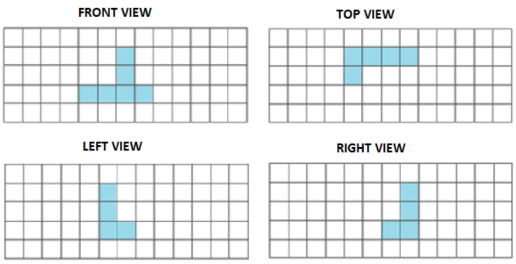
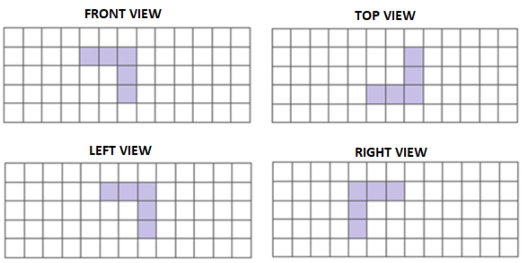
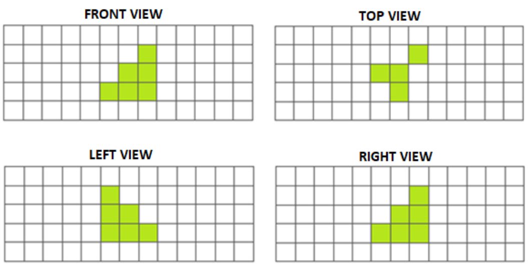
Orthogonal drawing questions asked students to draw the orthogonal views from the top, front, left and right on a squared paper. The first question was purposefully asked as an easy question as a warm-up and was therefore judged as easier than ones on the ministry exams (available online, Ministry of Turkish National Education, 2016a). What makes questions more or less difficult involves multiple factors. Two possibilities are the change in the number of cubes and the change in-depth, and therefore the orthogonal drawing questions are organised accordingly to increase in complexity according to these factors. Table 4.1 shows the shapes on the questions and summarises the height and the depth of the shapes and the least number of unit cubes needed to construct them.

Table 4.1. Orthogonal drawing question items

Question	Height	Depth	# of Unit Cubes
1 – 	1 unit	1 unit	4
2 – 	2 units	1 unit	5
3 – 	2 units	2 units	6
4 – 	2 units	2 units	8
5 – 	2 units	3 units	10

Isometric drawing questions asked students to construct isometric drawings corresponding to given polycubical shapes' orthogonal drawings from the top, front, left, and right on an isometric paper. Isometric drawing questions started with an easy question as warm-up as above. The number of cubes was fixed to seven after the first question and only the places of the cubes changed to create another isometric drawing question. The difficulty of these questions can also be increased in various ways. Two possibilities are increasing the difficulty by the changing height and depth with the same number of cubes and therefore, the isometric drawing questions organised accordingly. The isometric drawing questions have a fixed number of cubes with an increase in height. Table 4.2 demonstrates the questions and summarises the height and the depth of the shapes and the least number of unit cubes needed to construct them. For each question, students are scored out of four for both orthogonal and isometric drawing questions, and partial credits are given for separate views. The scoring is explained in Section: 4.1.3.2 Worksheets in more detail.

Table 4.2. Isometric drawing question items

Question	Height	Depth	# of Unit Cubes
1 – 	1 unit	1 unit	3
2 – 	2 units	3 units	7
3 – 	3 units	2 units	7
4 – 	3 units	3 units	7
5 – 	3 units	3 units	7

4.1.3. Data Generation

The data were generated through observations, worksheets, and interviews.

4.1.3.1. Observations

The participants in the study were observed for four lesson hours during their regular mathematics lessons while they were learning how to construct 2D representations of polycubical shapes. Each lesson was 40 minutes in duration. Thus, in total 16 lessons (four classes x four lessons), were observed by the researcher as a non-participant observer. The researcher sat at the back of the classroom with her laptop and took field notes in five-minute intervals. A sketch of each classroom was drawn prior to the field notes. An observation protocol with descriptive and reflective observation notes in separate columns was used to structure the field notes during classroom observations (Creswell, 2007). In the descriptive notes, observations related to the classroom environment and students' and teachers' actions were noted. In the reflective notes, the researcher noted her comments and opinions on the actions taken. Copies of the materials used during the lessons (presentations, activity sheets, and book pages) were also collected as additional data.

4.1.3.2. Worksheets

Students were asked to complete a worksheet after the last teaching sessions (Appendix E). They were given two lesson hours (80 minutes) to complete the worksheet. The worksheet questions were adapted by the researcher from the past ministry middle school exam questions. The questions were ordered in a manner where they were getting more difficult to construct in the next question. They were piloted with 16 students (8 females and 8 males) from one school and same age group and necessary changes made prior to the study.

4.1.3.3. Interviews

The interviews were conducted in students' schools after the students' completion of the worksheets and were designed to allow in-depth exploration of their perspectives on and understanding of the topic. The researcher acted as an interviewer during the interviews. Interviews were audio-recorded to allow for transcription to be used during the analysis. They took 15 to 30 minutes to complete.

The interview process was started with participants being asked to consent to be involved in the study in addition to their parents' consent. Students who have decided to take part were reminded that the interview would be recorded. Then, participants were re-informed of the purpose of the research and were given a general introduction about the researcher. After the introduction, participants were encouraged to talk on minor topics as a warm-up, such as what they have been doing on their favourite holiday and introducing themselves and their families. Some of the warm-up questions addressed students' background characteristics.

- Where were you born? Can you tell me about the city?

The main interview included two types of questions. The first half of the interview questions were prepared to explore students' opinions about and experiences with 2D representations of 3D shapes and the difficulties they faced when learning. For example, the following question asked students to rate the difficulty of constructing 2D representations of 3D shapes on a ten-point scale, in which one is too easy and ten is too difficult. After that, the students were asked to explain the reasons behind their choices and their suggestions to make the topic easier to understand if they found it difficult.

- For the last four lessons, you have worked on 2D representations of 3D shapes. Was it a difficult topic compared to other topics in mathematics? Let's assume a scale in which one is too easy and ten is too difficult, where about is this topic out of ten?
- You said you think it is a difficult topic to learn, why, what do you think makes it difficult to learn? What do you think should change to make the topic easier? OR You said you think this is an easy topic to learn but some parts are difficult, which parts are they? What do you think should change to make those easier?

There were also further questions about their understanding that aimed at exploring their verbalising of their understanding.

- If you were to tell what you have learnt about 3D shapes to somebody who has not studied this unit yet, what would you say?
- What could you tell the next year's class about the relationship between left and right views of the same 3D shape? How did you use this information during the last four lessons?

The second half of the interview questions were based on the students' drawings on the worksheets, completed after the last teaching session on 2D representations of 3D shapes. They were developed to help the researcher understand students' reasoning behind the incorrect answers. At this point, participants were reminded that this was not a test and they could say what they think without any hesitation. After that, they were provided with the worksheets they completed beforehand and asked to explain their strategies to draw the required 2D representations. The researcher took notes during the interviews to record the actions of the interviewees, e.g., pointing to a specific part of a shape. Below is an example statement which was used to start a discussion about a question in the interviews.

- Please talk me through how you have decided where and how many cubes to draw in this question.

At the end of the discussion of all problematic questions on the worksheet, participants were asked whether they have anything to add. The interviews ended with the participant being thanked for the involvement in the research.

All materials (consent forms, information sheets, worksheets, and interview questions) were translated into and used in Turkish which is the researcher's and participants' mother language.

4.1.4. Data Analysis

4.1.4.1. Observations and Interviews

The first research question answered how seventh-grade middle school students learn two-dimensional representations of 3D shapes in Turkish classrooms. To be more specific about the research addressed by this study, two sub-questions were defined.

- What are the students' difficulties in learning about three-dimensional shapes?

This question was necessary to understand students' difficulties so that the researcher could design better lessons to improve their understanding of the topic. The researcher followed a systematic and rigorous analysis of the observation and interview data. A thematic analysis of observation and interview data was carried out as the purpose was to capture student experiences when they learn two-dimensional representations of polycubical shapes following the approach described in Chapter 3. This included the inductive process of open coding to analyse and interpret data. As Marshall and Rossman, (2011) suggest, the analysis procedure followed the sequence of organising and coding the data, generating themes, testing understanding and searching for alternative explanations and writing the report. All interview transcripts were analysed for meaningful statements, significant phrases and sentences directly related to the interview questions, which were then contrasted with lesson observations and significant actions.

In order to preserve the validity of interviews, audio-recorded interview data were transcribed, in Turkish. The transcribed interview data were coded by the researcher, then 10% of it was blind coded and back-translated by another researcher in the field. Moreover, to help increase validity, peer evaluation and member checks were used in this study. The first strategy was peer evaluation. Despite the fact that the researcher started the coding independently, she worked with two other researchers to discuss the analysis of her study after the open coding stage. They listened to and commented on the researcher's evaluation of the data, and validated the emerging themes. Disagreements were solved through discussion. Member checking was used as the second strategy. During the interview, the researcher paraphrased the sentences of the interviewees to ensure that they shared the same understanding. The researcher was also aware of the possibility of reactivity, which may occur when the participants behave differently than they normally do with the awareness of being observed. The researcher attended their lessons for four hours prior to the actual observation and both teachers and their students had time to get used to her presence in their classrooms.

Moreover, although all of the participants knew that the researcher was interested in technology-enhanced geometry lessons, none of them attempted to use any digital software in their lessons and they followed their previously designed lesson plans so hopefully the change in their actions to impress the researcher was minimal. The researcher was also aware that she interpreted the data according to her knowledge and understanding and how she was receiving the data as a pattern.

The discussion of the findings is organised into two sections: Sections 4.2.2 Students' Perceptions of their Performance and the Challenges they faced and 4.2.3 Current Pedagogy and Students' Learning Experiences. Section 4.2.2 explains the difficulty theme where the difficulty of representing three-dimensional shapes in two-dimension through orthogonal and isometric drawings discussed. The particular difficulties appeared to be visualisation and drawing, which constructed the codes of this theme.

Section 4.2.3 describes the teaching practices theme where current pedagogy and teachers' existing teaching practices are described focussing on similar and different activities in each phase of the lessons. Table 4.3 presents themes, codes and subcodes of Study 1.

Table 4.3. Themes, codes and subcodes of Study 1

Themes	Codes	Subcodes
Difficulty	Visualisation	
	Drawing	
Teaching practices	Use of manipulatives	Teacher use Student use
	Exam-focused pedagogy	
	Use of technology	Teacher use Student use

4.1.4.2. Worksheets

The second sub-question addressed students' errors in representing three-dimensional shapes.

- What are the students' errors in representing three-dimensional shapes?

Students' worksheets were scanned to produce an electronic copy for the data analysis. A rubric with all possible correct drawings for each of 10 questions was used to analyse them (Appendix E). The completed worksheets were coded by the researcher. No points were given for either incorrect or not attempted answers and one point was given for correct answers for each aspect of an item. That is, each question is scored out of four for different views: front view (one point), top view (one point), left view (one point), and right view (one point). Therefore, for each question, students are scored out of four for both orthogonal and isometric drawing questions. Another expert in the field also coded 20 random worksheets out of 199. The agreement between the raters for the coding tested using *Kappa* and found to be Kappa= 0.97 (p<.001), suggesting a very high agreement. The worksheets were also coded for the nature of the mistakes (described in Section 4.2.1 and exemplified in Figure 4.2 and Figure 4.4).

There were only 10 different descriptions out of coded 124 student mistakes in the jointly coded 20 worksheets, and disagreements between the raters were solved through discussion. The interview data were also used as complementary data to explain students' reasoning and solution strategies when completing the worksheets.

4.1.5. Ethics

The University of Nottingham approved the research ethics of this study on May 23rd, 2017; ref: 2017/64 (see Appendix C). The researcher considered all issues related to anonymity, privacy and data security. All named participants were given pseudonyms.

4.2. Results

This section starts with the analysis of students' worksheet performance with a further focus on students' common errors in orthogonal and isometric drawings of 3D shapes. It continues with students' perceptions of their performance and their challenges. It then represents current pedagogy which could have caused these. The section ends with the summary of the findings which leads to the proposed model.

4.2.1. Analysis of Students' Worksheet Performance

Before going into the detailed investigation of the errors in each type of drawings, a mixed measures ANOVA was conducted to examine the effect of question type (orthogonal and isometric) and gender (female and male) on students' performance. The dependent variable was the students' performance which was the summed scores of the orthogonal drawing questions and isometric drawing questions, with a possible range of 0-20 (see Table 4.4).

Table 4.4 shows the participants' mean scores split by question type and gender. The ANOVA results show a significant effect of question type on students' performance, $F(1,197)=265.255$, $p<.001$, $\eta_p^2=.574$. The analysis revealed a significant difference in students' orthogonal drawing scores and isometric drawing scores with a large effect size, that means students performed better on the orthogonal drawing questions ($M=11.92$, $SD=6.31$) than the isometric drawing questions ($M=5.27$, $SD=5.58$). However, there was no significant effect of gender, $F(1,197)=2.108$, $p=.148$, $\eta_p^2=.011$ and no significant interaction between question type and gender, $F(1,197)=2.239$, $p=.136$, $\eta_p^2=.011$.

Table 4.4. Test scores for orthogonal and isometric drawing by gender

		Orthogonal drawing(/20)		Isometric drawing(/20)	
Gender	n	*M*	*SD*	*M*	*SD*
Female	107	12.14	6.54	6.06	5.98
Male	92	11.66	6.06	4.35	5.42
Total	199	11.92	6.31	5.27	5.78

Students found it challenging to construct both orthogonal drawings and isometric drawings –even though they performed better in orthogonal drawings. Students' most common mistakes in orthogonal drawings were categorised as redrawing the given shape as the front or a part of it as a side view (E1), drawing the cubes at the back onto another column (E2), drawing the part only at the very front (E3), swapping the left and right views (E4) and drawing the view upside down (E5). All of these mistakes are exemplified below in Figure 4.2 using sample student answers to Question 3 in orthogonal drawings on the worksheet. This question was purposefully chosen to present as it has a medium level of difficulty out of five questions. As a reminder, Question 3 in the orthogonal drawings asked students to draw orthogonal views (i.e., the views from the front, top, left and right respectively) of the blue polycubical shape, whose correct answer is in Figure 4.1.

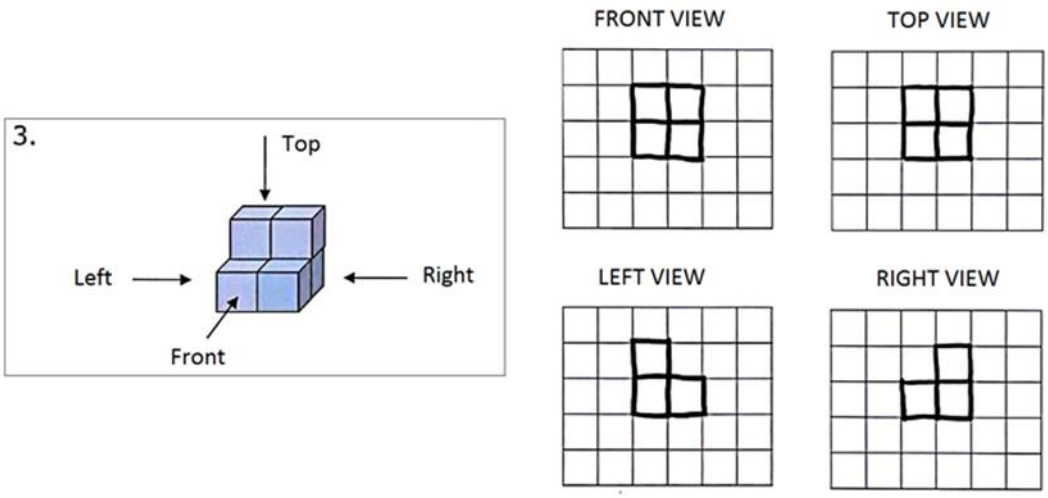


Figure 4.1. Q-3 in orthogonal drawings and its correct answers

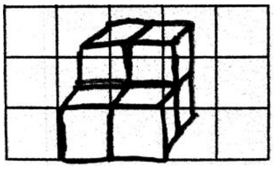
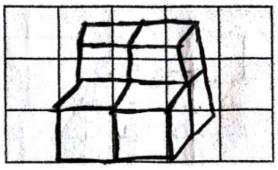
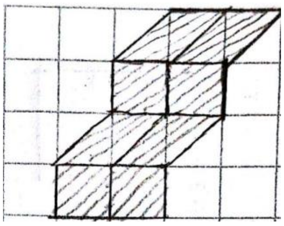
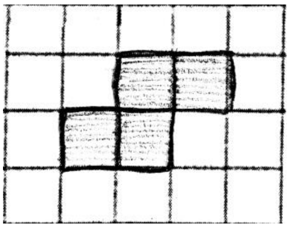
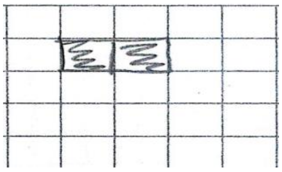
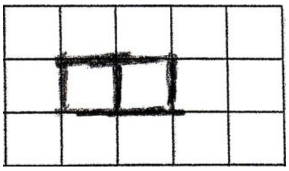
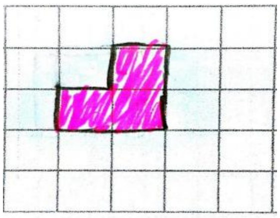
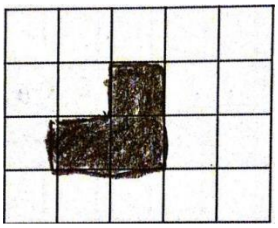
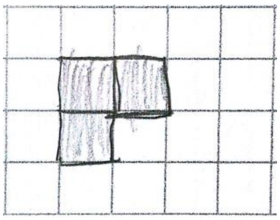
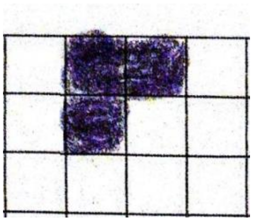
Description	Example student mistakes	
Redrawing the given shape as the front or as a part of it as a side view (E1)	Front View 	Front View 
Drawing the cubes at the back onto another column (E2)	Front View 	Front View 
Drawing the part only at the very front of the shape (E3)	Top View 	Top View 
Swapping the left and right views (E4)	Left View 	Left View 
Drawing the view upside down (E5)	Left View 	Left View 

Figure 4.2. Orthogonal drawing errors

Table 4.5 shows the number of errors for orthogonal drawing questions by above error types and by gender. The numbers next to the questions show the number of incorrect answers out of the number of students attempted (e.g., 50 students answered Q1 incorrectly out of 199 students attempted to draw it). The sum of the number of errors (E1 + E2 + E3 + E4 + E5) may not be equal to the total number of students who made these errors (e.g., 61 ≠ 50 for the Q1, 140 ≠ 124 for the Q2). The reason for this is that some students made more than one type of error and their incorrect answers coded for all error types. Moreover, given research reviewed in section 21.4 on gender, the table presented the number of errors made by females and males separately. As presented in Table 4.5, both genders made a similar number of errors in different error types in orthogonal drawings.

Table 4.5. Number of errors for orthogonal drawing questions by error type by gender

		n	E1	E2	E3	E4	E5
Q1 (50/199)	Female	107	16	N/A	9	N/A	6
	Male	92	13	N/A	11	N/A	6
	Total	199	29	N/A	20	N/A	12
Q2 (124/194)	Female	104	12	18	30	N/A	9
	Male	90	14	20	26	N/A	11
	Total	194	26	38	56	N/A	20
Q3 (106/192)	Female	104	10	6	31	12	3
	Male	88	12	7	16	13	2
	Total	192	22	13	47	25	5
Q4 (117/185)	Female	101	5	17	38	4	3
	Male	84	8	20	36	3	3
	Total	185	13	37	74	7	6
Q5 (151/188)	Female	100	8	23	21	10	13
	Male	88	7	22	28	5	21
	Total	188	15	45	49	15	34
Totals*	Female		51	64	129	26	34
	Male		54	69	117	21	43
	Total		105	133	246	47	77

*Totals were calculated by adding the number of mistakes in a particular error type in all questions; for example, total female error in error1 was calculated by adding the number of females' errors in Q1 to Q5 (16+12+10+5+8=51).

Redrawing the given shapes (E6), drawing only one of the views isometrically (mostly the front view) (E7), and constructing a drawing which combines the given views side by side either orthogonally or isometrically (E8) were found to be the most common errors in the isometric drawings (see Table 4.6). In addition to these, there were many mistakes because of the linking problems in drawings, which were also important to mention (E9). The following figure illustrates two student drawing examples of each common mistake and also linking problems (Figure 4.4). While the first column of the figure shows descriptions of the errors, the second and third columns show sample student errors. Similar to the choice of orthogonal drawing question, sample student answers of Question 3 on the isometric drawings was purposefully chosen to present as Q-3 has a medium level of difficulty out of five isometric drawing questions.

As a reminder, Question 3 in the isometric drawings asked students to construct an isometric drawing which combines given blue shaded orthogonal views (i.e., the views from the front, top, left and right) in Figure 4.3. This figure also shows possible correct answers to the question.

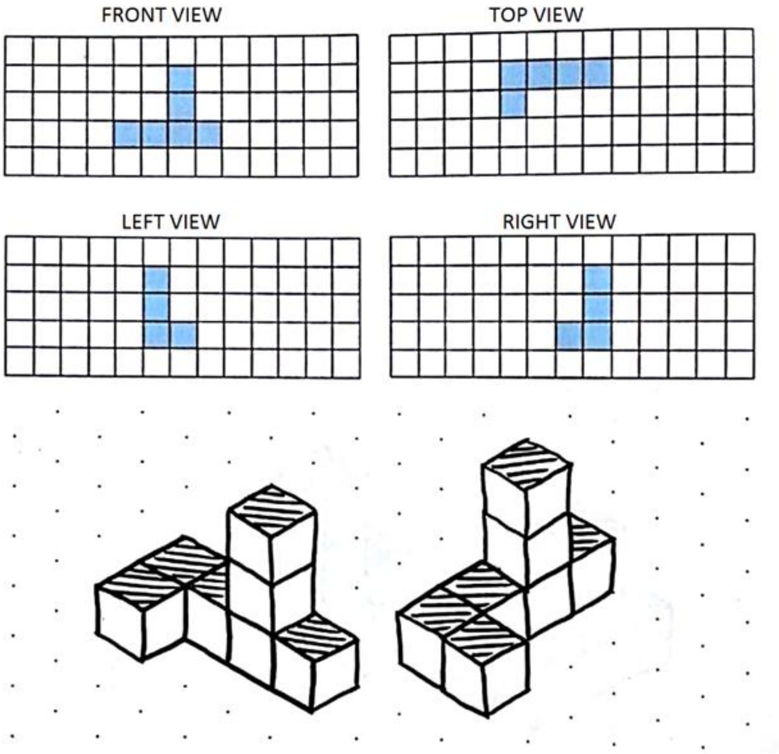
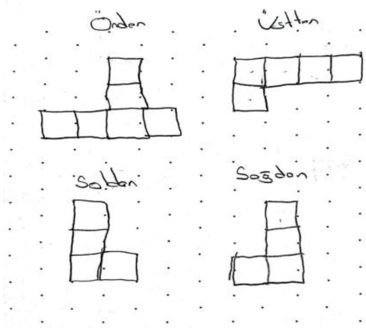
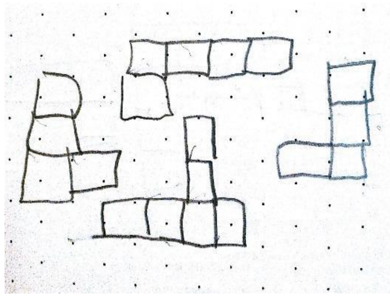
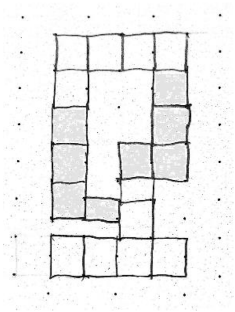
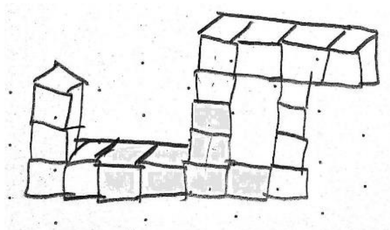
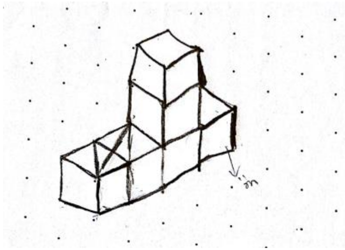
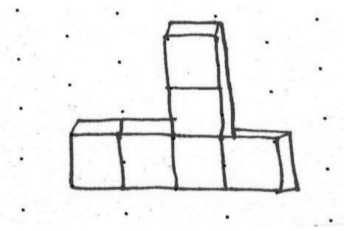
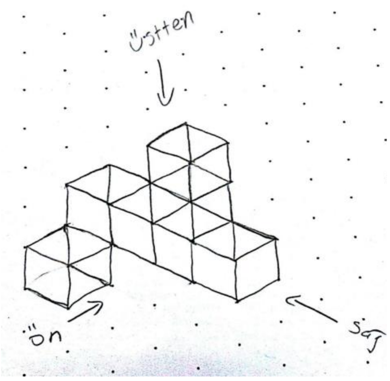
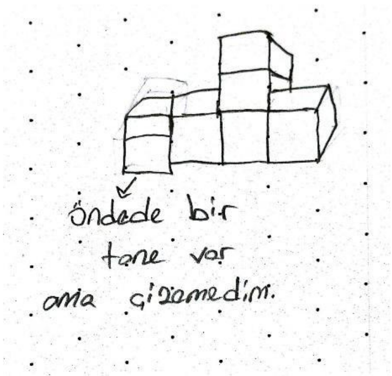


Figure 4.3. Q-3 in isometric drawings and its possible correct answers

Description	Example Student Mistakes	
Redrawing the given shapes (E6)		
Combining the orthogonal views side by side (E7)		
Drawing only one of the views (E8)		
Linking problems (E9)		

*Önde de bir tane var ama çizemedim (TR). = There is one more (cube) in the front but I couldn't draw it (EN).

Figure 4.4. Isometric drawing errors and linking problems

Table 4.6 shows the number of errors for isometric drawing questions by error type and by gender. Similar to the orthogonal drawings, the numbers next to the questions shows the number of incorrect answers out of the number of students attempted (e.g., 56 students answered Q1 incorrectly out of 190 students attempted to draw it). The sum of the number of errors may not be equal to the total number of students who made these errors as above. Again, similar to the orthogonal drawings, given the interest in gender, the table presented the number of errors made by females and males separately. As presented in Table 4.6, both genders made a similar number of errors in different error types in isometric drawings. The biggest difference was with 80-64=14 (more errors made by male students) in error 6, which corresponds to redrawing the given orthogonal drawings.

Table 4.6. Number of errors for isometric drawing questions by error type by gender

		n	E6	E7	E8	E9
Q1 (56/190)	Female	103	8	N/A	11	8
	Male	87	12	N/A	11	9
	Total	190	20	N/A	22	17
Q2 (136/175)	Female	93	18	14	22	18
	Male	82	13	19	21	16
	Total	175	31	33	43	34
Q3 (131/167)	Female	85	14	10	26	19
	Male	82	18	8	29	14
	Total	167	32	18	55	33
Q4 (115/151)	Female	79	13	10	23	11
	Male	72	19	14	18	16
	Total	151	32	24	41	27
Q5 (136/155)	Female	80	11	19	15	23
	Male	75	18	20	15	16
	Total	155	29	39	30	39
Totals	Female		64	53	97	79
	Male		80	61	94	71
	Total		144	114	191	150

Moreover, the mistakes in both orthogonal and isometric drawings were not consistent across all answers of a student and they were sometimes not consistent within the answers of one orthogonal drawing question of a student, probably since some students have two or more errors in a question at the same time.

4.2.2. Students' Perceptions of their Performance and the Challenges they faced

In this section, the main data were gathered from the observations and students' answers to the interview questions which were supported with the worksheets. As explained in Section 4.2.1, students found isometric drawing much harder than orthogonal drawing. This study also looked at students' perceptions to find possible reasons for why this might happen. The findings showed that students experienced two main difficulties in learning 2D representations of 3D shapes.

First of all, it is important to mention that it was observed that many students had difficulty to understand, imagine and construct orthogonal and isometric two-dimensional representations of 3D shapes during the observed 16 lessons. Most of the students (69%) reported that the topic is difficult for them to understand and gave scores on a ten-point scale between five and ten as their perceived difficulty of the topic. Below are two students' explanations for their scoring, they were chosen from those who scored high and low in the test respectively. One of the students who gave seven out of ten to the difficulty of the topic said in the interview that

> *Well, seven. It is not as difficult as ten but I have to admit that I still find it pretty difficult.*

Here, it could be concluded that students who scored higher than the average on the test may still consider the topic considerably challenging. In contrast, one of the students, who scored lower than the average on the test gave two out of ten to the difficulty of the topic, added that

> *I'm giving two. I am bad at mathematics. I think I couldn't do most of the questions on the worksheet correctly. I rarely answer my maths teacher's questions correctly but I think '3D shapes' is an easy topic for others.*

Thus, it could be concluded that students who rated the difficulty as low still may not consider the topic as an easy one for themselves.

Interviews about students' worksheet answers confirmed the observation results where mental visualisation of the shape (all of the students) and drawing the visualised shape on a paper sheet (half of the students) came up as students' two main difficulties to learn these drawings.

Students were unanimous in stating that mental visualisation of 3D shapes is difficult for them. They clearly explained their difficulty in combining separate views to build a 3D shape in their minds. One of them who tried to create separate 3D shapes for each orthogonal drawing said that

> *I just couldn't combine them [orthogonal drawings] in my mind, you know, for me, all orthogonal views separately create their own shapes but combining them in just one shape is so difficult [Figure 4.5].*

In the worksheet, rather than combining given orthogonal drawings in an isometric drawing to draw one polycubical shape, this student tried to draw separate 3D shapes for each orthogonal drawing. Regardless of these drawings' being incomplete or incorrect even given the separate conditions, the main difficulty of the student appeared to be the mental visualisation.

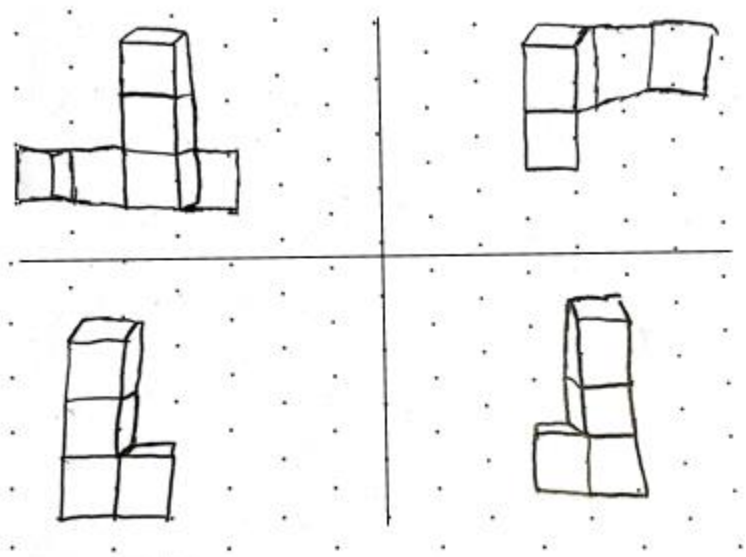


Figure 4.5. Sample student drawing for the Q-3 in isometric drawings

Many students found it challenging to *draw the shapes in their minds on an activity sheet* during the observed lessons. Half of the interviewed students shared similar experiences with their friends. The quote below illustrates an example episode from one of the interviews.

> *Researcher: Please talk me through how you have decided where and how many cubes to draw in this question [Figure 4.6].*
>
> *Student: Hımm… I wanted to draw… Four, five, six… [Counts the cubes previously drawn onto the worksheet.] Seven cubes but I drew only six cubes.*
>
> *Researcher: Where should be the seventh cube?*
>
> *Student: In front of this one [Shows the cube at the very left], but when I draw it, it seems like it is on that cube not in front of it.*
>
> *Researcher: I see. Will the shape be complete after you draw it?*
>
> *Student: Yes, but I don't know how.*
>
> *Researcher: How many units are the height of this shape?*
>
> *Student: Three [Points the cubes one by one].*
>
> *Researcher: I think what you imagined is perfectly correct but the drawing is a little inclined which makes the shape difficult to interpret in the height you are saying. It sometimes can be difficult to draw what we imagined. ... Let's start from the beginning and draw it step by step together.*

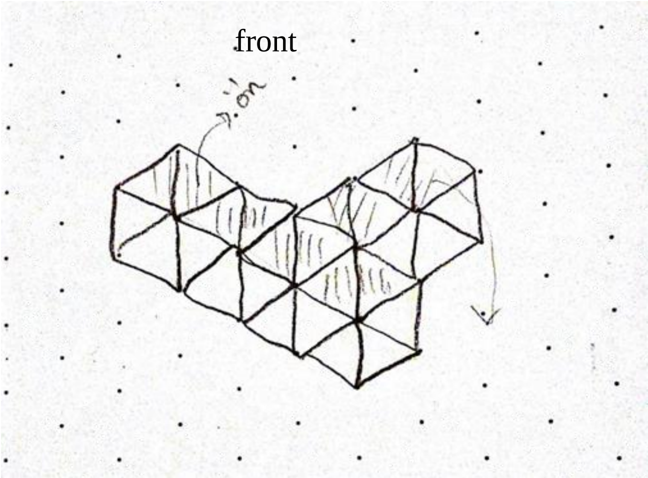


Figure 4.6. Sample student drawing for the Q-3 in isometric drawings

Above quote show that the descriptions of the student and the shape in her mind are perfectly correct although the drawn shape would not be interpreted as the required shape by many maths teachers not only because of its missing one cube but also because of the misuse of the isometric paper. This episode is only one of the examples out of recorded tens of them.

To sum up, mental visualisation and integration were found to be two difficulties students faced. The following section explores current pedagogy and students' learning experiences which could have caused this.

4.2.3. Current Pedagogy and Students' Learning Experiences

One of the purposes of this study was to understand students' experiences in learning 2D representations of 3D shapes. Therefore, a series of lessons by four teachers were observed to review the profile of current pedagogy. This section reports the result of the observation supported with analysis from students' interviews. It starts with a general description of the current practices and continues with the main features of lessons that could be the difficulty in understanding 3D shapes in the observed context.

As a reminder, the objectives of the observed lessons were (a) A student draws the orthogonal views (from the top, front, left and right) of 3D shapes constructed from unit cubes, and (b) A student constructs isometric drawings corresponding to given orthogonal drawings from the top, front, left, and right (the use of an isometric paper is suggested).

Existing practice is intended to achieve these objectives in four to six lesson hours. All teachers prepared their lesson plans individually using examples from various sources but with similar lesson structures.

The lessons started with an emphasis on prior knowledge and skills. While two of the teachers (Mr Abay and Ms Onay) mostly did this through summarising what was done in the previous lessons in their own words, the other two teachers mostly did this through reviewing the homework.

The lessons continued with/proceeded to the teachers’ introduction to the topic. All of the teachers introduced the topic through examples of their choice. The examples of Mr Abay and Ms Onay were mostly from an external textbook while Ms Semin and Ms Aras mostly used the workbook questions suggested by the ministry.

Once the topic was introduced, the lessons continued with the exercises. In this phase, students were given some time to work on a specific 2D representation question before their teachers explained the strategy to draw the correct representation on the board. The teachers followed different strategies while the students were working on their drawings. For example, Ms Aras constructed the corresponding 3D shape on teacher’s desk using cubes with 10 cm edges (Figure 4.7), and Mr Abay and Ms Onay walked between the desks to guide students with some hints. After a short time on task, all four teachers invited students to come and draw the correct representation on the board as a part of their lessons. If a student got it right, s/he explained and the teachers summarised her/his strategy to draw the correct representation. If not, the teachers either corrected the drawing or redrew it themselves, and then explained their strategy to draw the correct representation. If there were no volunteers, the teachers either chose a random student or constructed the shapes themselves and explained their strategies. In either case, whether the teachers or a student drew the correct representation, the teachers summarised a strategy to draw the correct representation, and then, gave some time to students for copying the correct answer to their notebooks or activity sheets. It is of note, there were only a few students who volunteered to construct shapes’ isometric drawings on the board while there were more students for the orthogonal drawings. Going along with their different exercise questions, teachers used different approaches to highlight important aspects of the content. Mr Abay and Ms Onay repeated the steps to draw a 2D representation over and over again whereas

Ms Semin and Ms Aras asked students to watch their hands while (and if) they were constructing a 2D representation.

The lessons ended with a summary of what was discussed. There were varieties of approaches to summarise the lessons. Mr Abay opened objective evaluation questions from the moodle and summarised what he taught with quick multiple-choice questions. Ms Onay showed summary videos from the moodle to conclude her lessons. Both Mr Abay and Ms Onay ended their lessons with the materials suggested by the Ministry of Education. Ms Semin and Ms Aras ended most of her lessons by giving homework from different sources. By the end of her lessons, Ms Aras was always in hurry to finish the questions planned for the lessons and she gave the remaining questions as homework. Ms Semin, on the other hand, completed her planned questions most of the time and gave homework from another source after summarizing what was done in the lessons in her own words.

Turning now from this description, it appears there are three main features of lessons that could be associated with students' difficulty in understanding 3D shapes in the observed context.

The first of these found to be the use of manipulatives – where rather than students learning actively with blocks their role was reduced to more passive observation. The teachers in the study were aware of the importance of using concrete material models in middle school maths lessons. Almost all of the observed lessons included concrete manipulatives such as unit cubes and linking cubes. However, teachers chose to use these themselves instead of offering students an opportunity to use them. For example, Ms Aras built a shape on teacher's desk using cubes with 10 cm edges. Then, she drew its orthogonal views onto the board whilst explaining her strategy to draw and asked students to copy the drawings to their notebooks.

There was absolute unanimity in the students' comments about the use of materials in the current classroom. All students reported that their mathematics teachers dominated the use of manipulatives and technologies in the classroom if any were present. These materials were mostly for the teachers' use and the lessons were mostly teacher-centred where teachers demonstrated how to construct shapes with unit cubes and explained their strategies to draw the constructed shape on board. Thus, similar to this example, students mostly observed their teachers and they did not have a chance to build shapes and explore the topic through concrete materials themselves.

Only one of the teachers, Ms Semin, introduced a task which requires students' use of concrete materials. She invited her students to work in groups to build 3D shapes with linking cubes. She divided students into four groups (Figure 4.7) and gave sheets having different orthogonal drawings and a number of unit cubes to each group (Figure 4.8). Yellow shaded students did not come to the lesson and students with blue arrows changed their desks for the group work. The drawings on the sheets were the views from the front, back, right, left and top, respectively. Students were asked to construct four 3D shapes with linking cubes and then draw them to their isometric papers. The teacher checked each group's construction one by one and gave them feedback on whether it's correct or not. If not, she helped them with their construction by first asking questions and then simply by taking the cubes and constructing the shape herself.

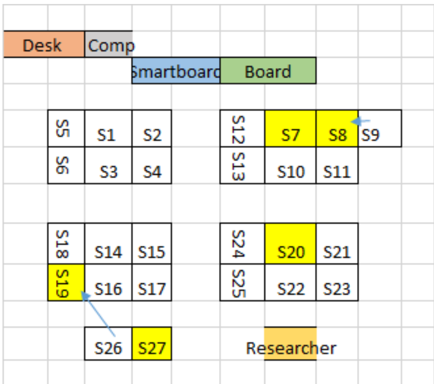


Figure 4.7. Sketch of the classroom

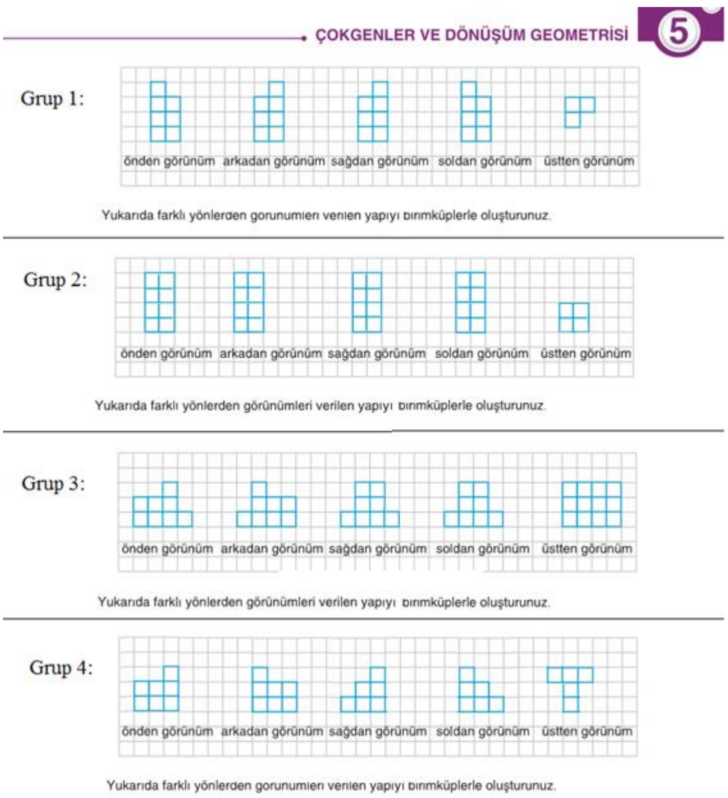


Figure 4.8. Sample worksheet for the group work

Secondly, it was observed that most of the maths teachers used *a very exam-focused pedagogy* with a little interaction. Ms Semin's example, above, was also the only task which requires collaborative interaction in the observed 16 lessons. When students' were asked about their experiences, all students replied that teaching was mostly conducted with a didactic approach as they have a very tight programme to cover within the fixed number of lesson hours due to ministry middle school exam. Teachers included past ministry exam questions in their lesson plans to increase students' motivation and to emphasize the importance of the topic. For example, Ms Onay asked two questions from the ministry exams in 2008 and 2010 (Figure 4.9). While the first of these was concerned orthogonal drawing of the right view of the given shape, the second question asked children to ask about the removal of which cube changes the front view. She said that these were the most difficult questions about the topic in the exams so far and if her students could do these, then they could do other questions easily.

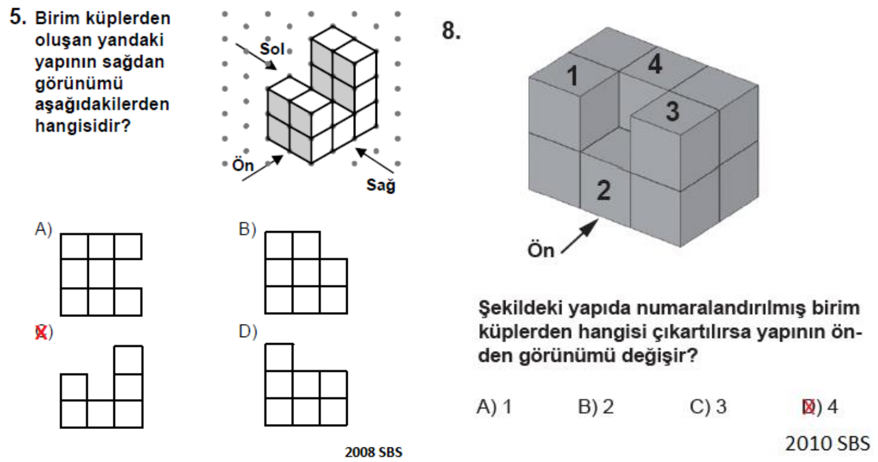


Figure 4.9. Ms Onay's choice of ministry exam questions

Other teachers showed a tendency to find and ask difficult questions in the lessons to prepare students for the ministry exam. Similar to Ms Onay, Mr Abay said he found the questions in the maths book too easy for this topic and he wanted to ask difficult questions. He brought copies of a page from an external source and distributed these to the students (Figure 4.10). These questions asked orthogonal views of the given polycubical shapes. Half of the questions had shapes that had more than twice as many cubes as the ministry exam questions and they were so hard for the students that almost nobody in the class was willing to answer them. Students appeared to be afraid of losing face when they made mistakes in front of the class. It was also observed that students tended to accept what was being said by their teachers instead of questioning and exploring the topic. For example, in Mr Abay's second lesson, almost nobody volunteered to draw the views on the board (Figure 4.10). He called student 19's name and invited her to the board to draw it. The student drew the front view incorrectly. Mr Abay asked student 16 to correct it, explained the correct answer with his own words and asked whether everybody understood. Student 19 attempted to say something and looked like she did not understand but preferred to keep silence.

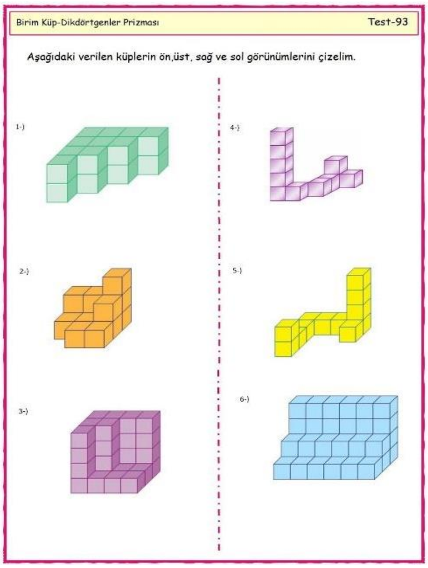


Figure 4.10. Mr Abay’s choice of questions

Finally, it was observed that technologies were used by all four teachers in only limited ways. Students were not encouraged to use applications on their tablets for visualising the shapes in any of the lessons. While half of the teachers also showed videos/questions from the Ministry of Education’s Moodle at the end of the lessons to summarise what they have taught (Figure 4.11), all of the teachers mostly only used the smartboard as a technology to solve questions. They mostly taught memorisation of their strategies as if it was the only way to learn this topic and suggested repetitions (i.e., answering more questions) as a learning strategy to perform better and faster. Therefore, they prepared many questions prior to the lessons, answered as many of them as possible during the lessons and gave the remaining questions as homework. Moreover, these questions were answered on the board either by teachers or students but in either case without using the properties of the smartboard by only drawing with a regular board marker.

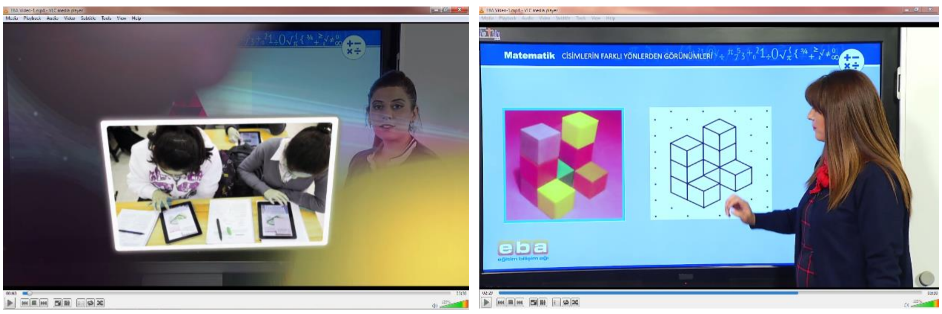


Figure 4.11. Screenshots from Ms Onay’s selection of videos

Students also came up with some solution strategies to increase their learning opportunities of orthogonal and isometric representations of 3D shapes. These solutions were their active use of concrete manipulatives (10 out of 16 students), integration of various digital technologies to enhance their visualisation of the shapes (nine out of 16 students) and use of real-life examples (two out of 16 students). While the first two were already discussed in the features of the lessons above, real-life examples were not a part of any of the lessons. It is observed that students related learning three-dimensional shapes neither to other things they learnt at the school nor to their daily life so they had almost no interest in learning the topic. Hence, it was not surprising that only two of them stated the use of real-life examples as a solution strategy in the interview saying that these examples could be helpful for them to better connect orthogonal and isometric drawings with their practical life.

4.3. Summary of Findings and Discussion

To sum up, this study looked at how the seventh grade middle school students learned two-dimensional representations of 3D shapes in the sample, specifically focusing on investigating students' experiences in learning three-dimensional shapes and students' errors in representing three-dimensional shapes. Both female and male students found it harder to make isometric drawings than orthogonal drawings. This study also looked at students' perceptions to find reasons for why this might happen. Main difficulties were found to be mental visualisations of 3D shapes and drawing the shapes in their minds on an activity sheet. When it comes to the pedagogical rationale could have caused these, it was found that observed geometry lessons in Turkey were dominated by teacher-centred pedagogy. It was rare in these regular classrooms to see hands-on activities and discussions of real-life examples when students were learning representations of 3D shapes. Interviews revealed a lack of teacher motivation for teaching the topic without an exam-focused instruction as well as children's views of the inherent difficulty of the topic made the teaching of 3D shapes more problematic.

Positively, the opportunity that the FATIH Project offers to innovate in geometry teaching in Turkey provides a context in which to overcome these problems (MoNE, 2012). Therefore, a series of lessons were designed by the researcher to tackle known teaching problems and challenges employing both physical and digital tools. The underpinning theoretical model is called the RETA Three-dimensional Shapes

Teaching Model and emphasizes four design principles for three-dimensional shape learning in geometry lessons: Realistic, Exploratory, Technology-enhanced and Active.

5. THE RETA 3D SHAPES TEACHING MODEL AND LESSON PLANS

5.1. Introduction

This chapter describes the RETA 3D shapes teaching model and the lesson plans that are the result of responses to the second research question.

- What principles can inform two-dimensional representations of three-dimensional are best taught to Grade 7 students in Turkish middle schools?

The responses are explained in the two sections below which correspond to two sub-questions.

- What are the important elements of three-dimensional shapes lesson plans?

This question can be answered by a model, which is called the RETA (Realistic, Exploratory, Technology-enhanced, and Active) three-dimensional shapes teaching model. The four design principles were chosen according to the researcher's judgement and understanding of the reviewed literature and the results of the first study. The principles are designed for the Turkish FATIH Project context, explained in Section 1.2, by the researcher drawing on the theories, her experience as a Turkish mathematics teacher and observations and interviews reported in Chapter 4.

- How can specific activities be designed to teach three-dimensional shapes?

The response to this question is the lesson plans that were designed based on the RETA principles and investigated in Study 2, Study 3 and Study 4. Lesson plans were prepared by embedding the RETA principles and were revised after each study in order to better fit the needs of the students. Section 5.3 presents the earliest version of the lesson plans; Chapter 6 and 7 report modifications made to the lesson plan after each earlier cycle.

The researcher needed to revise the plans following the same principles as a natural process of design-based research methodology as such it is not the intention of the researcher to claim these lessons are suitable for all context and that all researchers would produce the same designs. Hence, this thesis provides a possible answer to the above questions for the specific context explored.

5.2. The RETA Model

This section describes a model of teaching three-dimensional shapes. The model has four principles: Realistic, Exploratory, Technology-enhanced and Active (RETA). Although these words are polysemic, each principle is described within the context of the model, and the meanings associated with these principles will be the ones described here. As these are widely-used terms commonly encountered in the literature, this section defines them more specifically for use in this thesis.

5.2.1. Realistic

The first principle, ***realistic*** lessons, refers to the intent of integrating real-life examples and contexts into the lessons. Real-life examples provide concrete and real-world applications of the knowledge and skills learned in the classroom (Gravemeijer, 1994). This is intended to enhance students' awareness of the importance of the topic in their daily life, make inferences about the concepts' real-life relations, enhance motivation for learning three-dimensional shapes and help them solve real-world problems in their future. As explained earlier, the immediate proximal aim is to do better at the geometry exam that the Turkish government set but the researcher is hoping that there is more for doing this than passing the exam of the Turkish government. The following looks at spatial and mathematical aspects (importance) of realistic lessons and then explains their applications in the designed lessons.

Firstly, spatial thinking is used in daily life in many ways. For example, while preparing a suitcase and packing it into a car trunk, we inherently use spatial thinking (Liben, 2007). It is also necessary for educational curricula - from the use of molecular models in chemistry (Barke & Engida, 2001; Pribyl & Bodner, 2006) to understanding the layers of mountains in geography (Lee & Bednarz, 2009; Robertson et al., 2009). Spatial thinking has also been considered as an important skill which many disciplines look for in their works, such as science, engineering and mathematics.

Many professions ranging from radiologists to product designers are required to think in both two- and three-dimensions. For example, as a part of their occupation, doctors examine two-dimensional images of a body created by radiologists in order to diagnose the problems of the patient. An example closer to geometrical drawings is from architects who sketch their building projects on a paper or a computer screen to represent these in two-dimension to work with. They draw plans and elevations of the buildings in order to simplify those three-dimensional shapes into a series of two-dimensional pieces for many reasons, for example, solar energy saving and heating and electricity structures (Matusiak, 2017). Plans are useful representations to understand proximity and spatial relations between the rooms of a building, whereas elevations are important to understand key dimensions such as wall lengths and height and to show openings such as doors and windows; thus, they both are useful in real-life contexts. Plans are called top views, and elevations are called the views from the front, back, left and right in middle school geometry curricula that the researcher is interested in developing lessons on teaching them. Including such real-life examples into designed lesson plans could increase students' interest and engage them in learning the topic (Fredricks et al., 2017).

The second (mathematical) aspect of realistic education takes English realistic mathematics education as an example. Realism is important for teaching mathematics to make students aware of the real-life use of the topics that they learnt in the classroom (Gravemeijer, 1994; Van den Heuvel-Panhuizen, 2003). This stance originated from the Freudenthal Institute in the Netherlands in the 1970s to improve the quality of maths teaching in Dutch schools (Freudenthal, 1971, 1973). Freudenthal (1987) argued that there is a worldwide need to apply mathematics and realistic mathematics education is a way to achieve that. According to him, realistic mathematics education makes mathematics of human value by starting and staying in reality while teaching mathematics. It connects the mathematics to the reality to stay close to the children so that motivates them to study mathematics (van den Heuvel-Panhuizen, 2003).

It is easier for children to find this connection if it is more obvious; for example, it is much easier to relate the study of population in geography or study of the motion of objects in physics with the real life; however, many children and even adults struggle to relate mathematical topics with real life (Cornell, 1999; Larkin & Jorgensen, 2016). They either consider mathematics an abstract science or relate some components of maths to real life but lack explaining the broader use of components in real life (e.g., Mulero, Segura, & Sepulcre, 2013; Reid, Petocz, Smith, Wood, & Dortins, 2003). For example, Mulero et al.'s (2013) study with 94 university students found that more than half of them did not relate architecture to mathematics and did not know any architect who is leading a mathematical contribution to architecture. Whilst this is the case, it is important to provide realistic mathematics education that provides an opportunity to raise students' awareness of the connection of mathematics to real life as early as school years.

In realistic mathematics education, children learn mathematics based on activities they may encounter in their daily life; they have the opportunity to construct their knowledge themselves through group work, discussion and reflection (van den Heuvel-Panhuizen & Drijvers, 2014). These conditions are in line with constructivist theories (Cobb, 1994; Cobb & Yackel, 1995; Gravemeijer, 1994; Simon, 1995) and match with the other principles, such as exploratory and active, of the model which will be explained in the following sections.

The UK was one of the earliest countries to adopt and use realistic mathematics education (De Lange, 1996). English realistic mathematics education, specifically geometry, is a good example of an effective intervention which supports the researcher's approach to teaching mathematics (Cooper & Harries, 2009; Dickinson & Eade, 2005; Dickinson & Hough, 2012). Realistic mathematics education in English classrooms was trialled with Grade-7 students in a local school in Manchester in 2003 and reaction to designed realistic education materials were very positive (Dickinson & Eade, 2005; Eade & Dickinson, 2006). The research continued in over 20 schools over a three-year period and evidence showed that students' understanding of mathematics and their approach to solving problems improved. The improvement was not only in those who actively engaged with the topic but also in lower attaining students. Consistent findings about improvements in the mathematics achievement in various topics, including geometry in mathematics, has been found over years (Blum et al., 2019; Dickinson et al., 2010; Hough et al., 2017; van den Heuvel-Panhuizen, 2019). This realistic stance in maths education has integrated into middle school maths lessons in the UK since then and set the basis of the realistic principle of the present research (Dickinson & Hough, 2012).

Realistic mathematics education is not without its critiques. Firstly, there is a criticism which says it has been overstated and abstract mathematical principles are much better than realism (Keune, 1998, 1999). Keune (1998) described realistic mathematics education as flat as a pancake. Gemert (2015) cited Keune's (1999) position that says "There is a need for giving more attention to abstraction and logical reasoning to better make use of the Dutch mathematical talent that would be lost because of the realistic approach" (p. 365). However, it is important to note here that Keune's (1999) speech which further explains his position about realistic maths focused more on what realistic mathematics education became in practice in the Netherlands rather than what it is intended to be. Moreover, realistic does not imply there is no room for abstraction neither for teaching from examples (Gemert, 2015). This PhD research used worked examples in addition to integrating realistic mathematics.

Secondly, there is a criticism of realistic mathematics education that says it provides realistic but not real contexts hence it is reductionist (Verstappen, 1994). Verstappen (1994) argued that the problems in realistic mathematics education provide simplified *realistic* contexts which may later cause problems in handling *real* problems formally in mathematics and in later life. Gravemeijer (2001) responded to this by emphasizing that the problems in realistic mathematics education *can but does not have to* deal with the authentic everyday life situations that are more complex for children with many variables than what the realistic mathematics education proposes. According to him, what is essential in realistic mathematics is to provide a familiar context in which students can act smartly to understand the mathematics in it.

Finally, there are studies which show that sometimes just putting things in a realistic context does not make them easier to learn for children. For example, Chandler and Kamii's (2009) study with 98 children found that it is more challenging for students to understand the place value when teachers use coins as a real-life example to teach it. They found that although it is simple for teachers as adults to think one dime as ten pennies, it is hard for students and even more challenging when teachers include monetary examples to the teaching of ones and tens. Monetary examples are how just putting something in a realistic context may actually make things more challenging therefore real-life examples should be chosen wisely according to the needs of the students. The following paragraph explains techniques to incorporate realistic mathematics in the lesson plans in this thesis.

Videos and photos were chosen as suitable methods for illustrating these ideas. This is, videos and photographs are used to bring these ideas to life by asking students to articulate real-world experiences. Real-life videos of the mathematical content's real-life use and discussions around the videos are suggested by many researchers to provide students with an insight into that what they learn might be useful in their future life. However, some critics have argued that videos are perceived by students as entertainment rather than as informative, leading to students to not make as much use of the videos as had been anticipated (Salomon & Perkins, 2005). Consequently, this motivated the design choice to not simply show relevant content in the videos, but also to provide a student-centred environment for discussions around them. Peer discussions and a whole-class discussion following them were designed to help students make connections between the knowledge and skills learned in the classroom and their real-world applications. Moreover, almost all of the unit cube constructions that are designed for students to build are representations of realistic images of buildings (e.g., a castle and a school). Of note, the lessons include pictures that the children may not have the necessary experience in like castles and yet castles are in picture books and these pictures are very familiar to children from television and books. This is aimed at providing them with a consistent real-life experience throughout the lesson, not only during the discussion on a video clip. In conclusion, the first principle of this model aims to relate the content of three-dimensional shapes in geometry with its real-world applications.

5.2.2. Exploratory

The second principle is described by the term ***exploratory*** and refers to the use of worked examples in lessons that support students in exploring the topic. Examples play role in the teaching of geometry and have found a place in many teaching and learning theories (Bruner, 2017; Marton, Booth, & Booth, 2013; Marton et al., 2004; Skemp, 2012; Watson, Mason, & Mason, 2006; Wilson, 1986). Worked examples can be described as instructional devices that provide somebody else’s solution for a student to study. In geometry instruction, it is common to have students solve problems. However, solving problems might not be very beneficial when students have just started learning before they gain some level of understanding of the topic (Renkl, 2011; Salden et al., 2009). Worked examples, on the other hand, found to be effective for initial skill accusation and be more productive when compared to solving problems at the beginning (Kalyuga et al., 2001; Renkl, 2014, 2017). Furthermore, learning geometry by critiquing, comparing and discussing multiple solutions brings many benefits from increasing student engagement with the examples to helping students effectively integrate their previous knowledge into their current learning processes (Pierce et al., 2011; Silver et al., 2005).

Moreover, some of the designed worked examples include specifically designed mistakes for students to diagnose and remediate and to discuss possible reasons for them similar to those of Durkin and Rittle-Johnson (2012) and Evans and Swan (2014). For example, Evans and Swan (2014) supported eight secondary school mathematics teachers in the UK and more than 20 in the US to integrate worked examples with designed mistakes. They designed lessons with these worked examples, which are available online on the Mathematics Assessment Project's official website: http://map.mathshell.org/. These teachers provided worked examples when their students struggled with a geometry problem. Evans and Swan (2014) found that providing examples with mistakes and asking students to critique them have the potential to support students' development of their own strategies for problem-solving in geometry. They reported as a limitation that some students were focused on correcting errors rather than making holistic comparisons. The present study included peer discussions followed by a whole-class discussion to help students build their conceptual understanding of holistic issues regarding the topic. The researcher was aware of discussions being a) more demanding for both teachers and students because of the skills needed for critique and discuss the ideas behind the solutions and b) very different than how teaching occurs in the observed context. In conclusion, the second principle of this model aims to provide an exploratory mathematics education where students engage with worked examples and diagnose, remediate and discuss designed mistakes.

5.2.3. Technology-enhanced

The third principle proposes a ***technology-enhanced*** education which refers to the strategic use of dynamic geometry tools in teaching three-dimensional shapes to provide multiple representations of them. The following looks at two aspects of technology-enhanced education: spatial and mathematical aspects, and then discusses their applications in the lessons.

People are required to think about three-dimensional shapes despite their difficulty in reasoning about three-dimensional shapes when working from the two-dimensional forms, thus making this process challenging (Reisberg & Heuer, 2005). People often need to work in two-dimensional forms of three-dimensional shapes in order to give meaning to them. Specifically, they may be asked to draw, interpret or transform two-dimensional representations in order to work with three-dimensional shapes. Two-dimensional representations can be integrated in the human mind in many ways, for example, by means of landmarks based on features of the shape or by reference to other shapes (Tversky, 2003, 2005). Reisberg and Heuer (2005) point out that "mental images seem to be represented from a determinate viewing angle and distance … since visibility from a perspective and occlusion seem to play a role in those data" (p.39). In their review, this statement is illustrated with a depiction study, which indicates that when people are asked to describe a cat in a shown picture their responses to the questions are quicker than when they describe particular parts of a cat, and they again are quicker to describe large and visible parts like the head of the cat in the picture than parts which are small and hidden from view such as whiskers and claws. In geometry lessons on polycubical shapes, cubes which are behind other cubes might be partly visible or invisible, and therefore it is not easy to represent them in two-dimensional forms, for example, orthogonally or isometrically. Providing a dynamic technology to make all views available and easy to find or discover by simply dragging the constructed shapes has the potential to enhance students' mental visualisation, which they struggle with, and to improve their understanding of two-dimensional representations.

Although the introduction of technology to geometry education has historically met with resistance (see Bolt, 1991), many researchers have found that dynamic geometry tools can help students in representing three-dimensional shapes in two-dimension (e.g., Oldknow & Tetlow, 2008; Simpson, Hoyles, & Noss, 2006). The Royal Society's working group on teaching and learning geometry 11-19. The Royal Society (2001) recommended not only greater attention to learning three-dimensional shapes but also learning them through more effective use of technological tools in classrooms. Taking this into consideration, Oldknow and Tetlow (2008) worked in small-scale pilot schools on the introduction of a 3D geometry software and assessment of its effectiveness and, then, extended their study to a larger project in a group of Hampshire schools in the UK. Their study showed that work in such software —which allows students to create two-dimensional representations of 3D shapes— provides students with both meaningful understanding of three-dimensional shapes and a good source of opportunities for active involvement, collaboration and for increasing confidence. Widder and Gorsky (2013) who further examined the use of these tools by the students in three-dimensional shapes lessons found that students used them according to their needs. Students having with different pre-test performances used tools for different purposes; those with low spatial abilities used the tool mainly for measurements of their constructed representations while those with high spatial abilities used the software primarily for self-examination (e.g., 50% more constructions and number of operations than their counterparts with lower spatial abilities) and for shortening mental processes such as (re)constructing and rotating the shapes. They all benefitted from the representations these 3D tools provided.

Although every representation has strengths and disadvantages (Friedlander & Tabach, 2001), many researchers agree that learning from an appropriate combination of representations with the help of technology is more beneficial than learning from a single representation (e.g., single-use of verbal, numeric, symbolic or graphical representations) and geometry software packages provide an environment for this combination (Ainsworth, 2006; Hoyles & Noss, 2003; Kaput, 1992; Pape & Tchoshanov, 2001; Pierce et al., 2011).

However, the outcomes/applications of integrating dynamic tools are not entirely positive. Such lessons are more difficult to design (Grandgenett, 2007), especially since they require the use of more student-centred methods with which some Turkish teachers struggle, and these lessons are in general more challenging for teachers to manage (Bates, 2005). Moreover, a lack of teacher knowledge in teaching geometry with technology and inconsistent beliefs and goals regarding the use of technology make it much harder to design technology-based lessons since teachers' knowledge, beliefs and goals affect the way they teach with technology (Ball et al., 2008; Mishra & Koehler, 2006; Niess, 2008; Shulman, 1986). For example, while some teachers see dynamic geometry tools as distractions rather than learning tools and find them time-consuming, some others perceive these as a very helpful facilitator and effective way of teaching (Saralar & Ainsworth, 2017; Yu-Wen & Andrews, 2009). In the present study, lessons were strategically integrated with a free dynamic tool GeoGebra that is available to use both individually on tablets and collaboratively on interactive boards for manipulating 3D shapes. As explained previously in the Introduction, these choices were partly pragmatic as 1.5 million tablets were distributed to students and all middle school classes were provided with the interactive whiteboards in Turkey from 2011. To conclude, the third principle of this model aimed at providing students with technology-enhanced experiences of an appropriate combination of 2D representations of 3D shapes with the help of a dynamic geometry software.

5.2.4. Active

The fourth principle refers to the ***active*** learning environments where students themselves have control of the use of concrete manipulatives instead of them watching teacher's constructions and copying drawings on the board—as found in Study 1. Active lessons are proposed as more than just contrast with passive lessons as described by Chi (2009) and Schank (1994). This thesis chose to use the term active as this principle refers to the involvement of learners as active participants – although activities involved could be described as constructive (e.g., students' building cube-constructions for themselves) and sometimes interactive (e.g., looking at students' cube-constructions and solutions and getting feedback on them) in the literature (see Chi, 2009). The following describes the use of concrete manipulatives in this thesis, summarizes claims on the use of these manipulatives and their benefits in geometry and discusses their applications in the designed lessons.

Historically, teachers relied on workbooks and memorization to present middle school mathematics. However, for over twenty years educators (e.g., Cain-Caston, 1996) have argued that these approaches are "ineffective and outdated" (p.271). Many researchers who have compared traditional teaching methods with an alternative method found that students perform better with alternative methods, such as those of studying teaching with concrete manipulatives (Driscoll, 1983; Sowell, 1989; Suydam, 1984). Concrete manipulatives are "objects that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered" (Swan & Marshall, 2010, p.14). An earlier definition actually includes my proposal of making students active, by defining concrete manipulatives as "models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around *by students*" (Hynes, 1986, p.11). Concrete manipulatives, in this thesis, refers to unit cubes, such as multilink cubes or linking cubes, and unifix cubes.

There is no consensus on the use of concrete manipulatives in the literature. While some academics argue against them (e.g., Ross, 1989; Uttal & DeLoache, 1997), some others argue for the use of manipulatives (e.g., Moch, 2001; Van de Walle, Karp, &

Bay-Williams, 2010). The following looks at general claims about concrete manipulatives, both anti and pro and then discusses the specific benefits in geometry.

Manipulatives are claimed not to offer a magical advantage to enhance mathematics learning (Bartolini-Bussi & Mariotti, 2002; Chandler & Kamii, 2009; Fuson et al., 1997). Most people who argue against manipulatives use them to teach symbolic relations and discuss the use of blocks (any base-ten material such as dienes blocks) to represent addition and subtraction to teach place value. Bartolini-Bussi and Mariotti (2002), Baturo (2000) and Fuson et al. (1997) argue that the use of manipulatives in the classroom does not guarantee that students conceptually understand the subject matter. Fuson (1990) claimed that even manipulatives with semiotic potential to present the idea of unit failed to achieve intended learning outcomes. Many of them underlined that it is challenging for students to pass from groups of ten unit cubes to tens, i.e., understand the concept of ten units as one ten-unit block and relate this to the place value (Baturo, 2000; Hiebert & Wearne, 1992; Ross, 1989).

Secondly, it was argued that children do not connect mathematical concepts with manipulatives that forces them to learn both mathematics and the use of manipulatives (Schoenfeld, 1986; Uttal et al., 1997). When manipulatives are used for representing symbolic relations, which often is the case in the literature, students need to learn both the mathematical concept and how to represent it using manipulatives. Research found that students did not understand that the intent of using manipulatives is to represent a mathematical concept (Hughes, 1986; Schoenfeld, 1986). Therefore, they may fail to construct a correct shape with manipulatives but they may understand the concept or vice versa. For example, Schoenfeld (1986) found that students who correctly represent a particular concept with manipulatives may fail to answer problems requiring very similar representations.

Proponents of the idea that manipulatives do enhance learning of mathematical concepts, on the other hand, argue that manipulatives are helpful not only in enhancing mathematics learning (Kennedy & Tipps, 1994; Tooke, Hyatt, Leigh, Snyder, & Borda, 1992) but also in increasing motivation and decreasing anxiety about mathematics (Battista, 1986; Martinez, 1987), Therefore they claim that manipulatives should be used in all levels and with students with learning difficulties to gifted students (Flores, 2009; Peterson et al., 1988). Their empirical studies found that use of manipulatives leads to better learning outcomes (Kennedy, 1986; Suydam, 1984; Suydam, 1986; Williams, 1988). Therefore, "Mathematics educators around the world have found that mathematics is better learnt, and therefore should be taught, by students experiencing it through manipulatives," says Tooke et al. (1992) to express their enthusiasm to teach maths with manipulatives (p.61). Similarly, Kennedy and Tipps (1994) recommend using manipulatives by claiming that they "make even the most difficult mathematical concepts easier to understand … [and] enable students to connect abstract mathematical concepts to real objects" (p.71). Moreover, many of them claim that using manipulatives has found to reduce mathematics anxiety (Battista, 1986; Martinez, 1987; Sherard, 1985; Zemelman et al., 2005). Finally, a meta-analysis showed moderate to large effects on retention, and small effects on problem-solving, transfer and justification in favour of manipulatives over maths symbols (Carbonneau et al., 2013).

The researcher agrees with those who argued after their empirical work that manipulatives work if certain conditions are provided. As Ball (1992) highlights with the following words that plastics cannot teach maths: "Understanding does not travel through the fingertips and the arm… Mathematical ideas really do not reside in cardboard and plastic materials." (p.47). Students do not readily acquire new mathematics from using manipulatives. Many researchers describe certain conditions which make manipulatives useful, for example, effective instruction and aim for meaningful learning (Carbonneau et al., 2013; Furner & Worrell, 2017; Moch, 2001). For example, instruction is emphasized as a key factor. Effective use of manipulatives depends on teachers' ways of teaching with them. The meta-analysis of Carbonneau et al. (2013) with 7237 students from 55 studies on the efficacy of teaching mathematics with concrete manipulatives found that manipulatives provide better learning than traditional methods which provide only abstract maths symbols. This relationship between students' learning and use of concrete manipulatives found to be moderated by instruction. It also depends on students' link of manipulatives with what they represent as explained above, and that partly depends on teacher's instructions and guidance during the activity in the classroom (Uttal et al., 1997). According to theories in the literature, concrete manipulatives facilitate learning when they support certain instructional characteristics by any of the followings:

a) Affording opportunities for student-centred investigation of the topic (Kirschner et al., 2006; Mayer, 2004; Papert, 1980),
b) Facilitating students' abstract reasoning (Bruner, 1964; Montessori, 1964; Piaget, 1962),
c) Providing physical enactment (Biazak et al., 2010; Engelkamp et al., 1994; Kormi-Nouri et al., 1994) and
d) Stimulating students' real-life experience (Baranes et al., 1989; Rittle-Johnson & Koedinger, 2005; Tindall-Ford & Sweller, 2006).

Another example of these conditions is environments in which manipulatives are used for meaningful learning by building on existing knowledge and requiring reflection and thought of students (Baroody, 1989; Furner & Worrell, 2017). They emphasize that manipulatives help students when students relate their existing knowledge with the intended learning outcomes through inquiry. Use of manipulatives in teaching leads even better learning outcomes when students have prior experience in using the material (Marzolf & DeLoache, 1994) and use them consistently over extended periods of time (Sowell, 1989).

In geometry instruction, it is common to see teachers teaching three-dimensional shapes with unit cubes. Swan and Marshall (2010) study with 249 teachers in New South Wales found that cubes are the most used manipulative in presenting three-dimensional shapes and the third most used manipulative in mathematics lessons (after blocks and polydrons), unifix cubes with 66.5%, and multilink cubes with 43.3%. Although using these manipulatives has strengths and disadvantages, many researchers (e.g., Canny, 1984; Clements & Battista, 1990; Fennema, 1973; Skemp, 1987; Suydam, 1984) have agreed that the use of concrete manipulatives is very beneficial for teaching and learning mathematics and that it enhances students' understanding of mathematics by allowing them to discover the topic presented by their teacher. Students' use of concrete manipulatives can enhance students' visualization of shapes and thus improve their mathematics learning. "The relevant application of manipulatives to … classroom situations helps students visualize and develop problem solving strategies", says Moch (2001, p.83). Moreover, concrete manipulatives, specifically cubes, can contribute to students' more meaningful mathematical thinking and reasoning by giving them chances to construct and compare quantities so that students develop interconnected understandings of mathematical concepts (Stein & Bovalino, 2001). In other words, students can integrate and connect a variety of concepts and gain a deep understanding of them through their experience with concrete manipulatives. Concrete manipulatives also provide students with tangible and investigative experiences for two-dimensional representations and help them embody the problem-situation by touching and moving them around and finding the correct construction (e.g., see Carroll & Porter, 1997).

However, teaching three-dimensional shapes with these manipulatives might not be very beneficial if teachers dominate the use of them and students are not offered opportunities for actively engaging with the manipulatives by touching and moving them around. The way teachers integrate concrete manipulatives into their lessons is the key to students' performance, as it is in teaching other topics in maths (Alfieri et al., 2011; Wearne & Hiebert, 1988). Study 1, reported in Chapter 4, found that in classes, teachers dominated the use of manipulatives, and as a consequence, it appeared that students disengaged with the lesson content. This may even be a crucial reason for students' low performance in tests of three-dimensional shapes' understanding. In order to increase students' engagement and improve their understanding of shapes, three-dimensional shapes teaching must become more student-centred with classrooms where students actively engage in rich mathematical activities during which they have the control of manipulatives.

The designed lessons allow students to discover polycubical shapes through not only student-centred activities with unit cubes but also with the designed opportunities to reflect on these activities through teacher-guided discussions.

Moreover, the discrepancy of the use of manipulatives in the literature suggested a very specific argument for why they would be helpful in the type of problems in this research. Most people who argue against manipulatives use blocks to represent abstract concepts, for example, addition and subtraction to represent place value and percentages (e.g., Bartolini-Bussi & Mariotti, 2002; Chandler & Kamii, 2009; Fuson & Briars, 1990). Whereas, what the RETA principles are doing is much more similar to how concrete manipulatives are used in chemistry. In chemistry, students are building a model with atoms and structures so they do not have to constantly mentally imagine 3D structures; they externalize these into a 3D model (see Hegarty, Stieff, & Dixon, 2013). Similarly, in the designed lessons students will build cube constructions to externalize them so that they will not constantly need to imagine them to draw their orthogonal and isometric views. Constructions will not represent something else in mathematics (e.g., abstract symbols), i.e., they will only be externalized in order to facilitate their own drawing by students. Therefore, the researcher's intent behind using manipulatives is not the use of traditional maths concrete manipulatives which aims to teach symbolic relations. To conclude, active lessons aim to provide students with learner-centred environments where students engage with concrete manipulatives.

5.2.5. Summary of the RETA Principles

To sum up, these principles of the RETA model can be applied to a variety of topics in three-dimensional shapes in various contexts. It will still be the same model with different questions in different contexts to teach various topics. It is not claimed that the RETA principles are sufficient, other principles can be developed or enacted but the proposed principles are necessary for the teaching three-dimensional shapes in the researched context. Moreover, lesson developers do not necessarily need to use all of the principles of the model in a lesson plan, rather use them in their overall teaching on three-dimensional shapes as exemplified in the lesson plans in Section 5.3. This thesis developed the idea and investigated it empirically when teaching orthogonal and isometric representations of polycubical shapes in the context of seventh graders in two middle schools.

It is of note that the frameworks in section 2.2.3.3 influenced the way the RETA principles are designed. Particularly, Johnston-Wilder (personal communication, June 8, 2019) who is one of the designers of the ALIVE framework commented on the RETA principles at a conference by saying she thinks the RETA is very much like the ALIVE, and she could see how the RETA is picking up elements from their framework.

5.3. The Three-Dimensional Shapes Lesson Plans

The process of designing lesson plans was based on the RETA model, the findings of Study 1 and the reviewed literature. The lesson plans had to be between four to six hours as teachers in Turkish middle schools have a very tight curriculum to cover with a fixed time for teaching the topic. It should also be made clear that the lessons were designed according to the needs of the students and teachers in this context and as such do not aspire to be the best solution in other school contexts.

The following are the designed lesson plans. Each lesson plan included a lesson abstract including how the RETA principles are enacted in the lesson plans, a lesson structure including the allocated time for each of the activities, and descriptions of the activities in the format of a guide for teachers. To note, classes had different dynamics and profile of students, which made it hard to predict how things will go in different classes (e.g., how students will respond to a given activity). Thus, the time allocated for each activity was not intended to be rigidly followed by the teachers.

5.3.1. Lesson 1

Lesson abstract

Students focus on the issue of why we need two-dimensional drawings (orthogonal and isometric drawings) of three-dimensional objects. They develop an awareness of drawing views from the top, front and sides and how isometric drawing are related to real-life practices. They engage with several real-life examples and consider how these may be represented mathematically (realistic). This is, after engaging with the real-life examples, they construct polycubical shapes corresponding to pictures of 3D objects (realistic) with linking cubes (active). They create their concrete polycubical shapes in the authoring tool, created through GeoGebra. They explore the view from the front by manipulating their representation in GeoGebra (technology-enhanced). They develop an awareness of how the views change when they manipulate the shape. Lesson 1 focused particularly on a realistic principle of the RETA model where students articulate real-world experiences through videos and photographs on engineering, architecture and a drawing tool AutoCAD.

Lesson structure

- Agenda and aims (5 minutes)
- Real-life examples: Engineering, Architecture (10 minutes)
- Tools for drawing: AutoCAD, GeoGebra (15 minutes)
- Activity: Constructing buildings and drawing their front views (15 minutes)
- Conclusion and feedback (5 minutes)

Agenda and aims

Introduce yourself, if necessary.

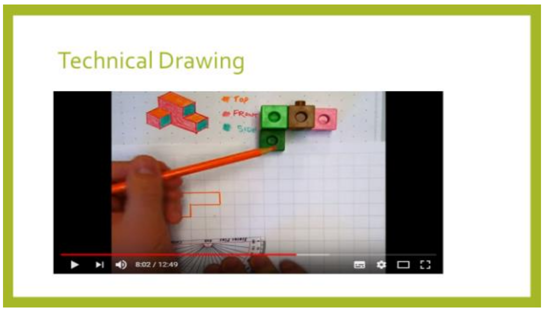
Explain to students that during the next four lessons we are going to discover different types of drawings and drawing tools, and relate our learning with their use in real-life situations.

Ask whether students have any question before starting to the first lesson, and answer their questions if any.

Explain to students that in this lesson we are going to consider some real-life examples to understand why people need two-dimensional representations of three-dimensional objects.

Engineering

Show a two-minute part of the technical drawing video. Explain why we might need to learn such drawings and how these can be related to mathematics.



Explain that this is how engineers start to draw multi-views of the shapes. They use different views to design different parts of the machines. All of the machines we use in daily life, such as computers, mobile phones, hairdryers and fruit squeezers, are composed of small parts, which were designed and drawn by engineers. Ask whether anyone's mother or father is an engineer or whether anyone has seen such a drawing before. Invite students to share their ideas as well.

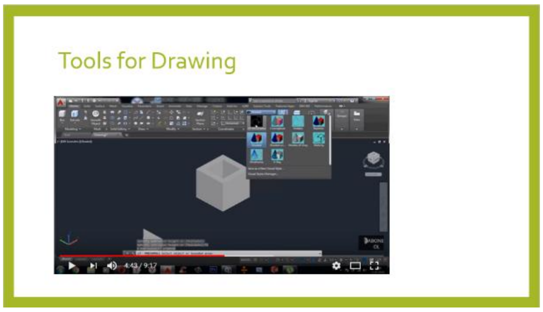
Architecture

Show the whole class the architecture photo and repeat the same procedure. Discuss why architects need to learn these drawings and how these can be related to mathematics. Invite students to share their ideas as well. Summarise the discussion with a few sentences.

Explain that the drawing in the video is the first step to draw plans of the houses, or draw new interior designs of the houses. All architects similar to engineers learn how to draw multi-views and prepare projects based on this knowledge. Such drawings are natural parts of their jobs. Architects also try to draw these shapes more clearly as their drawing should be easy to understand by the people who ask their help or their customers.

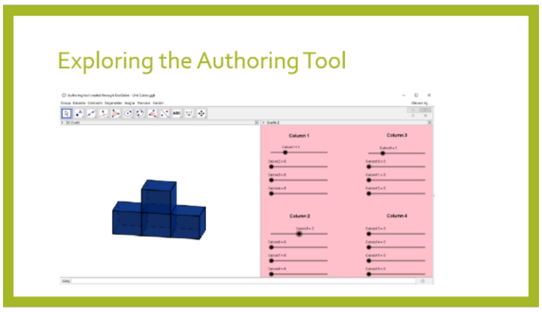
Tools for drawing

Show whole class a two-minute part of the engineering video. Explain that this is how engineers use a tool to construct their shapes. Discuss how such tools might help them draw three-dimensional polycubical shapes in two-dimension. Invite students to share their ideas as well.



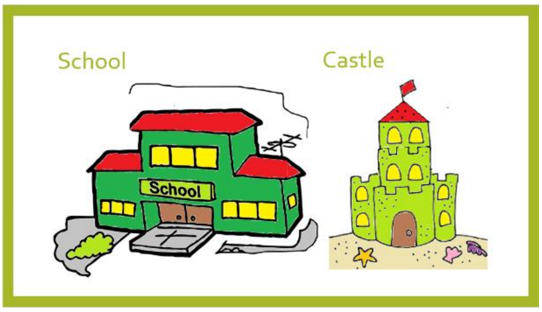
Exploring the authoring tool

Explain to students that there are some tools which help us in drawing shapes similar to engineers.



Introduce the authoring tool. Ask students to turn on their tablets and run the authoring tool created through GeoGebra. Give them some time to explore how it works. Move to construction examples after the discovery of GeoGebra.

Constructing buildings – Activity 1



Organise students into groups of 2-3.

Give a box of linking cubes to each group. Show the school picture and ask them to construct the school using linking cubes. Check their constructions and discuss their answers. Repeat the same procedure for the castle.

Possible correct answers include the following:

Figure 5.1. Possible correct constructions from linking cubes – I

Students may construct the castle totally different than each other as only one view (front view) of the castle is seen from the pictorial representation. Here, explain that we need more than one view to construct the exact shape. Then, ask at least how many views we need and why?

Show the slide – Castle.

Ask them to construct the castle from linking cubes. Following that ask them to construct the same castle on the tool (by now they are familiar with both the polycubical shape and its pictorial representation).

Note: If there are students who constructed with a different depth, remind the point discussed before: Having only one view of a shape is not enough to decide its all dimensions, therefore we may not construct the exact shape only having its one view.

Ask them to remind you at least how many views we need to construct the exact shape and why?

After they all construct the shapes on the authoring tool, give a copy of the Activity Sheet Castles and focus on the front view of the first representation.

Invite the groups of students to manipulate their GeoGebra constructions to decide how to represent the front view. Ask all students to draw the front view on the dotted paper individually.

Do not forget to ask students to save their files before moving to the next question on the sheet.

Please note that it might be useful or easier for students to see the depth if we use the angles of the isometric paper on the tool (30^{o}-60^{o}), so before giving the authoring tool to students set the angle accordingly. After they manipulate the shape they may decide which angle they would like to use.

The following are a possible correct GeoGebra construction of the castle and its front view:

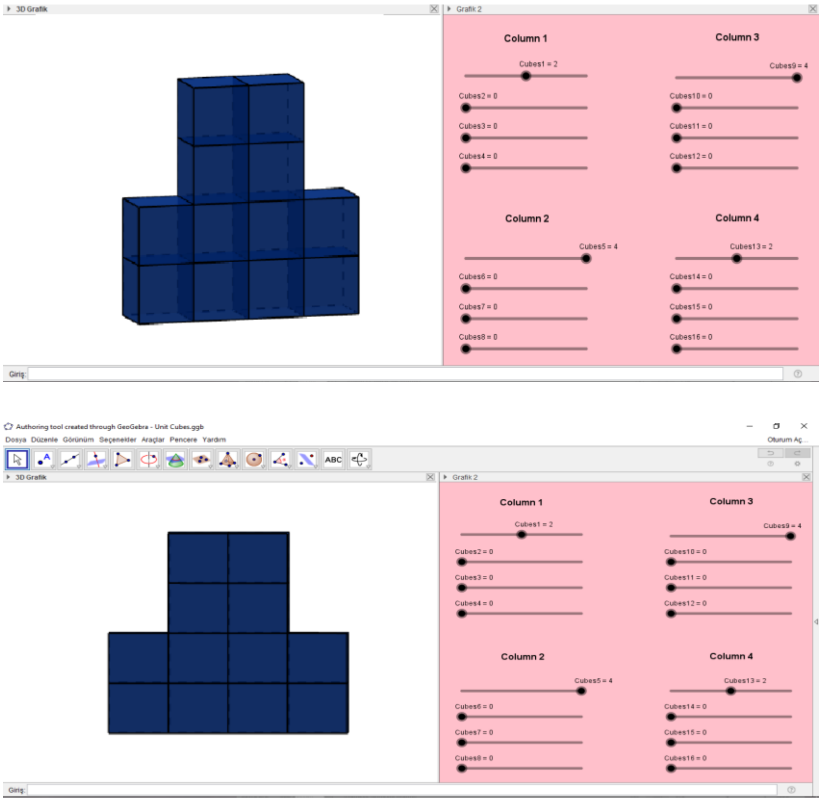


Figure 5.2. A possible correct GeoGebra construction and its front view – I

<u>Constructing buildings – Activity 2</u>

(1) Show the next slide having pictures of the buildings which require relatively more complex constructions. Point the school picture and ask them to construct the school using linking cubes or/and GeoGebra. Check their constructions and discuss their answers. Repeat the same procedure for the castle. Collect students' constructions to give them back in the next lesson.

Some of the possible correct answers could be the following

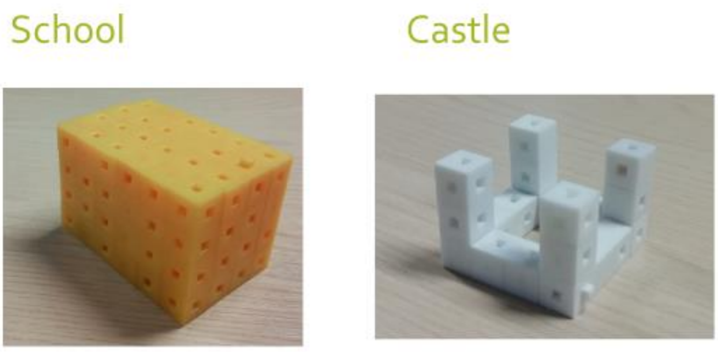
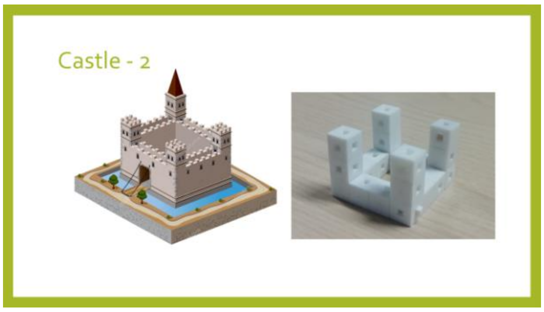


Figure 5.3. Possible correct constructions from linking cubes - II

(2) Show the slide – Castle – II.



Ask students to construct the castle, which they constructed from the linking cubes, on the tool. After they all construct the shapes on the authoring tool, invite them to focus on the front view of their Castle II representation on the tool.

Invite the groups of students to manipulate their GeoGebra constructions to decide how to represent the front view. Ask all students to draw the front view on the dotted paper individually.

The following are a possible correct construction of the castle and its front view:

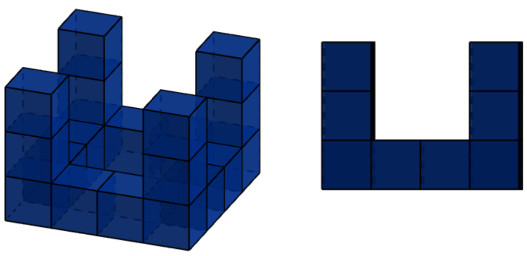


Figure 5.4. A possible correct GeoGebra construction and its front view - II

(3) Show the slide – Question – III.

Follow the same procedure for the third question of the worksheet as well. This time, do not ask students to construct the shape with linking cubes. However, students who need concrete construction may continue constructing with them.

Do not forget to ask students to save their files in GeoGebra before moving to the next question on the sheet as they will use them during the next lessons while exploring the views from top and sides.

The following are a possible correct construction of the third representation and its front view:

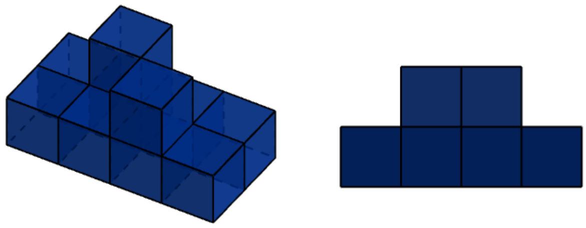


Figure 5.5. A possible correct GeoGebra construction and its front view - III

Ask students to construct other shapes having the same front view. Say that students can choose to use linking cubes or GeoGebra to construct the shape. The following are possible answers.

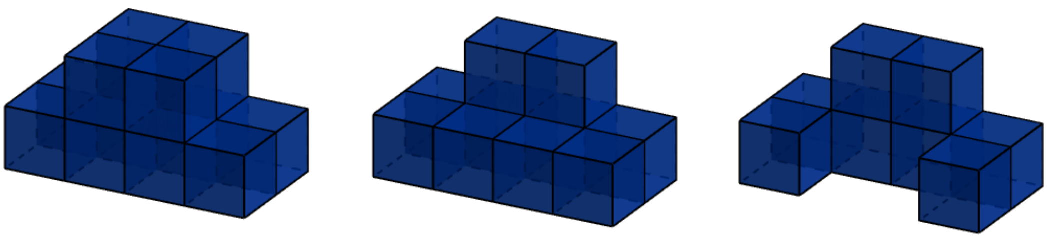
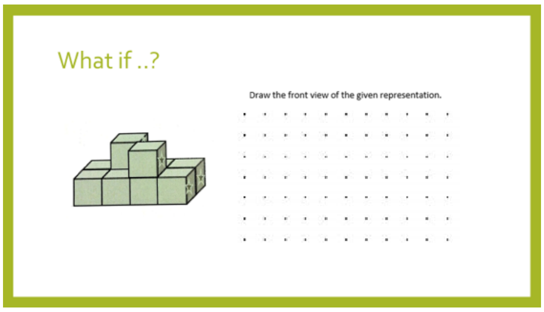


Figure 5.6. Possible correct GeoGebra constructions

Conclusion and feedback



Conclude the lesson with a question. Show the slide What if, and distribute the sheets of paper to students and ask them to draw the front view of the given shape on the dotted paper.

This question is different than the other questions and the stars indicating the front view is not in the front perspective. The aim is to raise awareness that front views of polycubical shapes can change according to the perspective we look at.

Some students might tend to draw the front view similar to the front view of the third question since the shape actually is the same and the only change is that the stars indicating the front view. Ask them to go back to their GeoGebra constructions and manipulate their constructed shapes to indicate the front view.

The following are a possible correct construction of the given representation and its front view.

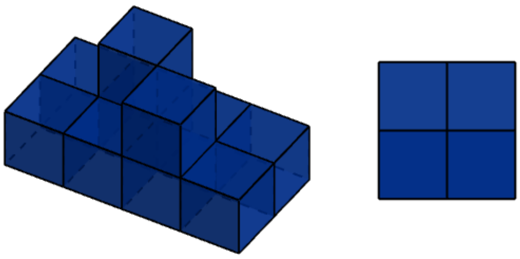


Figure 5.7. A possible correct GeoGebra construction and its front view - IV

Explain to students that in this lesson we looked at real-life examples where orthogonal drawings are used, and a real-life example where a dynamic tool used for drawing. Then, we constructed 3D shapes from linking cubes to represent some buildings in the given pictures mathematically. We explored an authoring tool and represented polycubical shapes on it, and used the tool to draw the view from the front on the dotted paper. The following lesson will be about the views from the top and sides.

5.3.2. Lesson 2

Lesson abstract

Students focus on drawing the views from the top and sides. They engage with two real-life examples where they discuss how mechanical and earthquake engineers use drawings of top and side views (realistic). They use polycubical shapes they previously constructed from linking cubes and in GeoGebra. They explore these views successively, by manipulating their representations in GeoGebra (technology-enhanced). They develop an awareness of how the views change when they manipulate the shape. First linking cubes, then GeoGebra fades away by the end of the lesson. They diagnose and remediate and discuss possible reasons for worked examples with designed mistakes (exploratory). In this specific lesson, they also compare and contrast the views from left and right, and discover symmetry. For lesson 2, this had a particular focus on a technology-enhanced part of the RETA model where students drag and manipulate their representations in GeoGebra to discover top and side views.

Lesson structure

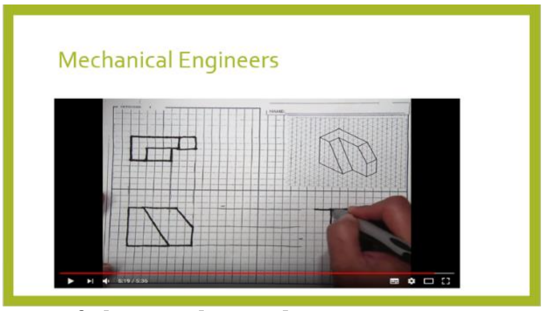
- Agenda and aims (5 minutes)
- Real-life examples: Mechanical and earthquake engineers (10 minutes)
- Activity: Learning to draw top views (15 minutes)
- Activity: Learning to draw side views (15 minutes)
- Discovering symmetry (5 minutes)
- Conclusion and feedback (5 minutes)

Agenda and aims

Remind students that this lesson is about two-dimensional representations/drawings of the shapes that they have started in the previous lessons. In the first lesson, we learnt how to draw the view from the front on the dotted paper. The first half of this lesson will be about the view from the top. Then, we will continue with the views from the left and right.

Mechanical engineers

Open the slide – Mechanical engineers.



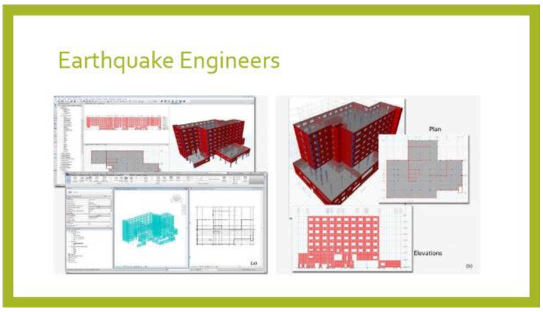
Explain to students that in this lesson we are going to consider some real-life examples to understand why people need two-dimensional representations of three-dimensional objects. Show a two-minute part of the mechanical engineers' video on design drawing.

Ask students to discuss the video focussing on different strategies in the drawings in this field and why such drawings might be important for the mechanical engineers.

Explain why we might need to learn such drawings focussing on the top and side views and how these can be related to mathematics.

Earthquake engineers

Open the slide – Earthquake Engineers.



Explain that the view from the top can also be called a plan in professional life. Earthquake engineers, for example, need to draw plans in order to design characteristics of the buildings.

Discuss (1) where else we see plans in real life, (2) whether they needed to use a plan before.

Possible Student Responses

Shopping centres' floor plans, earthquake plans, and buildings' fire plans etc.

Yes, classroom plans, and while learning sketches and scaling in geography classes

Learning to draw top views – Activity 1

(1) Show the slide – Castle.

Distribute their first castle polycubical shapes they constructed from linking cubes. Ask them to open their corresponding files on the authoring tool.

After they all open their representations on the authoring tool, give a copy of the Activity Sheet: Learning Top Views and focus on the top view of the first representation.

Invite the groups of students to manipulate their GeoGebra constructions to decide how to represent the top view. Remind students that they need to face the front view to decide the required top view (for the ministry exam questions). Ask all students to draw the top view on the dotted paper individually.

Do not forget to ask students to save their files before moving to the next question on the sheet.

The following are a possible correct GeoGebra construction of the castle and its top view:

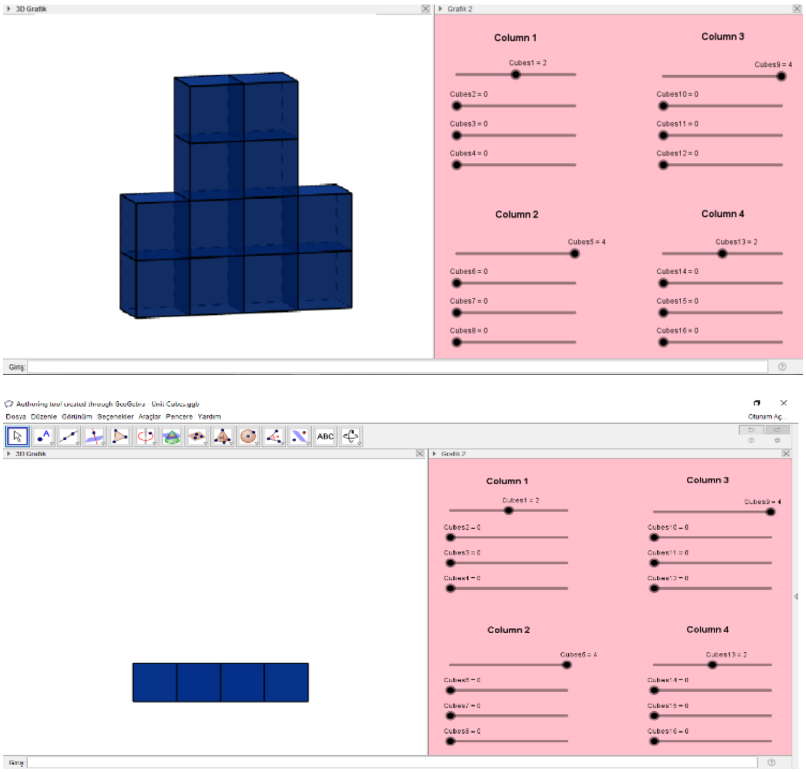
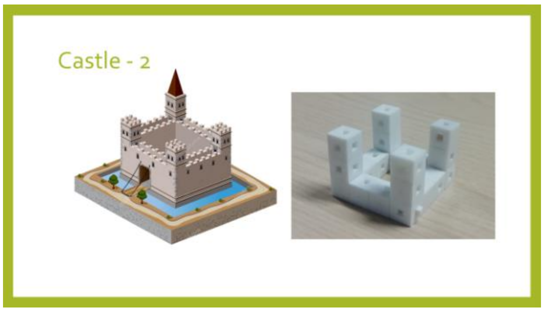


Figure 5.8. A possible correct GeoGebra construction and its top view - I

(2) Show the slide – Castle – II.



Ask students to open their corresponding files on the tool. After they all open their representations on the authoring tool, invite them to focus on the top view of their Castle II representation on the tool.

Invite the groups of students to manipulate their GeoGebra constructions to decide how to represent the top view. Ask all students to draw the top view on the dotted paper individually.

The following are a possible correct construction of the castle and its top view:

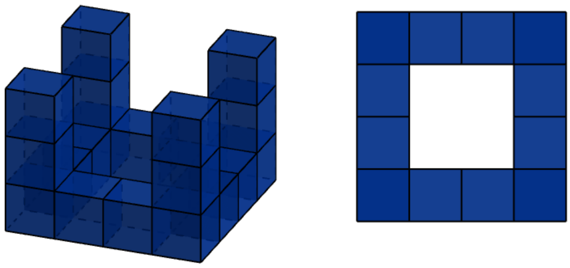


Figure 5.9. A possible correct GeoGebra construction and its top view - II

Ask students to construct other shapes having the same front view. – Could you do another shape having the same top view? Say that students can choose to use linking cubes, GeoGebra or both to construct their shapes.

Possible answers

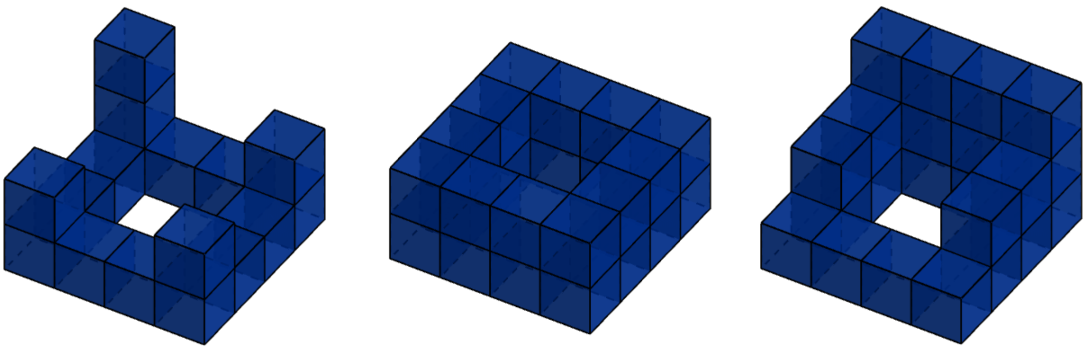
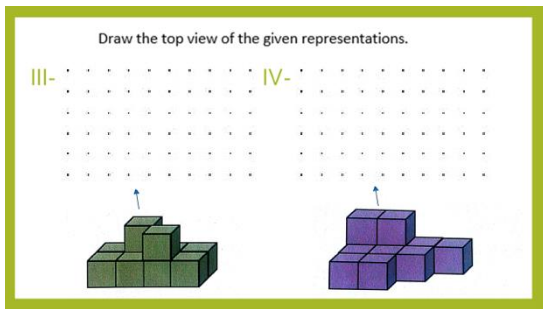


Figure 5.10. Possible correct GeoGebra constructions

(3) Show the slide – Question – III.



Follow the same procedure for the third and the fourth questions of the worksheet. Invite groups of students to discuss the top view of the shapes. Ask all students to draw the top view on the dotted paper individually.

Note that students do not have concrete polycubical shape constructed from linking cubes for the third and the fourth questions, they only have authoring tool representation of the third one.

Again note that students have neither the concrete polycubical shape constructed from linking cubes nor the GeoGebra authoring tool representation of the fourth question. If they need, provide GeoGebra authoring tool for help.

Do not forget to ask students to save their files (if they use them) before moving to the next question on the sheet because they may use them during the second half of the lesson while exploring the views from sides.

The following are a possible correct construction of the third representation and its top view:

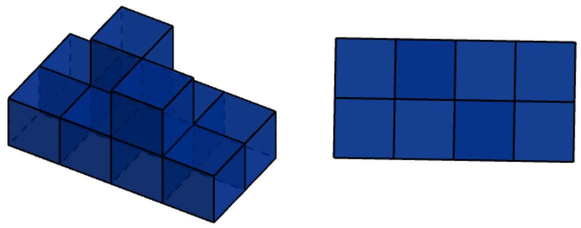


Figure 5.11. A possible correct GeoGebra construction and its top view - III

Repeat the same procedure for the fourth question.

The following are a possible correct construction of the fourth representation (if needed) and its top view:

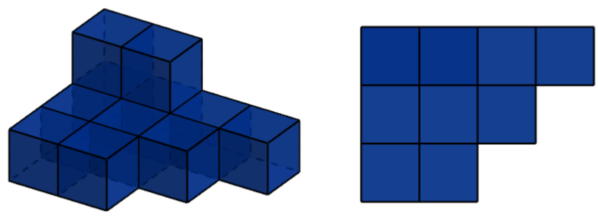
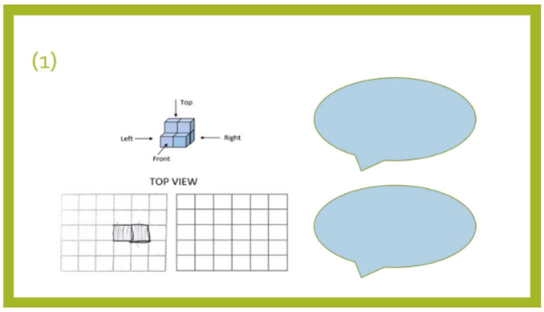


Figure 5.12. A possible correct GeoGebra construction and its top view - IV

<u>Learning to draw top views: Activity 2 – Finding the mistakes</u>

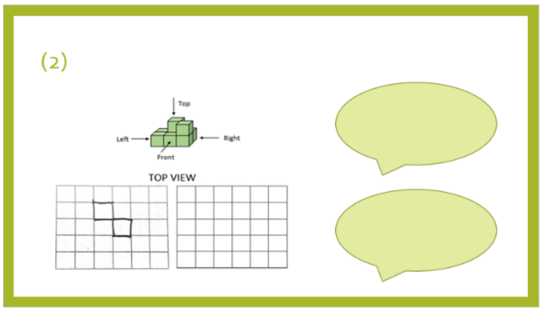
Show the slide – Find the mistake and discuss why – I. Provide one activity sheet to each student.



Say that here is a student’s work. Ask “what is the mistake and why is that?”

Ask students to discuss why it's wrong in their groups. Invite students to share their ideas with the whole class. Ask them to draw correct representation individually.

Show the slide – Find the mistake and discuss why – II.



Follow the same procedure with the first question for this one as well.

Ask students to discuss why it's wrong in their groups. Invite students to share their ideas with the whole class. Ask them to draw correct representation individually.

Explain to students that in this lesson so far we looked at a real-life example where top view used. We discussed where else we see plans in real life, and whether they are needed to use a plan before. We drew top views or plans of the provided isometric representations. Now, we focus on the views from sides, with a special focus on the relationship between the views from the left and right.

<u>Learning to draw side views</u>

(1) Show the slide – Castle.

Distribute their first castle polycubical shapes they constructed from linking cubes. Ask them to open their corresponding files on the authoring tool.

After they all open their representations on the authoring tool, give a copy of the Activity Sheet: Learning Side Views and focus on the left and, then, the right view of the first representation.

Invite the groups of students to discuss how to manipulate their GeoGebra constructions to decide how to represent the left and the right views on the dotted paper. Remind students that they need to face the front view when deciding the required views. Ask all students to draw the side views on the dotted paper individually.

The followings are a possible correct GeoGebra construction of the castle and its views from the left and the right:

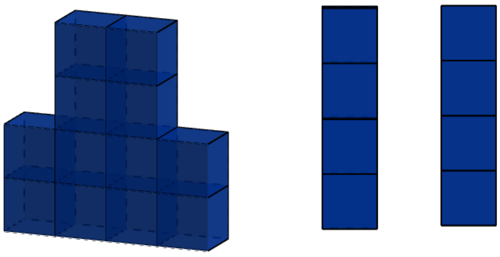
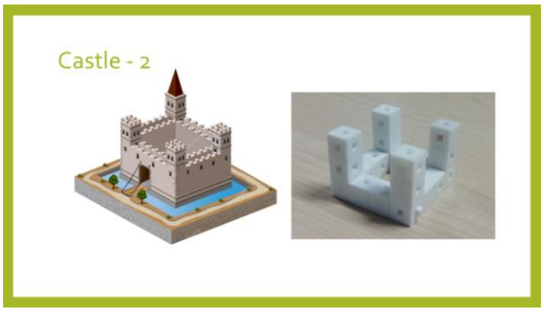


Figure 5.13. A possible correct GeoGebra construction and its left and right views - I

(2) Show the slide – Castle – II.



Ask students to open their corresponding files on the tool. After they all open their representations on the authoring tool, invite them to focus on the left, then the right view of their Castle II representation on the tool.

Invite groups of students to discuss how to manipulate their GeoGebra constructions to decide how to represent the side views on the dotted paper. Remind students that they need to face the front view when deciding the required views. Ask all students to draw the left, then right views on the dotted paper individually.

The following are a possible correct construction of the castle and its side (left and right) views:

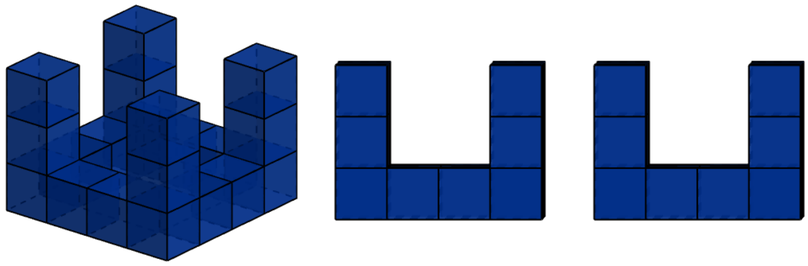
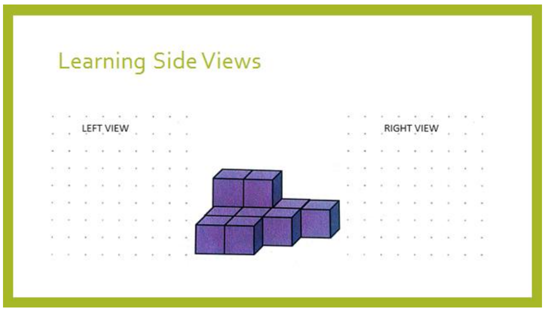


Figure 5.14. A possible correct GeoGebra construction and its left and right views - II



(3) Show the slide of the third question. Follow the same procedure with the first two questions for this question of the worksheet.

Invite groups of students to discuss the left and then the right view of the shape. Ask all students to draw those views on the dotted paper individually.

Note that students do not have concrete polycubical shape constructed from linking cubes for the third question, they may only have authoring tool representation of the third shape (if they chose to construct it for the top view). This is aimed to fade the concreteness away by the end of the lesson for the views from the left and right.

The following are a possible correct construction of the third representation and its views from the left and right.

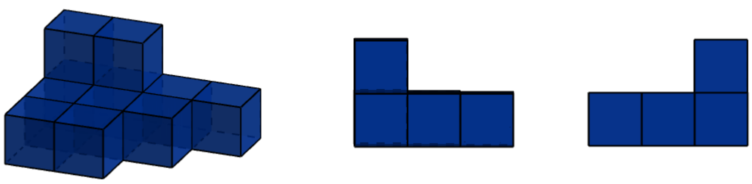
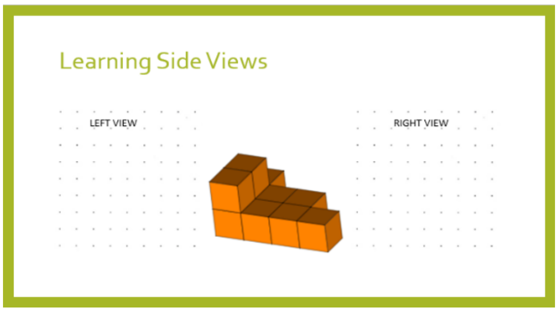


Figure 5.15. A possible correct GeoGebra construction and its left and right views - III

(4) Show the slide including the fourth question. Follow the same procedure for this question of the worksheet, too.



Invite groups of students to discuss the left and the right views of the shape. Ask all students to draw those views on the dotted paper individually.

Again note that students have neither the concrete polycubical shape constructed from linking cubes nor the GeoGebra authoring tool representation of the fourth question. If they need it, provide GeoGebra authoring tool for help.

The following are a possible correct construction of the fourth representation and its views from the left and right.

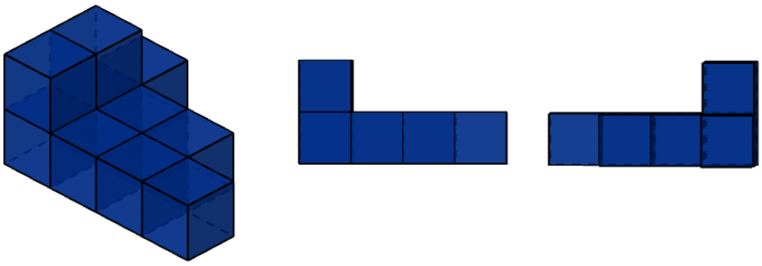
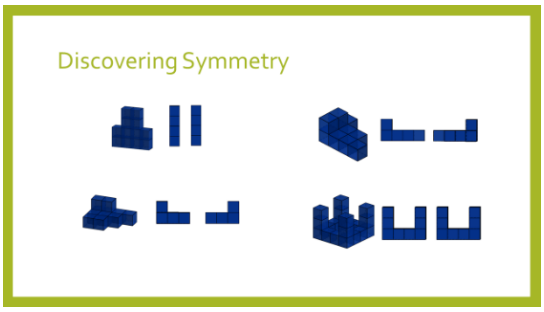


Figure 5.16. A possible correct GeoGebra construction and its left and right views - IV

<u>Discovering symmetry</u>

Show the slide – Discovering Symmetry.



Explain that we now will discover the symmetry and describe it with our own words. To do that, ask students whether they see a relationship between the view from the left and right.

Possible Student Responses

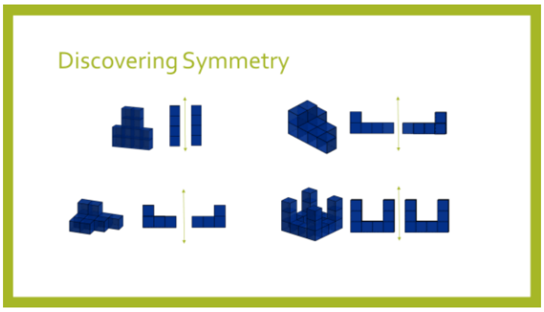
They are the same for two of the examples (blue examples on the worksheet) but not for the other two.

They are the same but face/look to opposite directions, see orange and purple representations on the worksheet, they are actually the same, having the same amount of squares.

They are symmetric to each other, it is something like a mirror; when we look in the mirror if we wave our left hand we saw in the mirror that we wave our right hand.

After all responses, explain to students that symmetry means one shape becomes exactly like another when you move it in some way, in our case, around a line. To be symmetric, two shapes must be the same size with one shape having a different orientation from the other one. Views from the left and right are always symmetric to each other.

Show or draw the symmetry lines.



Next year we will learn variations of symmetry, the specific name of the symmetry we see in the left and right views is called the line symmetry. You may also come across with the names mirror symmetry and reflection symmetry, they both refer line symmetry. Line symmetry happens when a shape is reflected across a line (like looking in a mirror).

If students ask other symmetry types, you may name point symmetry and rotational symmetry without details.

<u>Conclusion and feedback</u>

You already summarised the first half the lesson which is about drawing top views. Explain to students that in the first half we learnt the top views and in the second half of the lesson we looked at two-dimensional drawings of the views from the left and the right. We drew the left and the right view representations of the provided shapes. We discussed the relationship between opposite views focussing on the views from the left and right, and we called it line symmetry.

Explain to students that this is the last lesson about orthogonal drawings, which are the views from the front, top, left and right. The following lesson will be about isometric drawings of the polycubical shapes when their orthogonal drawings are available. We will discuss real-life uses of isometric drawings and then, draw shapes on an isometric paper.

5.3.3. Lesson 3

<u>Lesson abstract</u>

Students focus on isometric drawings corresponding to the given representations (four views – front, top, left and right) of three-dimensional objects. They develop an awareness of real-life uses of isometric drawings and how an isometric drawing and drawing views from the top, front and sides are related to each other (realistic). They engage with several isometric drawing examples and consider how these may be represented mathematically. This is, they construct polycubical shapes, from unit cubes, corresponding to four views of three-dimensional objects (active). They are given an opportunity to construct these shapes in GeoGebra before drawing them to an isometric paper as a two-dimensional isometric representation (technology-enhanced). Lesson 3 emphasised the active principle of the RETA model where students used manipulatives themselves to build polycubical shapes to draw them isometrically so that they do not need to constantly mentally imagine.

<u>Lesson structure</u>

- Agenda and aims (5 minutes)
- Real-life examples: Architecture and engineering (10 minutes)
- Activity: Constructing isometric drawings (15 minutes)
- Activity: From elevations to isometric drawings (15 minutes)
- Conclusion and feedback (5 minutes)

<u>Agenda and aims</u>

Explain to students that drawings we have drawn so far are also called elevations (front elevation, right elevation, left elevation etc.). Note that the top elevation also has a special name: a plan.

Remind students that in the previous lessons we had a 3D-like representation of a shape and we drew its four views: elevations and plan.

Explain to students that in this lesson, we have four views (the elevations and plan) to construct shapes to make their 3D-like drawings. There are different strategies to draw 3D-like shapes. We will first discuss how architects and engineers draw them. Then, we will construct our 3D-like drawings on a special paper, drawings which are more similar to how engineers draw.

<u>Architecture</u>

Explain to students that in this lesson first, we are going to consider some real-life examples to understand why people need two-dimensional representations of three-dimensional objects.

Show the whole class the first two-minute of the architecture video.

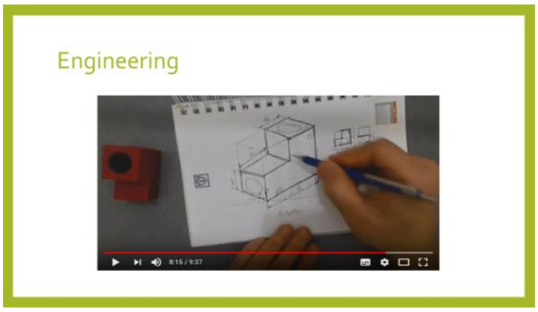
Describe the drawing strategy of the architects and how the drawing looks 3D-like.

Allow students to comment on it if they want to. Then, continue with the next video. A deep discussion about the first one, at this point, is not necessary because there will be a comparison between two strategies used in these two videos/occupations to make 3D-like drawings.

<u>Engineering</u>

Show the whole class the engineering video.

After the second video, explain why architects and engineers need to learn these drawings and how these can be related to mathematics.



Invite students to share their ideas as well. Then, move to the drawing activity.

<u>Drawing activity</u>

Explain how to draw a cube to an isometric paper (teach both ways).

Explain how to add cubes to their drawings

(a) to the front, to the back
(b) to the left and to the right
(c) to the top and to the down

of their drawings by drawing with them simultaneously.

Give them some time to try and find their own strategy to draw as well. Students may use linking cubes or GeoGebra to construct the same shape. Ask them about elevations.

How many squares can you see from the front/top/side?

What about when we add one more to top of/next to it? What about now?

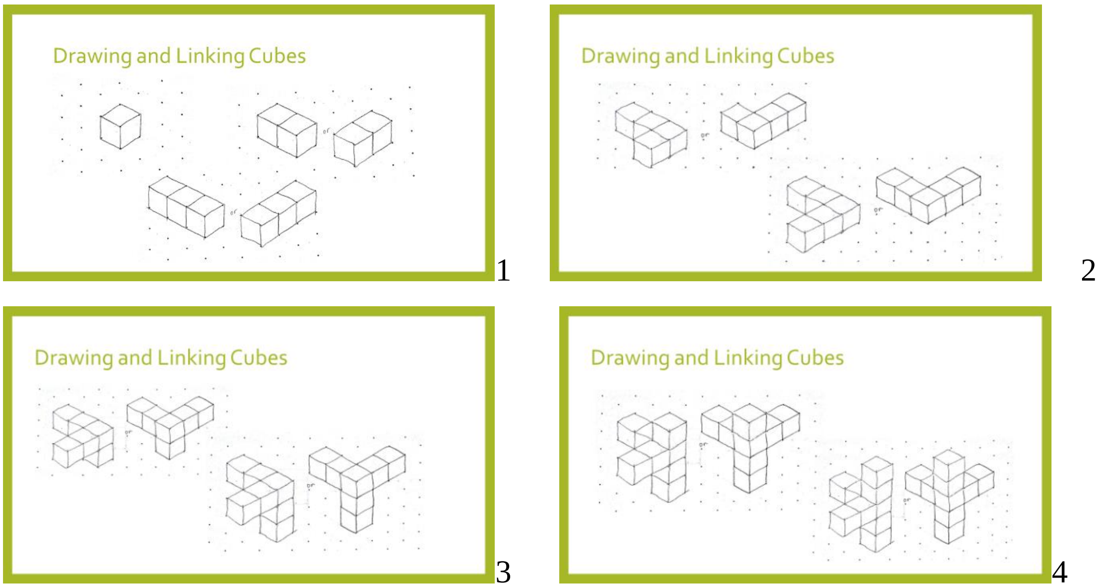


Figure 5.17. Step by step isometric drawings

<u>From elevations to isometric drawings</u>

Distribute the activity sheets (provide students elevations) and ask them to construct the shape using linking cubes, then in GeoGebra. May continue without linking cubes if you feel students do not need these.

Ask students to draw the shape they constructed from linking cubes and/or created in GeoGebra on the isometric paper individually.

Guide them when they construct their shapes and when they draw corresponding isometric representation on the isometric paper.

Then show the shape constructed in GeoGebra – which are on the same slides but come with a further click.

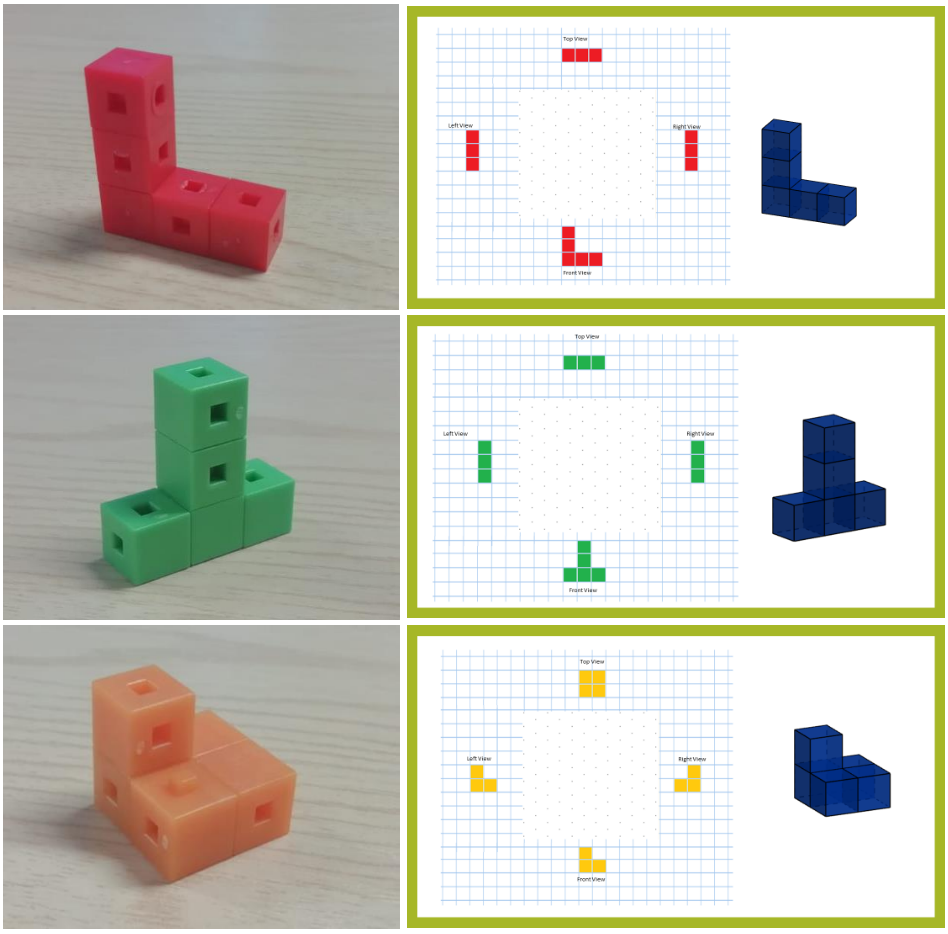


Figure 5.18. Possible cube constructions and isometric drawing slides for lesson 3

<u>Conclusion and feedback</u>

Explain to students that in this lesson we looked at some drawing examples from real life. We started constructing isometric drawings of the polycubical shapes we built with linking cubes and/or in GeoGebra.

Explain to students that the following lesson will be about further isometric drawings. We will then discuss some real-life examples to see what might happen if people misdraw these representations. We will finish the lesson by reviewing what we have done in the last four lessons through worked examples.

5.3.4. Lesson 4

Lesson abstract

Students focus on isometric drawings corresponding to the given four views – front, top, left and right – of three-dimensional objects. They engage with several isometric drawing examples and consider how these may be represented mathematically. The questions in this session have more cubes in different dimensions to challenge students. Students focus then on the issue of why two-dimensional drawings (orthogonal and isometric drawings) of three-dimensional objects are important. They develop an awareness of how orthogonal and isometric drawings are related to real-life practices (realistic). They engage with several real-life examples and consider how some of these are misrepresented mathematically and might be the possible reasons for them. They discuss some worked examples where they had been purposefully misdrawn (exploratory). The emphasis of Lesson 4 was the exploratory principle of the RETA model where students worked with designed mistakes in worked examples to diagnose and remediate and to discuss possible reasons for them.

Lesson structure

- Agenda and aims (5 minutes)
- Real-life example: Interior architects (5 minutes)
- Activity: Challenge yourself (15 minutes)
- Real-life examples: Exploring real-life examples (10 minutes)
- Activity: Finding the mistakes (15 minutes)
- Conclusion and feedback (5 minutes)

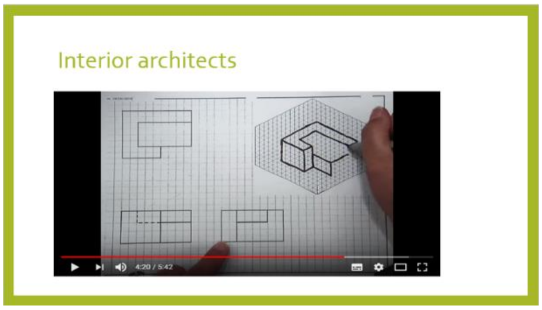
Agenda and aims

Explain to students that this is the last lesson about two-dimensional drawings. In this lesson, we will be dealing with examples in a similar way we did last week/lesson. We will discover another job which uses isometric drawings. We will draw isometric representations of the 3D shapes whose elevations and plans are given. These questions are modified from past ministry exam questions. Let’s try to draw them!

Explain to students that the lesson will continue with exploring common errors in drawing views from the front, top, and sides and in an isometric drawing. We will also discuss some real-life examples to see what might happen if people misdraw or misinterpret these representations. Then, we will discuss and critique some students' work where they were given isometric drawings and asked to draw elevations and plan and vice versa, and we will draw the correct representations.

Interior architects

Open the slide – Interior architects.



Explain to students that in this lesson we are going to consider another real-life example to understand why people need two-dimensional representations of three-dimensional objects. Show a two-minute part of the interior architects' video on design drawing.

Ask students to discuss the video focussing on different strategies in the drawing and why such drawings might be important for the interior engineers.

Explain why we might need to learn such drawings focusing and how these can be related to mathematics.

Drawing activity time - Challenge yourself

Distribute the activity sheets having four modified past ministry exam questions from 2008 and 2010 (MoNE, 2016a). Follow the same procedure with the previous lesson. Ask them to construct the shape using linking cubes, then in GeoGebra. May continue without linking cubes and/or GeoGebra if you feel students do not need these.

Ask students to draw the shape they might have constructed from linking cubes and created in GeoGebra on the isometric paper individually.

Guide them when they construct their shapes and when they draw corresponding isometric representation on the isometric paper.

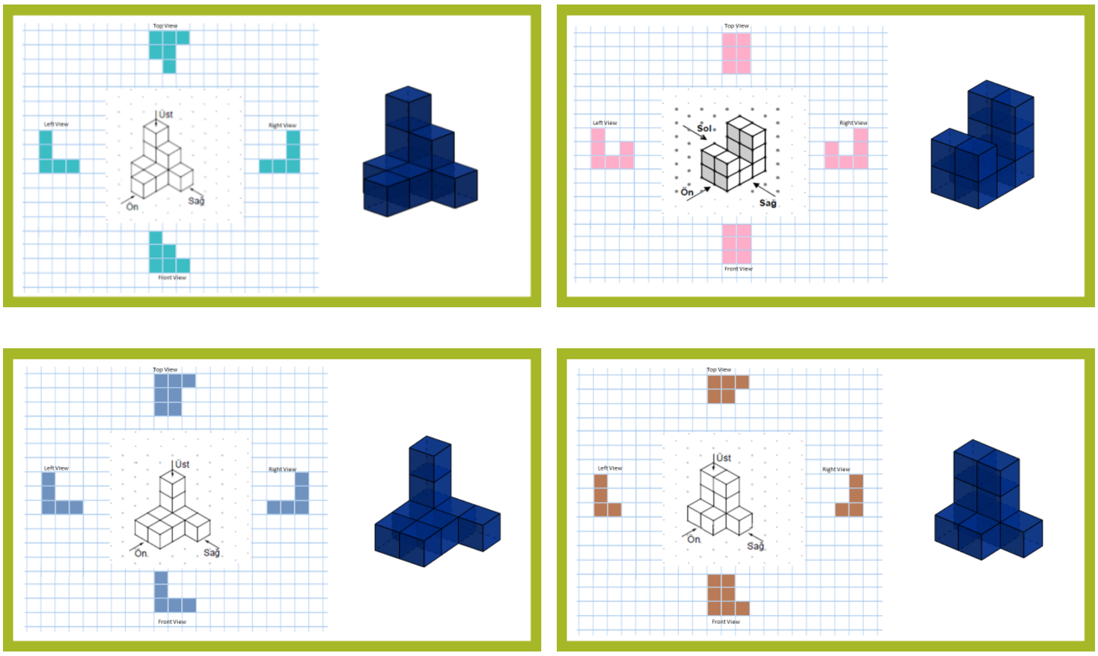


Figure 5.19. Isometric drawing slides for lesson 4

Slides have animations so the answers will appear step by step in each click. The following is an example to these steps – this applies to all isometric drawing question in this lesson.

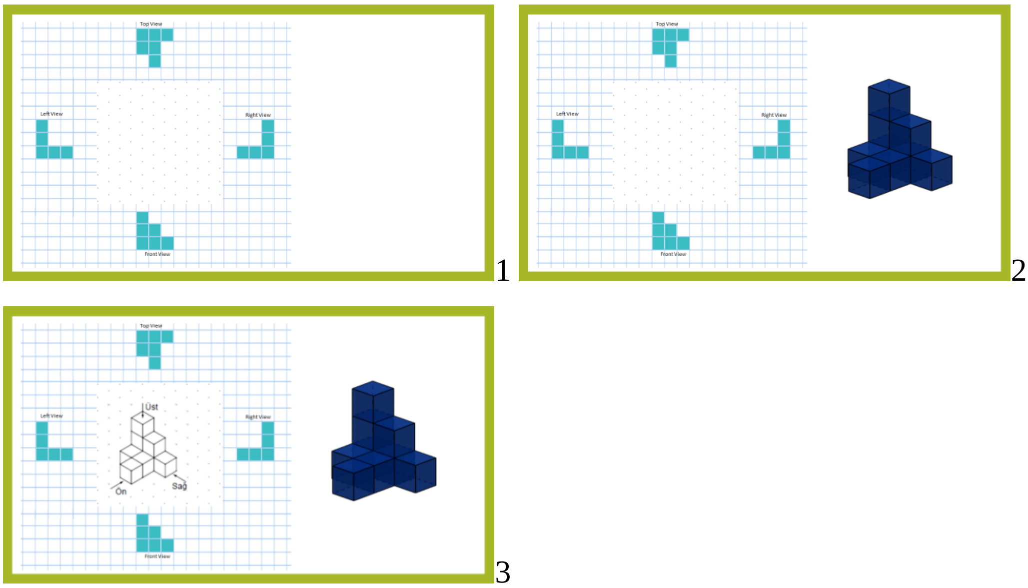


Figure 5.20. Isometric drawing slides' animation steps

<u>Exploring real-life examples</u>

(a) Organize the class into groups of 2-3 students.

Ask to discuss what might happen if elevations are drawn incorrectly in real life, and how these mistakes might affect our lives. Invite students to share their opinions with the whole class.

(b) Show the following photos one by one and ask students to discuss what might be wrong in the drawings or in their interpretations so that the result might cause these problems.

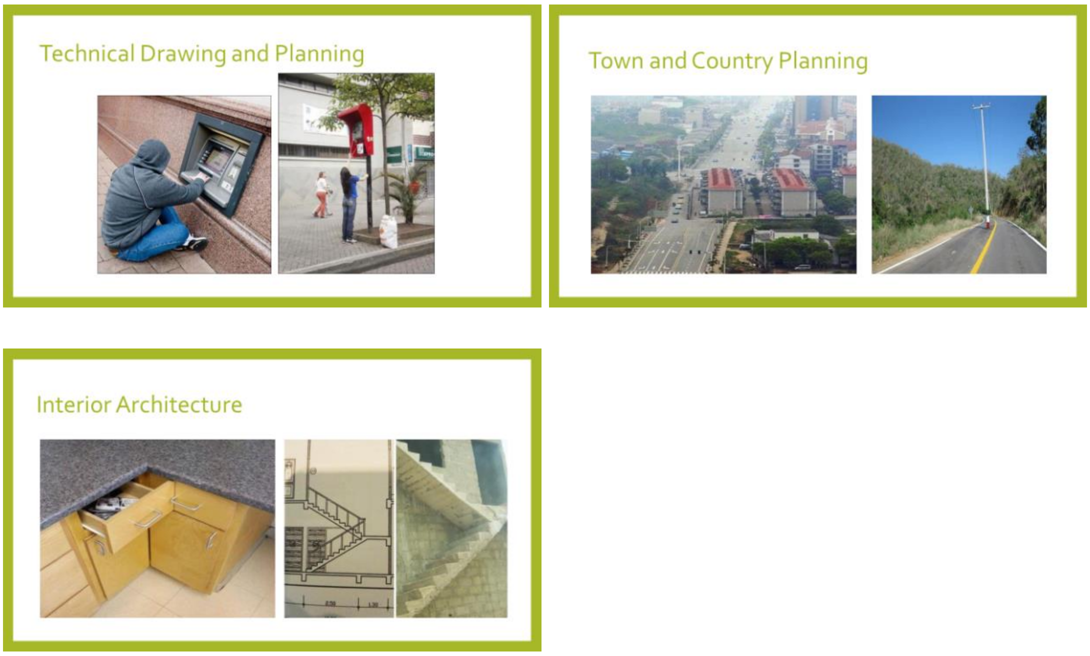
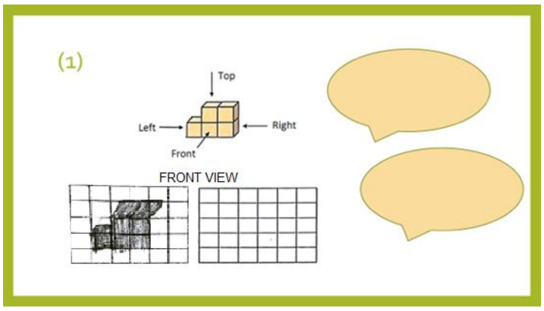


Figure 5.21. Real-life examples of lesson 4

<u>Activity: Finding the mistakes</u>

Provide one activity sheet to each student. Activity sheets have five worked examples, specifically designed mistakes. Explain to students that now we will explore common student errors in orthogonal and isometric drawings.

(a) Show the slide – Find the mistake and discuss why – I.



Say that here is a students' work. Ask them the following question. What is the mistake in the first drawing and why is that?

Ask students to discuss why it’s wrong in their groups and to note the reason for it to their worksheets.

Invite students to share their ideas with the whole class. Ask them to draw the correct representation individually.

The following is the correct drawing for the first question.

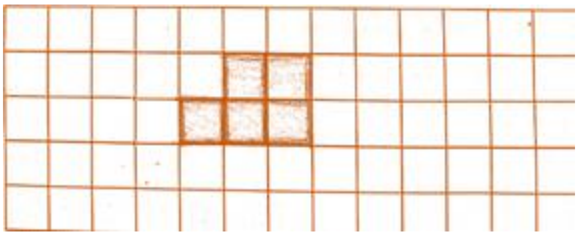


Figure 5.22. A possible correct drawing for the worked example - I

(b) Show the slide - Find the mistake and discuss why – II, III, IV, V.

Follow the same procedure with the first question for these as well.

Ask students to discuss why it’s wrong in their groups.

Invite students to share their ideas with the whole class and to note the reason for it to their worksheets. Ask them to draw correct representation individually.

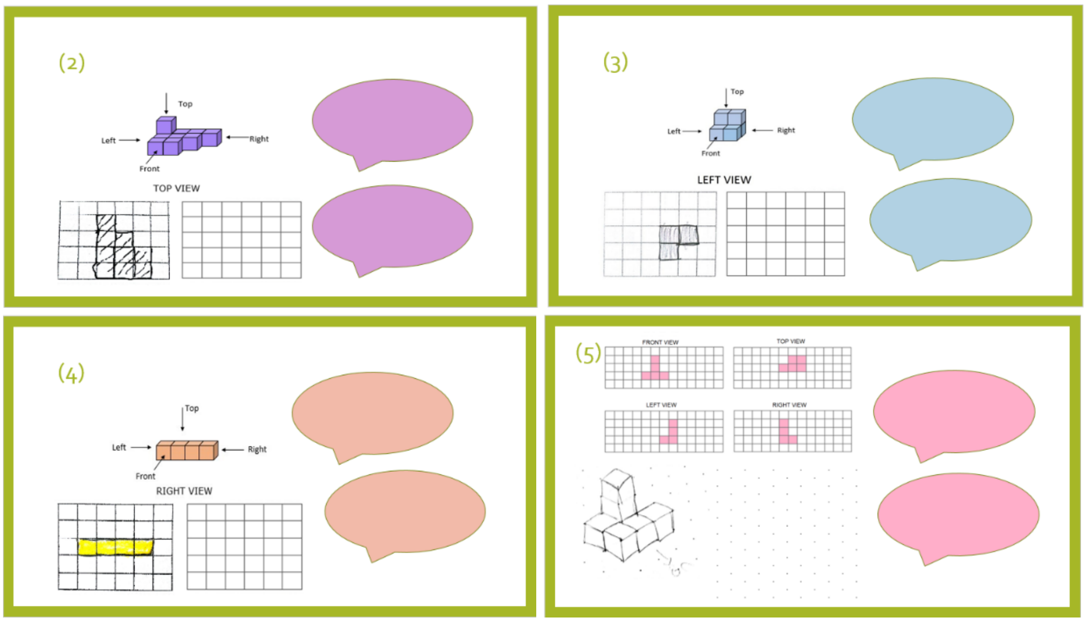


Figure 5.23. Worked examples – II, III, IV and V

Conclusion and feedback

Explain to students that in this lesson we practised isometric drawings, which were very similar to the past ministry exam questions. We discussed real-life examples where these two-dimensional drawings were used incorrectly. Then, we looked at and critiqued a student's work which had been misdrawn and corrected mistakes.

5.4. Conclusion

In conclusion, this chapter has summarised the RETA principles and shown how they were implemented in the first version of the lesson plans. Chapter 6 discusses how they were utilised in the first empirical study.

6. STUDY 2: 3D SHAPES AFTER-SCHOOL LESSONS

The main objective of this study was to see whether the RETA-based lessons are engaging and effective and to look for opportunities to improve them where they were found not to be. The researcher acted as an after-school mathematics teacher and taught the RETA-based lessons to explore how they worked in practice and what needs to be improved. In order to achieve this objective, this chapter describes the first implementation of the designed lessons and tries to answer two research questions:

- How do seventh grade students experience the RETA-based lessons?
- What are the outcomes of the RETA-based lessons for these students?

This is an iterative cycle to see how the RETA-based lessons worked (i.e., how the RETA principles worked in the lessons) and both questions are about informing that wider objective. The researcher evaluated how the RETA principles have been embedded in these lessons and how they worked through how they impacted students' experiences and outcomes. This study is based on the findings of the previous study which was reported in Chapter 4. The early work was about developing the RETA model (Chapter 5) and the subsequent work is about the implementation and evaluation of the RETA model. This study could also be considered a microcycle of the next study, which is reported in Chapter 7.

This is the closest implementation to the RETA-based lessons that are described in Section 5.3. Even so, the researcher realized that she needs to make minor changes to the designed choices when she acted as the teacher, which affected how the RETA-based lessons were implemented in this cycle. As the lessons were delivered by the researcher who designed them, this was the closest version of the implementation of the RETA model therefore the difference between the intended model and enacted model was small. But even so, there was an adaptation made about pair work. The lesson plans suggested students' work in pairs; however, the researcher needed to decide how the pairs were composed (or not at all). In this cycle, students worked in different pairs in different tasks. Pairs were allowed to choose themselves as they came in and this changed in different lessons, regardless of gender. All there combinations (two females, two males, a female and a male) were observed during the pair-work of the lessons.

6.1. Methods

6.1.1. Participants

This study was conducted in one of the public middle schools described in Section 3.2.1.1 Sampling. Mathematics teachers invited their seventh grade students, aged 12-14, to participate in the 3D shapes after-school lessons. Eight seventh graders, four girls and four boys, were chosen by their teachers from those who volunteered to be in the study. Although all students were volunteers, it should be noted that the sample is disproportionately drawn from the high achieving end of the distribution as perhaps may be expected from those who volunteer for extra mathematics lessons.

6.1.2. Data Generation

The data were generated through observations, worksheets and interviews.

6.1.2.1. Observation

The participants of the study were observed and audio-recorded for four lesson hours during the 3D shapes after-school lessons. Each lesson lasted approximately an hour and included completing lesson evaluation forms and discussions of students' experiences. The researcher acted as the mathematics teacher and delivered lesson plans based on the RETA 3D Shapes Teaching Model in the after-school lessons (the RETA-based lessons). An observation protocol was used to take field notes during and after the lessons. Most field notes were written immediately after each lesson as the researcher was the teacher of the lessons. This also prevented any confusion with other lessons.

6.1.2.2. Lesson Evaluation Form

This cycle was the only one which included completing lesson evaluation forms. Students were asked to determine how much they liked or disliked different components of the lessons on a scale of 1 to 6 as a part of the evaluation form in that 1 was strongly dislike and 6 was strongly like. On the lesson evaluation form, 1 was represented with a sad face at the very left and 6 was represented with a happy face at the very right (see Figure 6.1).

Strongly dislike					Strongly like

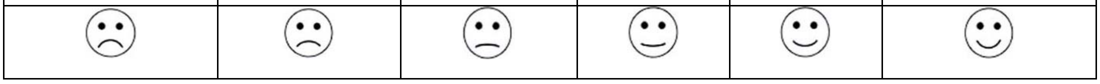


Figure 6.1. Evaluation scale

These forms also included ordering the difficulty of the different parts of the lessons on the list. For example, students were asked to order the following items related to worked examples from the easiest to the hardest.

a) Finding the mistake
b) Describing possible reasons for the mistake
c) Drawing the correct answer

In this cycle, lesson evaluation forms were a part of the lessons and were completed at the end of each lesson. Thus, these were added to the lessons for research purposes and would not normally be implemented in typical lessons.

6.1.2.3. Worksheets

In order to find out whether the lessons were successful and to improve their design, it was important to see how students did (or did not) improve at the target objectives. Thus, participants of the study were asked to complete a worksheet consisting of ten questions, as described in Section 4.1.2. The worksheets were completed by the students before the first lesson and after the last lesson in order to measure how they represented two-dimensional polycubical shapes.

6.1.2.4. Interviews

All eight volunteer students (four girls, four boys) were interviewed three times during the study; a pre-lesson interview, a group interview, and a post-lesson interview. Interviews were conducted in students' schools and were audio-recorded to allow for transcription to be used during the analysis. Interviews took 10 to 15 minutes to complete. The pre- and post-interviews were one-to-one whereas the group interview was a group discussion in groups of two to three students.

The aim of the pre-interview was to learn about students' perceptions of and past experience with three-dimensional shapes. The pre-interview process started with a short introduction of the researcher and warm-up questions. Then the following questions were asked.

- What comes to your mind when you think of three-dimensional shapes?
- How three-dimensional shapes are currently taught in the school?
    - What do you like about it?
    - What do you dislike? Is there anything you would like to improve?
- How does the mathematics teacher use technology in classes?
    - How about you and your friends, how do you use technology in classes?
- Do you think we need three-dimensional geometry in real-life other than mathematics classes? Why?
    - If yes, can you think of any examples of three-dimensional geometry's real-life use?

The post-interview questions were prepared to deepen understanding of students' experiences of the lessons and to capture their reflections about them. Although the process was the same, post-interview questions were slightly different and focused more on the differences and similarities between the regular lessons and the after-school lessons.

- What comes to your mind when you think of three-dimensional shapes?
- How were the teaching sessions different from your regular classes? Do you like the change(s), why (not)?
- Which of the sessions is your favourite? Why?
- Which of the sessions is your least favourite? Why?
- Did you notice how I use technology in the lessons to teach? How did you find the way I did that?
- Did it differ to how you expected me to use it or how your teacher normally uses it?
- How about when you used the technology yourself in the lessons? Did it differ to how you expected to use it or how you normally use it the classroom?
- Has your opinion about the use of three-dimensional geometry in real-life changed in any way?

Students were also invited to a group interview about redesigning the lessons at the end of the final lesson. The group interview was considered as an informal group discussion. Students were organized into groups of 2-3 and asked to work in their groups to discuss

- What they have learnt during the after-school lessons and
- How they prefer to change the lessons if they were the teachers who plan them.

Groups were asked to share their ideas with other groups and to comment on each other's ideas after the initial group-discussion.

6.1.3. Data Analysis

6.1.3.1. Observations and Interviews

A thematic analysis of observation and interview data was carried out similarly to Study 1, as described in Section 4.1.4.1 (Braun & Clarke, 2006). This analysis looked for themes rather than specific practices in conversation, therefore interviews were transcribed to note only what the participants said while not paying attention to the conversational context in which they said. The researcher transcribed the data herself. In coding, she specifically looked for the experiences of and outcomes for students, particularly focusing on the RETA principles. That is, she looked for whether and how the RETA principles are working in the lessons through how they impacted students' experiences and outcomes. Individual extracts of data (a sentence or a sentence group) were coded in as many different themes as they fit into therefore some extracts coded only once, some more than once and some were uncoded. Table 6.1 shows themes and codes and sub-codes of this study. As before, the interview data were transcribed and coded in Turkish. 10% of the interview data were blind-coded and back-translated by another researcher in the field. Peer evaluation and member checking were used to helping increase validity.

Table 6.1. Themes, codes and sub-codes of Study 2

Categories	Themes	Codes
Experience of students (Aspects of experience)	Realistic (Real-life videos)	Positive Negative
	Exploratory (Worked examples)	Positive Negative
	Technology-enhanced (GeoGebra)	Positive Negative
	Active (Student-centeredness/unit cubes)	Positive Negative
Outcomes for students	RETA (Students' drawing performance)	
	Realistic	Real-life examples
		Terminology
	Active	

6.1.3.2. Worksheets

Students' worksheets were scanned to produce an electronic copy for the data analysis. A rubric with all possible correct answers for each of 10 questions was used to analyse them, explained in Section 4.1.4.2 (Appendix E). Another expert in the field coded 4 random worksheets out of 16, and both researchers agreed on marking. The agreement between the raters was very high (Kappa = 0.95, p<.001). The worksheets were also coded for the nature of the mistakes. There were only 3 different descriptions out of coded 52 student mistakes in these worksheets, and disagreements between the raters were solved through discussion.

6.1.4. Ethics

The University of Nottingham approved the research ethics of this on October 19th, 2017; ref: 2017/94 (see Appendix D). As discussed in more detail in Section 3.3 Ethical Issues, the researcher was sensitive to issues related to anonymity, privacy and data security as well as the challenge of interviewing students about her own teaching.

6.2. Results

This section starts with the students' experiences of the RETA principles. Section 6.2.1 represents students' experiences with a further focus on positive and negative experiences. It continues with the outcomes of the RETA-based lessons for students in Section 6.2.2. The section ends with a summary of the findings and discussion.

6.2.1. Students' Experiences of the RETA Principles

This section answers the third research question: How do seventh grade students experience RETA-based lessons? It is mainly concerned with the specific activities described by the students when they were asked to talk about what they experienced in the lessons. These answers were supported with the researchers' observation notes and photos of the students' constructions. In general, the RETA-based (realistic, exploratory, technology-enhanced and active) lessons worked because of how the students engaged with these designed activities based on these principles in the lessons.

6.2.1.1. Realistic

As explained earlier, videos were chosen to present the real-life use of 2D representations of 3D shapes in order to apply the realistic principle. The videos followed by the discussions to make the lessons not only entertaining but also informative and reflective by articulating students' perspectives on videos in a peer and classroom discussion. The aim of the discussions after the real-life videos was to help students benefit more from the videos than solely watching them.

All students appreciated the video experience and found the videos relevant and entertaining. They found watching videos as a good experience and expressed that videos provide a more enjoyable way of learning than how they normally learn 3D shapes. For example, when the researcher showed the architecture video in the third lesson, Enver excitedly said “See, how he is drawing a house as if it is three-dimensional!” and wanted the researcher to replay the video when it had ended (Figure 6.2).

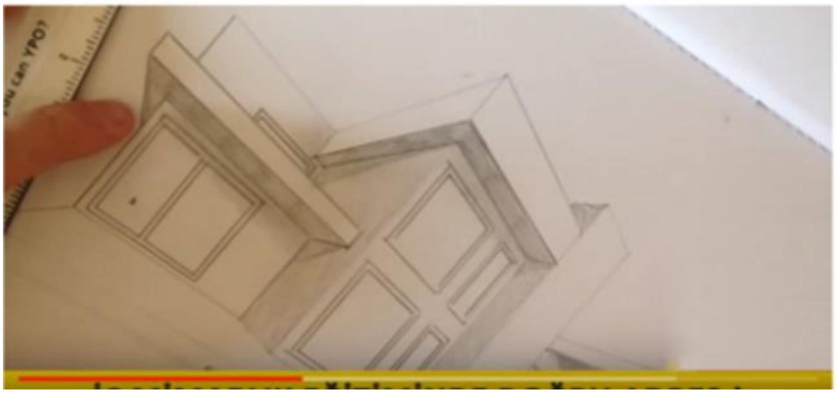


Figure 6.2. 3D drawing video

All students were happy with the researcher’s providing an online video content which made the lesson more accessible to them. Leyla and Melis asked the researcher to share links of the videos with them so that they can rewatch the videos after the lesson.

The students’ negative comments focused on the follow-up discussion after the videos. Despite the lessons being designed to give students a voice through discussions on real-life videos, it was observed in the lessons that students struggled to find particular terminology to use in discussions that made discussions difficult for them. They only gradually learned particular terms such as projection and isometry but did gain confidence in describing the shapes and ideas around them.

Students’ answers in the evaluation forms (the scale was illustrated in Figure 6.1) support the observation findings. Evaluation forms, which were completed after each lesson, asked students to determine how much they liked or disliked different components of the lessons (See Section 6.1.2.1). Table 6.2 shows students’ choices concerning four real-life discussions during the designed lessons.

Table 6.2. Students' choices of the discussions on real-life examples on the evaluation forms

Students	Discussions on real-life examples			
	Discussion 1	Discussion 2	Discussion 3	Discussion 4
Bugra	3	5	5	6
Enver	4	6	6	6
Fatih	1	3	5	5
Hande	5	5	6	6
Leyla	6	6	6	6
Melis	6	6	5	6
Nilgun	5	1	5	6
Utku	5	6	5	6

Evaluation forms showed that the students appeared to like real-life discussions more through the final lessons. Two one out of six were given, one in lesson 1 and one in lesson 2. After that point, there were no further dislikes of discussions. It seems like if people do have a pattern, it is to go up whereas Nilgun was the only student who did not follow this pattern. She did not seem to engage with Discussion 2 on the day. The reason for her scoring was found to be an irregular 3D shape in one of the real-life videos (Mechanical engineers), which was eliminated for the next cycle after the observation of the majority of students' struggle to understand it. Although this was the case, observations showed that they still did not enjoy real-life discussions as much as they did while watching the videos. Students' comments on the discussions in the post interviews supported these results where students explained that they enjoyed the real-life videos but did not enjoy the follow-up discussion as much.

> *I liked the videos but I would be happier if there would not be a discussion in the end.*

said Hande. Similarly, Bugra said

> *The videos were nice and they made the lessons entertaining. I liked them... I liked to see somebody while drawing these shapes in real life but discussing something I had already known did not entertain.*

To sum up, students' experiences with the realistic principle activities embedded in the lessons were mostly positive. They enjoyed watching the videos and they grow to like the discussion after some experience.

6.2.1.2. Exploratory

All students liked the activities with worked examples which require tinkering, finding out and describing designed mistakes, and discussing possible reasons for them and drawing the correct answers. They explored 3D shapes through worked examples with which they engage through intrinsic motivation. It was observed that all of the participants engaged with worked examples with designed mistakes during the lessons and especially enjoyed spotting the mistakes and correcting them. Moreover, six out of eight of the students indicated that activities with worked examples were their favourite.

Similar to the discussion on real-life examples, the participants did not enjoy describing the possible reasons for the designed mistakes in the worked examples. These were specifically designed mistakes for students to diagnose and remediate and to discuss possible reasons for them. As a reminder, Figure 6.3 shows a sample worked example together with the expected correct answer. In this specific question, students were provided with a designed mistake for the top view of the given shape and were asked to spot the mistake in the drawing and discuss possible reasons for the mistake in their groups.

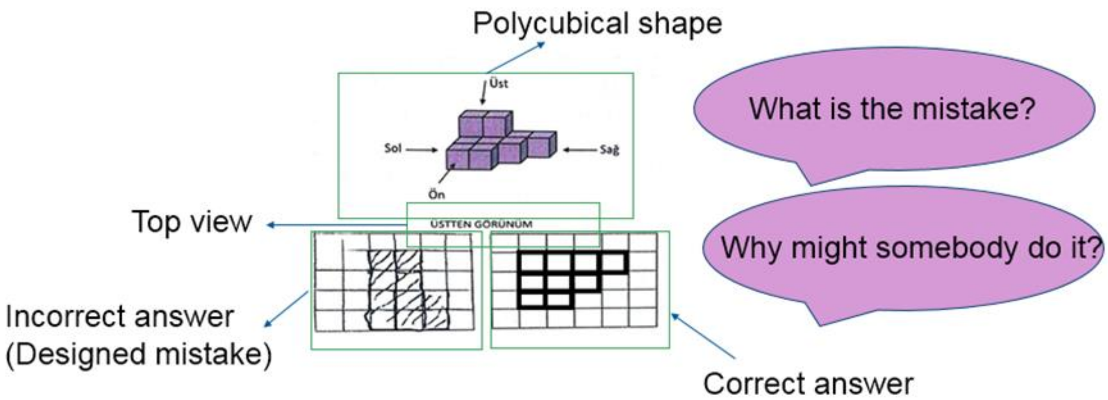


Figure 6.3. Sample worked example

Although this activity with worked examples was found to be the favourite activity by the majority of the students (excluding Nilgun and Utku), they found it challenging to describe possible reasons for the mistakes that they already have spotted and corrected. The researcher needed to facilitate the process with guiding questions. On the evaluation forms, students were asked to order the following items from the easiest to the hardest.

a. Finding the mistake
b. Describing possible reasons for the mistake
c. Drawing the correct answer

Seven students out of eight answered c, a, b: i.e., drawing the correct answer was ranked as easier for the students than finding the mistake and discussing possible reasons for it. The hardest action in this task for students was the description of possible reasons for the mistakes. When they were asked for further details on this in the interviews, Hande said that

> *The activity with the students' mistakes was my favourite. Students made some mistakes and we corrected them and discussed why they are wrong.*

and added

> *It is easier to find my own mistakes and express them. I sometimes could not actually express my understanding of why a student might do the mistake… It is difficult to understand why a student might do such a mistake. Although I liked the bit that I drew the correct answers, I did not like this why part of the activity.*

Utku's quote is a typical comment on the activity with worked examples. All students responded similarly and showed positive attitudes toward the activity regardless of whether it was their favourite or not.

To sum up, students experienced the exploratory principle positively. They enjoyed practising 2D representations through worked examples. However, it was difficult for them to answer "why might somebody do this mistake?" questions which require them to put themselves in somebody else's place while they had already drawn the correct answer.

6.2.1.3. Technology-enhanced

This section focuses on students' experiences with the technology-enhanced principle which refers to the use of dynamic tools in the lessons. In general, students enjoyed using a dynamic tool in the class and talked about their involvement in activities with GeoGebra positively. The majority of them said they found constructions in GeoGebra helpful for them in their two-dimensional drawings of the polycubical shapes. It was observed that with the help of the dynamic tool, they had more chances to be creative without thinking any issues related to the lack of materials such as the number of cubes required to construct a shape. For example, in order to construct the second shape in Row 2 of Figure 6.5, a pair would need 36 cubes. A lesson, therefore, requires 540 cubes for a class of 30. However, it is not realistic to assume all schools having this much material to give a class of students in a Turkish context. The dynamic tool which was used in the class, GeoGebra, made it available and easier for the students to construct a shape and reconstruct it, and possibly therefore, students built a variety of constructions in GeoGebra after they were prompted to build a second shape having the same front view of the given picture.

Figure 6.4. Sample castle picture used in the lesson

Figure 6.5 shows students' GeoGebra constructions for the picture in Figure 6.4. As a reminder, this picture was purposefully designed in order to hide the depth of the building. Students worked in pairs and discussed their answers with another pair (a group of 4) before sharing them with the whole class. They represented the castle picture in different directions and using different bases of GeoGebra. All were perfectly correct with one unit depth (Row 1) and they were the same. After the discussion on the minimum number of views needed to construct the exact shape, students became aware of other possible answers and their constructed shapes varied (Row 2).

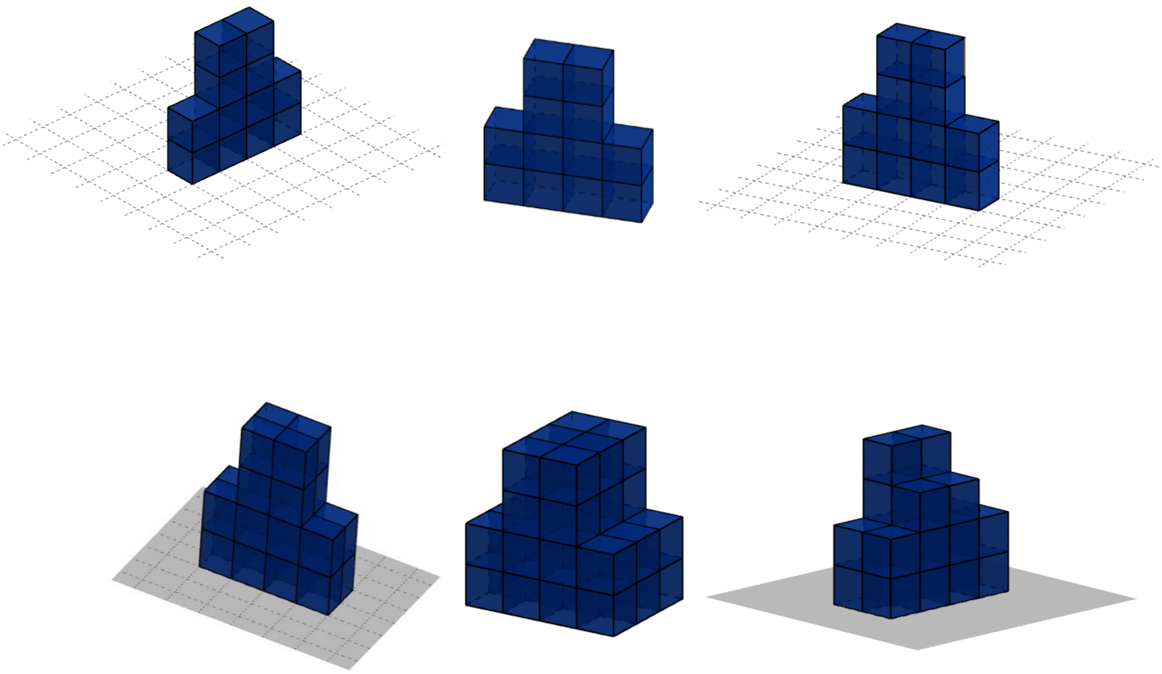


Figure 6.5. Sample constructions in GeoGebra for the picture in Figure 6.4

All of the students seemed to like and benefitted from working with the dynamic tool and creatively built a variety of correct constructions to represent given pictures. In the RETA-based lessons, students were no longer solely dependent on a teacher's representation, which was the case that was found in the previous study. They had immediate guidance from the teacher whenever they needed. In the post-interview, students started comparing the current approach with their previous learning experience. They discussed constructing shapes in GeoGebra, and the researchers' approach for teaching with the tool in addition to talking about their previous teachers' drawings on board. To illustrate, a typical perspective from the post-lesson interview was shown below:

> *Fatih: I liked the way you teach us introducing the concept via videos and slides, and then allowing us to explore it through [the constructions] in GeoGebra. ... Our teacher generally uses*

technology to demonstrate example questions to us; we don't use it. He also draws shapes to the smartboard and we lose so much time.

Most importantly, they were in charge of their own learning, and technology has facilitated this by providing them with an environment to present their understanding of three-dimensional shapes. They reported that their own constructions representing a shape in GeoGebra enhanced their understanding of the concept, and facilitated their drawings. For instance, Enver said

I mostly needed GeoGebra to visualise a shape as a whole before drawing it isometrically. [My constructions in] GeoGebra helped me to understand and draw the shape more easily.

Students' only complaint about the technology was constructing all of the shapes on the worksheets in GeoGebra. Students wanted to use the tool only whenever they struggle to visualise the shape and whenever they felt the need to use it. Related to this, four students out of eight commented in the post-interview that they found constructing some of the shapes in GeoGebra unnecessary. For example, Bugra said

I found it boring to construct all the shapes from unit cubes and in GeoGebra. Some were really easy.

On the other hand, it was observed that most of the students initially found constructing shapes in GeoGebra not very straightforward. Only Hande expressed his opinions about it by saying:

It was also challenging for me to use the tool at the beginning but then I discovered how to use it, and I got used to using it.

Although only one of the students complained about the effort spend on discovering how to use GeoGebra, it was observed during the lessons that students had difficulty in discovering the tool through creating random shapes and playing with it themselves with a little guidance.

To conclude, students' experiences with the technology-enhanced activities embedded in the lessons were mostly positive. Students built a variety of constructions in GeoGebra, some of which were not possible to build with the available concrete materials. They reported and the researcher observed that their experiences would be better if they would be more independent to choose when to use the tool and if they would receive more guidance on the initial discovery of GeoGebra.

6.2.1.4. Active

Active principle aimed at providing active learning environments where students themselves have the control of the use of concrete manipulatives instead of them watching teachers' constructions. In general, students enjoyed being active in the classroom and talked about their involvement in activities with unit cubes positively. As the RETA-based lessons were innovative and different approach to teaching geometry, students listed a number of differences between how their teachers teach and how the researcher teaches in relation to the active principle which supports a student-centred pedagogy, by giving control of manipulatives to the students. This section starts with students' reflections on how teachers traditionally teach and continues with how the researcher has taught.

Students described their learning activities as primarily copying teachers' drawings from the board and papercraft where they cut prism models to learn their nets, surface areas and volumes in the pre-interviews prior to the RETA-based lessons. These were similar to what was found in the first study.

One way students expressed their views on teaching practices was describing how teachers' resources and their use in the lessons. In the pre-interviews, students tent to report how their teachers dominated the use of manipulatives in the lessons and listed a number of teacher's resources and talked about their uses. Students were unanimous in stating that previously they used to learn three-dimensional shapes with mathematics teachers' presenting pre-drawn pictures and shapes on the board and copying them to their exercise books and notebooks. It was common to search 3D shapes online, to present 3D shapes such as prisms and pyramids from a webpage on a smartboard and to elaborate on these. Below is a sample student quote from the pre-interview about this.

> *Fatih: She [my maths teacher] showed 3D shapes like a cube, prism and cylinder on the smartboard, and taught them pointing to the drawings on the board and elaborating them.*

When students were asked to talk about the ways teachers taught, apart from one student, Utku, all seven students unanimous in the comments they gave on the lessons. While talking about their experiences of learning 3D shapes in the traditional approach, they discussed using the board and their teachers' drawings of 2D representations of these shapes on the board. In the pre-interview, Leyla described one of her previous/traditional lessons with the following words:

> *My [maths] teacher picked up a piece of chalk and began drawing on board. She explained the areas and volumes based on her drawings.*

Similarly, Nilgun discussed the teacher's drawings on the board by saying:

> *My [maths] teacher drew the three-dimensional shapes on the board and sometimes referred to her notebook while drawing the shapes in the questions and while drawing the answers of the questions.*

Exercise books were another resource whose use was described by the students in the pre-interviews. All students were expected to buy exercise books from those suggested by the ministry of education. Teachers' search for a good book, choosing and then following one in the lessons was not surprising for the interviewed students in the context of this school. A typical example of explaining this situation is the following:

> *Melis: We [My maths teacher, I and my classmates] bought an exercise book. She [my maths teacher] is showing one of its pages on the smartboard and solving problems on it. We are following questions from our books and she is following them from the board.*

Students' experience with 3D shapes in the traditional approach was not interesting enough to engage them with the topic actively. It was mostly passive and its description did not include any student-centred activity. Most of the students found lessons with traditional approach boring and time-consuming whereas the RETA-based lessons for them were mostly more interesting and were different than how they expected to learn 3D shapes.

To note, any activity might be driven from multiple RETA principles simultaneously. Representation construction activities started with the students' unit cube constructions (active), continued with GeoGebra constructions (technology-enhanced) and ended with students' drawings as constructions. The researcher enacted two principles, technology-enhanced and active, in a construction activity. Therefore, it was not surprising to get students' responses including both of them when students were asked for their experiences in the post-interviews.

In the post-interview, students started comparing the current approach with their previous learning experience. The active principle of the RETA offered students a student-centred environment where they explored the topic in their own pace. Students were aware of not only the change in the 2D representations but also the student-centred approach of the researcher to teach the topic through student-constructions from unit cubes (and with digital technologies). For example, Enver said that

> *You didn't use unit cubes and GeoGebra much, you gave them to us to use and asked some questions when we needed help. Our teacher usually uses tools himself. I liked your way.*

Melis added that

> *If only you would use GeoGebra, like our teacher used the tools, we would have copied from you and we could not learn much. I liked to use it myself because I did not copy what you constructed. I found [the shape] myself.*

Students all found the lessons, which are based on the RETA Model, as an unusual and exciting experience. It was observed that students enjoyed the lessons where the researcher acted as a teacher and guided students with provoking questions and hints while they were constructing their shapes. They were aware of the researcher's student-centred approach to the use of resources and were happy with it. None of the students mentioned any negative experience about them being active.

Finally, students gave feedback on the representations constructed as actors who manipulated the tools to build constructions which were designed to help them in their 2D drawings. Table 6.3 shows students' views on the difficulty of constructing different representations on a scale of 1 to 10, where 1 is too easy and 10 is too difficult. To note, while each student drawing on a worksheet coded out of four for each view (front, top, left and right) and the total score for five drawings for each type of drawing (orthogonal and isometric) was 4x5=20 in Table 6.4, it would be hard and limited for students to rate the difficulty of a question out of four. Therefore, they were asked to rate the difficulty on a scale of 10. This is the reason for the difference between the precision of Table 6.3 and 6.4. Analyses showed that students enjoyed the RETA-based lessons despite the apparent difficulty with the 2D drawings of polycubical shapes isometrically. Students found constructing all of the external representations of isometric drawings much harder than orthogonal drawings. This is, it was more difficult for them to build a shape when its parts were available (isometric drawing) than representing parts of a given shape when the shape was available (orthogonal drawing). On the other hand, mental visualisation of one view (e.g. left view, right view and top view) was harder for the participants than the mental visualisation of a whole shape.

In this context, external representations refer to constructions from unit cubes, constructions in GeoGebra and students' own drawings as constructions, while internal representations are the mental visualisations or mental images of shapes.

Table 6.3. Students' lesson evaluations for the difficulty of representation construction scale for orthogonal and isometric drawing lessons

Representation	Orthogonal drawing lessons (/10)	Isometric drawing lessons (/10)	Total (/20)
Unit cubes	4.4	5.6	10.0
GeoGebra	2.5	5.9	8.4
Mental visualisation	6.8	5.8	12.6
2D drawing	6.9	7.3	14.2

In general, students enjoyed the learning resources that could enable them to actively participate in the learning process, and allow them to explore the mathematical content instead of merely listen and observe the teacher. Their comments and evaluations on the use of manipulatives for representing 3D shapes were mostly positive. Their least favourite lessons were the lessons on orthogonal drawings while the favourite lessons were lessons on isometric drawings. When they were asked for the reasons underlying these decisions, they had specific comments on different representations (orthogonal and isometric) they constructed during the lessons. The reasons for least favourite lessons were difficulty levels of the questions and, related to this, time spent on each activity. Half of the participants spent less time than expected to complete designed activities and disengaged with the lessons after completing the tasks in the orthogonal drawing lessons. However, it was observed that isometric drawing lessons kept students invoked and potentially increased students' understanding and retention levels. In these lessons, they felt questioned, challenged and were awake and attentive.

To sum up, the active principle was the only principle which students shared only positive experience. Although students found external representations of isometric drawings much harder than those of orthogonal, they were happier to construct these representations themselves rather than passively copying their teachers and did not complain about their use of manipulatives.

6.2.1.5. Conclusion of Students' Experiences of the Lessons

To conclude, students experienced the activities based on realistic, exploratory, technology-enhanced and active principles mostly positively. Nevertheless, these experiences could be better with some design changes, which are addressed in Section 6.3.1.

6.2.2. Outcomes of the RETA-based Lessons for Students

This section answers the fifth research question: What are the outcomes of the RETA-based lessons for students? The RETA-based lessons overall are working not only because of how the students engaged with these principles in the lessons but also because they are leading to better learning outcomes. This section starts with students' orthogonal and isometric drawing performances prior to and after the RETA-based lessons. All principles potentially affected students' orthogonal and isometric drawing performances. In addition, there were some other learning outcomes that were particularly related to two of the principles. Thus, it continues with two sub-sections on how realistic and active principles affected learning outcomes.

6.2.2.1. Students' Orthogonal and Isometric Drawing Performance

The results suggested that students performed better after the RETA-based lessons. As there were only eight students, inferential statistics are not reported. However, the descriptive statistics do appear to show marked improvement, and this is particularly visible on the isometric drawings (Table 6.4).

Table 6.4. Test scores for orthogonal and isometric drawing

Test	Orthogonal drawing(/20)		Isometric drawing(/20)		Total (/40)	
	M	*SD*	*M*	*SD*	*M*	*SD*
Pre-intervention	17.4	4.3	10.5	8.1	27.9	11.8
Post-intervention	19.9	0.4	16.9	4.2	36.8	4.4

It is of note that any errors made by the students in this study were previously found and listed in Study 1. Therefore, the current study reported the improvements in drawing performances by referring to these errors and reporting whether the RETA-based lessons helped students overcome existing errors.

To illustrate students' performance, Question 5 of both orthogonal and isometric drawing questions are shown below. These two questions were purposefully chosen to present as they were designed to have the highest level of difficulty out of five questions. Mistakes of the participants were mostly in these later questions. As a reminder, Question 5 in the orthogonal drawings asked students to draw orthogonal views (i.e., the views from the front, top, left and right respectively) of the polycubical shape, whose possible correct answers are shown in Figure 6.6.

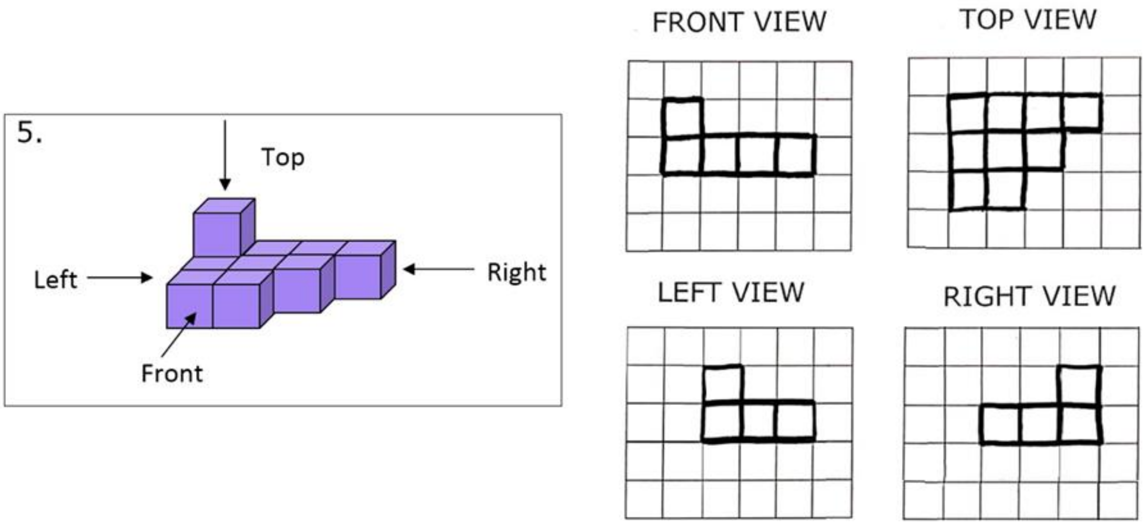


Figure 6.6. Q-5 in orthogonal drawings and its possible correct answers

When students were asked to make orthogonal drawings on the pre-test, their incorrect answers were about drawing cubes at the back to another column (E2), drawing the part only at the very front (E3) and drawing the view upside down (E5). These mistakes (see Figure 6.7) were expected since they were found to be the common mistakes in Study 1 and were listed as Error 2, 3 and 5 in *Figure 4.2.* Orthogonal drawing errors.

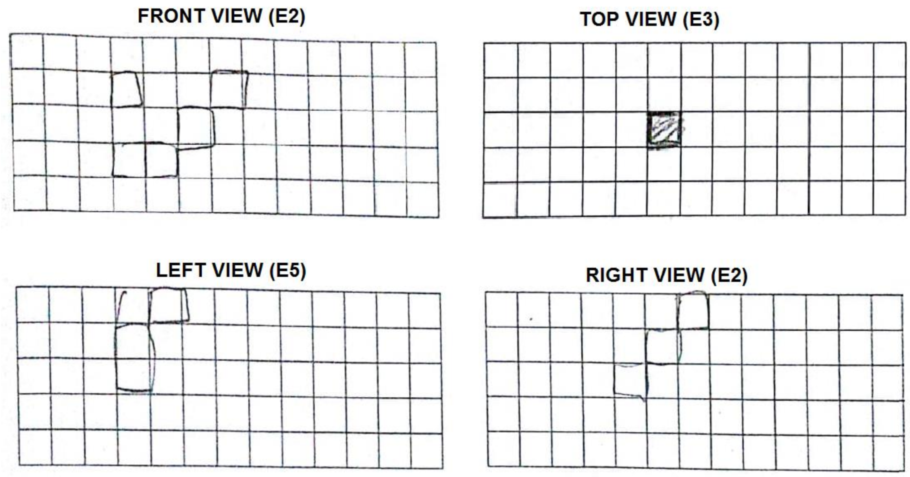


Figure 6.7. Sample pre-intervention student drawings for the Q-5 in orthogonal drawings

These above incorrect answers, whose error numbers were written in parentheses, were not observed after the intervention with the RETA-based lessons. Both shaded and non-shaded representations were accepted as perfectly correct. The analysis showed that almost all of the students answered orthogonal drawing questions correctly in the post-intervention test.

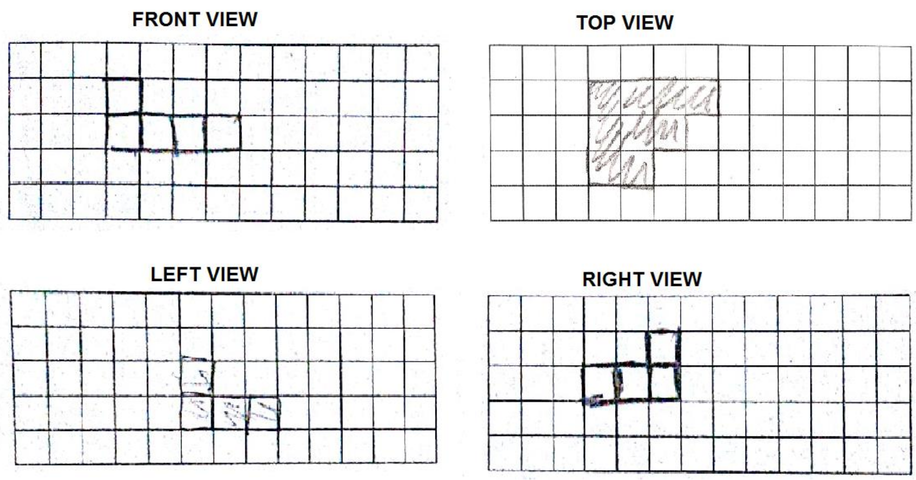


Figure 6.8. Sample post-intervention student drawings for the Q-5 in orthogonal drawings

As a reminder, Question 5 in the isometric drawings asked students to construct an isometric drawing which combines given shaded orthogonal views (i.e., the views from the front, top, left and right) in Figure 6.9. This figure also shows possible correct answers to the question.

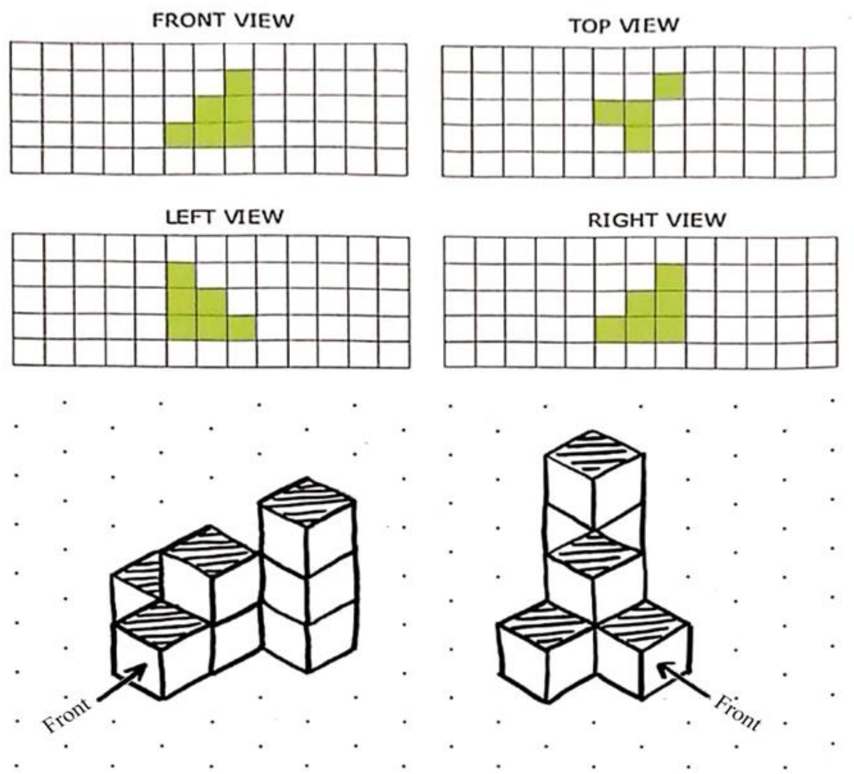


Figure 6.9. Q-5 in isometric drawings and its possible correct answers

When students were asked to make isometric drawings on the pre-test, their incorrect answers were either two-dimensional (E7) or did not follow the dots of the isometric paper (Error 9). Such mistakes (see Figure 6.10 for Hande's and Enver's drawings) were not unexpected and were listed as Error 7 and 9 in *Figure 4.4.* Isometric drawing errors and linking problems.

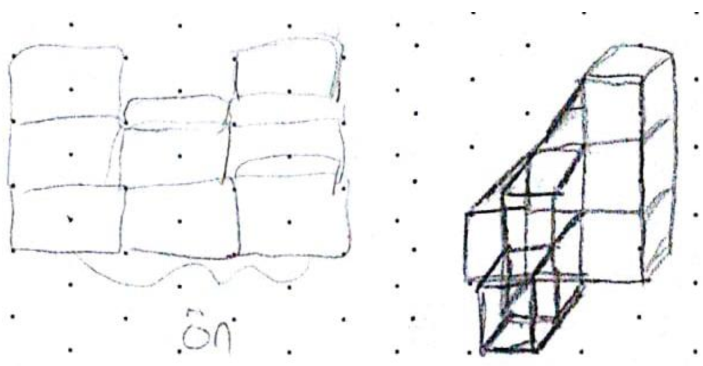


Figure 6.10. Sample pre-intervention student drawings for the Q-5 in isometric drawings

The analysis of the post-intervention tests found that all students learnt how to use isometric paper after the RETA-based lessons even if some drew incomplete or incorrect answers. Figure 6.11 shows the correct responses created by the same students as the incorrect drawings in Figure 6.10.

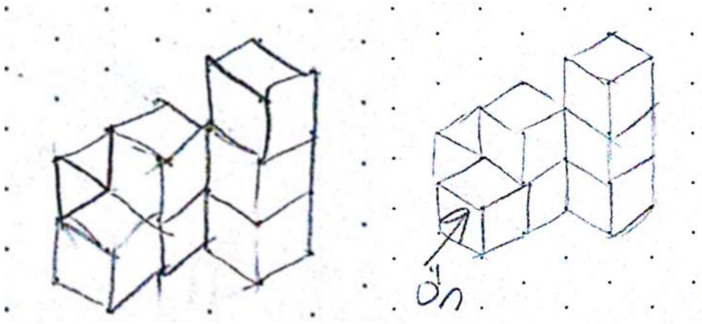


Figure 6.11. Sample post-intervention student drawings for the Q-5 in isometric drawings

To conclude, RETA-based lessons improved students' drawing of 3D shapes and overcame any pre-existing errors, although for orthogonal drawings it should be noted that their performance was already high before the intervention began.

6.2.2.2. Realistic

Learning outcomes were not limited to the drawing performances. The realistic principle of the RETA had an observable effect on students' understanding of 3D shapes and their applications in real-life.

6.2.2.2.1. Students' Terminology and Descriptions of 3D Shapes

Although students had experienced three-dimensional shapes, they did not know how to mathematically describe them. Therefore, they did not relate their experienced 3D shapes from real life with the 3D shapes they learnt in mathematics. In presenting the realistic mathematics, what the researcher was trying to do is to help them understand how these can be articulated and described.

Three different sub-codes were listed for students' terminology and descriptions of three-dimensional shapes: naming, describing, and giving everyday examples. While students were naming or describing geometrical shapes such as cube and cylinder in the pre-interviews, they were more likely to talk about everyday examples of 3D shapes in the post-interviews when they were asked what comes to their mind when we say 3D shapes.

Typical answers for naming found to be the following:

> *Utku: What comes to my mind when you say 3D shapes? I only think of cubes. Oh, and also rectangular prism… and cylinder, that's all.*

> *Hande: I think of a squared prism and a rectangular prism that we learnt in the fifth grade.*

Above answers considered to be *naming* answers where only names of the 3D shapes were listed by the students. *Describing* answers included some level of descriptions of the 3D shapes in addition to the names. Although there were some errors in some of the descriptions, answers coded as *describing* included at least one property of 3D shapes such as nets and numbers of faces.

> *Fatih: For example, imagine a cube, I know that it has 6 faces, 12 edges, and also 6? vertices.*

> *Melis: Last year, our maths teacher made us draw some shapes such as nets of a cylinder, we dealt with 3D shapes then.*

It was also observed that these answers in the pre-interview were either names or descriptions of the properties of common 3D shapes in mathematics but none of them was goods or objects from daily life. Students all perceived 3D shapes as mathematical models prior to the RETA-based lessons.

Students' answers in the post-interview started with the general descriptions of what they have done in the last few lessons when they were asked what comes to their minds when we say 3D shapes.

> *Bugra: I remember all the videos we watched, where, for example, engineers really use drawings from the top, left and right.*

> *Leyla: What first comes to my mind is drawing polycubical shapes from different views.*

They continued with everyday examples of 3D shapes which were distinct from how they talk about 3D shapes in the pre-interviews.

> *Bugra: 2D representations of 3D shapes remind me of drawing something by simply looking at anything in our daily life, designing*

their sizes as if they are real. And, also, buildings and castles were all real 3D shapes and we constructed them.

Leyla: We see and use 3D shapes in most of the places, in architecture, engineering, the design of houses, we use them everywhere.

None of the students talked about common 3D shapes in mathematics (e.g., prisms) or their descriptions in the post interviews. This suggests that students were able to consider three-dimensionality of the real world rather than only conceiving 3D shapes as mathematical models. It is possible that being present in the classroom which supported with a realistic principle encouraged this mode of thinking.

6.2.2.2.2. Real-life examples

With respect to 2D representations of 3D shapes used in real life, students struggled and did not find examples from real life in the pre-interviews. The students did not know what they were in real life and clearly said that they do not know what they were for. Students' answers were short, limited and confused in the pre-interview when they were asked whether and how we need 2D representations of 3D shapes in real life. While most of the students mentioned the topic is important and they use it in their daily lives, they did not give examples. In addition to this, one of them, Utku, said he learnt 3D shapes in maths for the sake of learning but they are not useful in real life. Below are typical quotes from the pre-interviews.

Enver: I think yes, of course [we need 3D geometry in real life]. ... Well, I don't know [an example].

Nilgun: It is very difficult to find something from real life. I don't know.

Utku: I don't think so [I don't think we need 3D geometry in real-life]. I think we learn these for maths. We don't use these in real life much.

The realistic principle of the lessons played a role in making them aware of the real-life use of geometrical ideas in engineering and architecture. There was an observable change in their response to the questions regarding the real-life use of three-dimensional shapes. Students grasped the importance of the topic without any apparent difficulty. In the post interviews, they showed no hesitation in explaining three-dimensional shapes using real-life examples when they were asked to talk about whether their opinions about the real-life use of 3D geometry changed in any way. A typical example is the following:

> *Fatih: It changed, I learnt where we could use 3D shapes in real-life. Also, people use 3d shapes' drawings in their occupations and they might sometimes make mistakes. Or, engineers and architects might perceive or draw the same shape differently and this may cause problems. If they only have a view from the top or only from the left et cetera, this may cause real-life problems.*

Only one of the students, Bugra, said his opinions about the use of 3D geometry did not change. However, the following quote from his post-interview is self-explanatory that the lessons had an effect on his opinions.

> *Bugra: No, I think it is the same. I liked 3D geometry and I still like it but of course, I do not like solving too many problems on 3D shapes. … 2D representations of 3D shapes remind me of drawing mistakes of architects and engineers and their consequences (in the photos). It was exciting and fun to discuss mistakes in real life.*

To sum up, students who did not relate 3D shapes in mathematics with real life prior to the lessons were able to give examples to these shapes from real life after the RETA-based lessons.

6.2.2.3. Active

The active principle also had an effect on representations constructed as an outcome of the intervention. When students were given the opportunity, they built various shapes for the same picture. Figure 6.12 shows sample student constructions from unit cubes to represent given building pictures. As a reminder, some of the pictures did not include a minimum number of elevations to represent shapes. Students were encouraged to discuss the number of minimum elevations needed to construct the exact shape and the reasons for this before constructing possible representations for them.

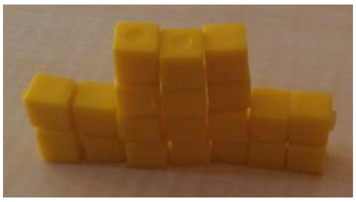
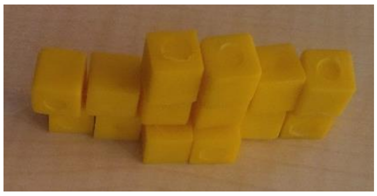
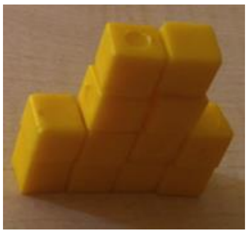
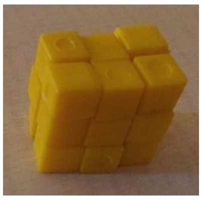
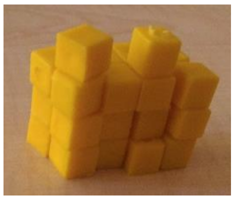
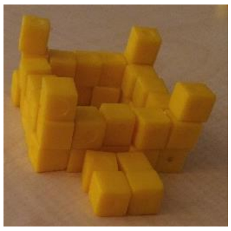
Picture		Example student constructions	
1-			
2-			
3-			
4-			

Figure 6.12. Unit cube constructions for the given pictures

Figure 6.13 and 6.14 show sample student strategies to construct a polycubical shape to represent the given picture with the unit cubes. For the purpose of illustration in this chapter, picture 1 was chosen to demonstrate students' strategies of constructing shapes in more detail. This was purposefully chosen to present as it was the best representative of the typical strategies found by the students for all constructions.

In the beginning, in the first study, observations showed that teachers were building only one construction from unit cubes and students were passive and they were observing and were trying to understand what the teacher's construction represents and to relate it to a 3D shape and its 2D representations. This study showed that students can have very different responses to the same construction when they actively use and engage with the manipulatives as can be seen in these two cases.

While constructing shapes, students were encouraged to explain their thinking to their pairs. One of the pairs, Leyla and Nilgun, for example, drew line segments to the right of the building (Figure 6.13, step 1). The vertical segments divided the windows on the right-hand side as 1+2 windows. The pair considered the number of windows as their benchmark and decided to construct the first floor of the school with 2+3+2=7 units in length, and 1 unit in width. Then, they constructed the second floor of the school with 3 units-length because there were 3 windows on the second floor. They decided to construct the height of the first floor as 2 units by dividing the right-hand-side of the floor into two parts with horizontal line segments, and estimating there are almost two halves (Figure 6.13, step 2). They, finally, considered the second floor as high as the first one and completed their unit cube construction (Figure 6.13, step 3).

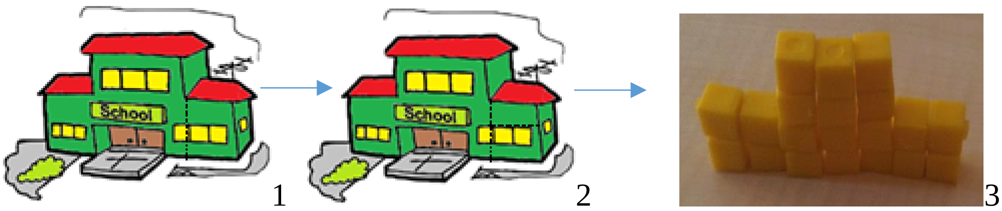


Figure 6.13. Sample student strategy to construct a building from unit cubes - I

Another pair, Fatih and Melis, started their construction by drawing two vertical lines to show the borders of the door. They considered the door as a benchmark for 2 units and showed their unit of measure with a red horizontal line segment on the top of the building. They divided the school picture into three pieces of 2 units in length (Figure 6.14, step 1). Then, they divided the school with two horizontal line segment groups to show that the height is approximately three times as high as the door (Figure 6.14, step 2). Therefore, they chose the height of the building as 3 units. Step 3 in Figure 6.14 shows the complete construction of this pair.

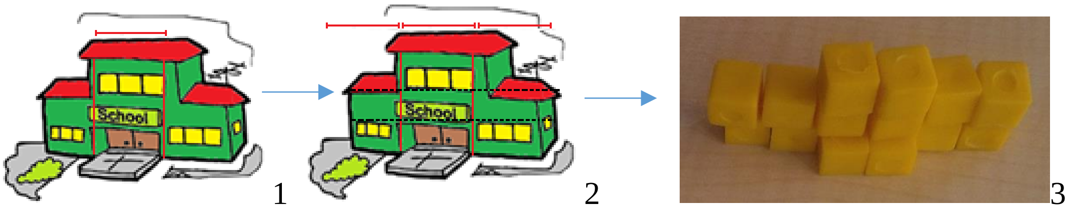


Figure 6.14. Sample student strategy to construct a building from unit cubes – II

To sum up, providing students with a chance to construct their own understanding of 3D shapes through unit cube models potentially helped them achieve learning outcomes.

6.2.2.4. Conclusion

To conclude, RETA-based lessons seemed to mostly achieve their learning outcomes. These outcomes were not only in the orthogonal and isometric drawings questions. Children also got better at finding and describing real-life examples of 2D representations of 3D shapes that they study in the RETA-based lessons (e.g., architecture and engineering examples).

6.3. Summary of Findings and Discussion

To sum up, the aim of this study was to explore students' reflections on the lessons and to investigate outcomes for the students so that lessons can be adjusted according to their needs before collaborating with a teacher in the next cycle. It was found that students' experiences with the lessons were mostly positive. Students commented on the improvable parts of the lessons only after they were prompted to talk about them with questions on, for example, their least favourite lessons and the reasons for these.

Section 6.2.1 examined students' experiences of the RETA-based lessons in order to find possible ways to improve them. All principles of the RETA model in the RETA-based lessons were experienced positively by the participants. They enjoyed watching real-life videos (realistic), working on designed mistakes in worked examples (exploratory), building constructions in GeoGebra (technology-enhanced) and from unit cubes (active). There were three main issues discussed by the students. First of these were discussions on real-life examples. Many students found the topic difficult to discuss possibly because of the struggle in finding the right terminology. Secondly, it was challenging for them to find possible reasons for the designed mistakes in worked examples. It might be because it was their first experience with this question type, it might be because they answered the question for the second time or it might be the question structure and wording which may make it hard for them to put themselves in somebody else's place. Finally, the participants commented on the difficulty of constructing representations, i.e., constructions from unit cubes, GeoGebra constructions, students' own drawings as constructions and mental visualisation as well as one student's comments on and the researcher's observation of their struggle with the initial discovery of GeoGebra.

Section 6.2.2 looked at students' outcomes. The RETA-based lessons helped students improve their two-dimensional drawings. Although there were insuffcent participants to make inferential tests a sensible choice, the descriptive statistics showed that students performed better on the worksheets after the lessons. All eight of the students, individually, scored better than they did before. It seems reasonable to conclude that RETA-based lessons have been successful at prompting new learning on three-dimensional shapes that leads to better drawings. Moreover, through a range of opportunities to develop practical awareness of three-dimensional shapes' real-life use, students communicated their understanding of shapes in mathematics with the real-life world. There were changes in students' terminology and descriptions of three-dimensional shapes and their real-life examples of these shapes. While students only named or described the properties of common 3D shapes in mathematics such as cube or cylinder in the pre-interview, they talked about real-life's being 3D and 3D tools used in daily life in the post-interview. Students who did not find an example to 3D shapes from real life in the pre-interview came up with several examples including those of architecture and engineering in the post-interviews. It might be concluded that the lessons, which were based on the RETA principles, might have increased their awareness of the three-dimensionality of real life that hopefully affected their engagement with this topic in mathematics positively. It is possible that being participating in lessons which included the realistic principle encouraged this mode of thinking. Finally, students were aware of the change in the teaching approach as well as the difference in the teaching resources. They all found new student-centred active approach a new and exciting experience and seemed happy with it.

6.3.1. Design Changes

The students' proposals had practical implications for the design of the lessons of the next cycle.

First of all, it was found that students did not engage with the follow-up discussion after the real-life videos. Therefore, the first change was decreasing the number of real-life videos and providing more time for and guidance on discussion of the contents on the ones used. Lesson evaluation forms completed by the students and interview data showed that students did not enjoy the discussions and did not engage with them in the way it was intended. This did not seem to affect their understanding of the real-life use of three-dimensional shapes; however, it is better to provide an environment for them to enjoy while learning. Therefore, videos which were marked as more challenging and complex to understand on the evaluation forms (design drawing videos in lesson 2 [Mechanical engineers] and 4 [Interior architects]) were eliminated and additional guidance questions and explanations for teachers were added to facilitate students' discussion of the remaining videos.

<u>Possible prompt questions which were added for "Tools for drawing" video in lesson 1</u>

What are the potential advantages and disadvantages of using a tool like this?

- What about accuracy?
- What about getting the detail right?
- What about time spent on construction?

Why do you think we both need tools and pen and pencil drawings? – A possible answer: Sometimes it is easier to sketch, sometimes tools help us to visualise the shape so that we can draw with pen and pencil

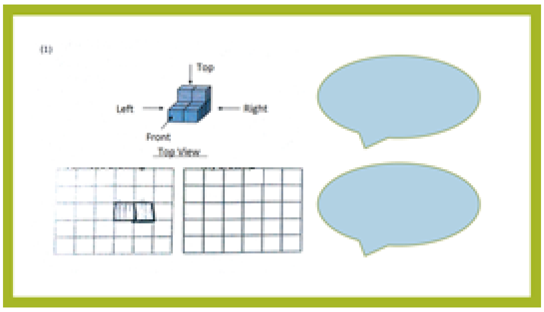
Do you think using a similar tool in this class help us visualise different views of a shape? How? – An expected answer: with the help of the manipulations the tool allows

<u>Possible explanations which were added for "Engineering" video in lesson 3</u>

More precisely, explain to students that architects do their drawings in the way we watched in architecture video as they need to show their drawings to people to communicate with them. Engineers do their drawings in the way we watched in engineering video as they need to communicate with each other. What we will do is more similar to what engineers do. We will construct the three-dimensional shapes (you can choose to use the tools you need to use: linking cubes or GeoGebra, or both) whose views from the front, top, left and right are given. Then, we will use a special paper to draw its 3D-like representation. This paper is called an isometric paper, and our 3D-like drawings will be called isometric drawings of three-dimensional objects.

The second change was in the worked examples. Some of the worked examples were specifically designed mistakes for students to diagnose and remediate and to discuss possible reasons for them. As explained in the findings, students found it challenging to describe possible reasons for somebody else's mistake for a question they have already answered correctly on the worksheet. The designed mistakes were actual student mistakes to the worksheet questions from Study 1. After this feedback, these questions were replaced with still similar but different questions than those of worksheet questions they had previously answered:

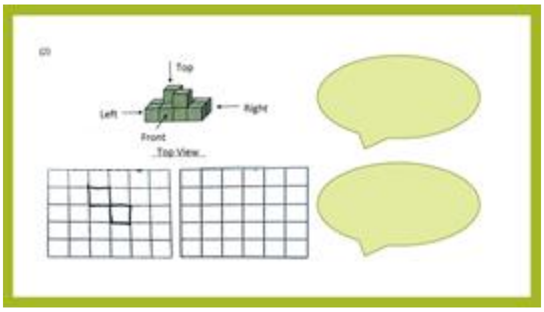
<u>Revised worked examples for the "Activity: Finding the mistakes" in lesson 2</u>



Show the slide – Find the mistake and discuss why – I. Provide one activity sheet to each student.

Say that here is a student's work. Ask "what is the mistake and why is that?"

Ask students to discuss why it's wrong in their groups. Invite students to share their ideas with the whole class. Ask them to draw correct representation individually.



Show the slide – Find the mistake and discuss why – II.

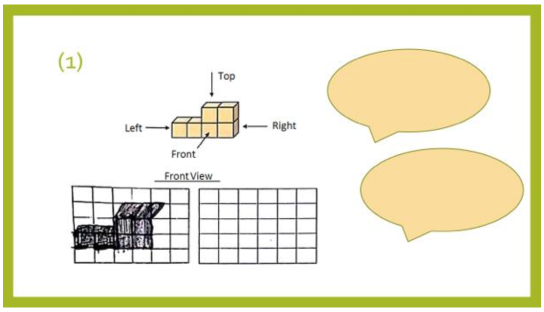
Follow the same procedure with the first question for this one as well.

Ask students to discuss why it's wrong in their groups. Invite students to share their ideas with the whole class. Ask them to draw correct representation individually.

<u>Revised worked examples for the “Activity: Finding the mistakes” in lesson 3</u>

Provide one activity sheet to each student. Activity sheets have five worked examples, specifically designed mistakes. Explain to students that now we will explore common student errors in orthogonal and isometric drawings.

(a) Show the slide – Find the mistake and discuss why – I.



Say that here is a students’ work. Ask them the following question. What is the mistake in the first drawing and why is that?

Ask students to discuss why it’s wrong in their groups and to note the reason for it to their worksheets.

Invite students to share their ideas with the whole class. Ask them to draw the correct representation individually.

The following is the correct drawing for the first question.

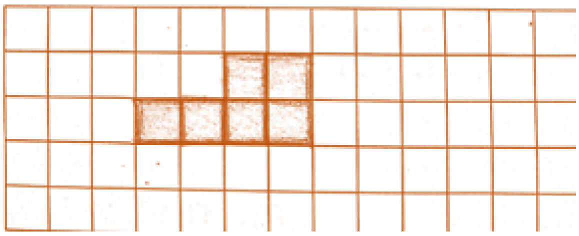


Figure 6.15. A possible correct drawing for the worked example I

(b) Show the slide - Find the mistake and discuss why – II, III, IV, V.

Follow the same procedure with the first question for these as well.

Ask students to discuss why it’s wrong in their groups.

Invite students to share their ideas with the whole class and to note the reason for it to their worksheets. Ask them to draw correct representation individually.

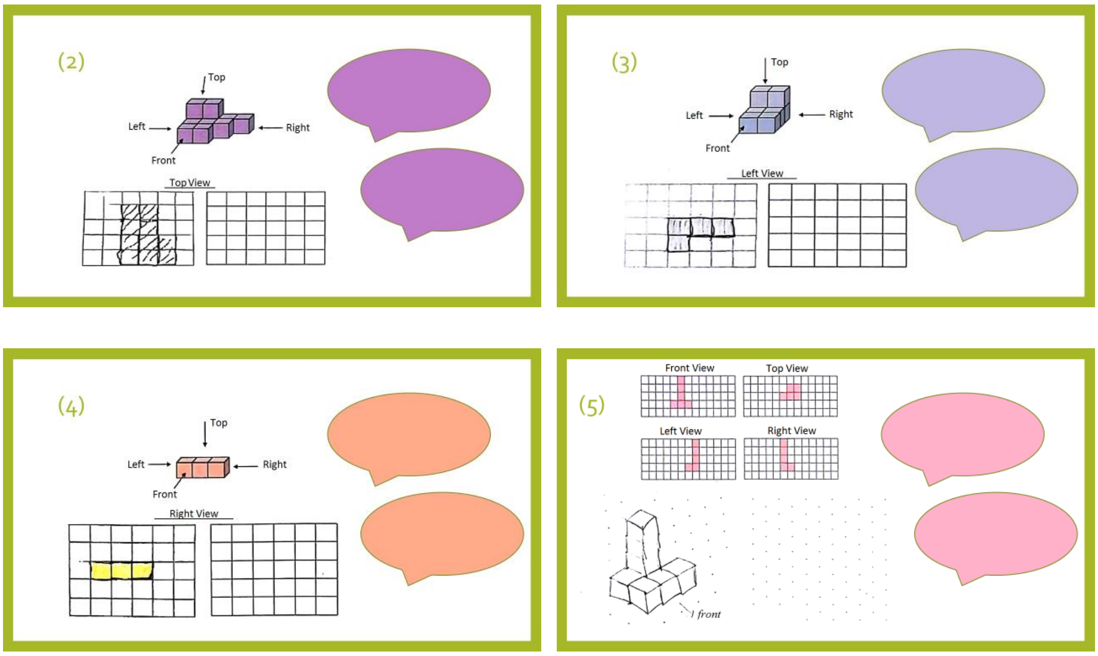


Figure 6.16. Slides for question II, III, IV and V

The followings are possible correct answers for the second, third, fourth and fifth questions.

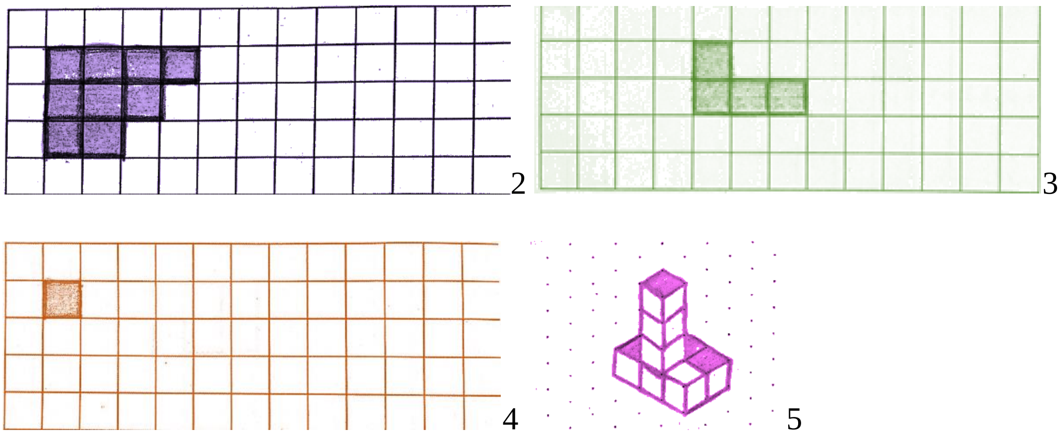


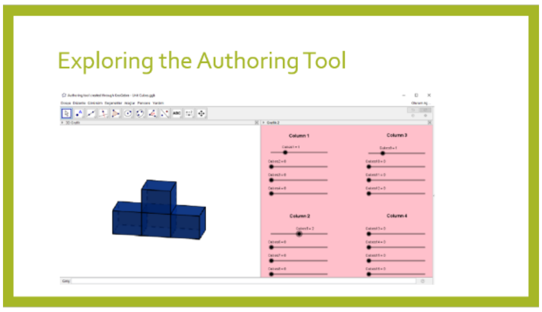
Figure 6.17. Possible correct drawings for worked examples II, III, IV and V

Thirdly, as explained earlier, it was observed that most of the students found GeoGebra difficult to use at the beginning. This was their first experience with GeoGebra and therefore the researcher noted that they need more time and structured guidance to explore the tool. The exploration of the tool was not structured but only guided with questions in this cycle. After this feedback, the researcher added a 15-minute 'Exploring the Authoring Tool' activity to stimulate their exploration of the tool.

Possible guiding questions were added to show how each button works and how the one manipulates the shape.

As explained earlier, related to this, half of the students commented in the interview that they found constructing earlier shapes in GeoGebra unnecessary. Volunteer students did not need to use GeoGebra to answer initial relatively easier questions about orthogonal drawings and probably, therefore, this affected their comments. The difficulty of the questions had been intended to increase in each question: i.e., question 3 was designed to be more difficult than question 2, which was designed to be more difficult than question 1. The researcher designed them in this way not only to facilitate students' learning of the drawings but also to help them discover the features of GeoGebra step by step. Because there will be a structured activity for exploring the tool in the next cycle, it was decided that he constructions with the unit cubes and in GeoGebra will be on a voluntary basis. A student, who does not need to use them, will be allowed to skip over this activity. Moreover, although all of the students engaged with GeoGebra in later questions, they did not talk much about the visualisation it provides during their paired activity. The reason for constructing a shape in GeoGebra was to explore different views of this shape through the visualisations it provides, therefore, the researcher aimed to develop a discourse to move GeoGebra to guide conversations about visualisations for the next cycle.

<u>"Exploring the authoring tool" activity in lesson 1</u>



Explain to students that there are some tools which help us in drawing shapes similar to engineers.

Introduce the authoring tool. Ask students to turn on their tablets and run the authoring tool created through GeoGebra. Give them some time to explore how it works.

Open the slide – Exploring the Authoring Tool. Give an example (use a simple construction similar to the one on the slide) to show how buttons work and how we can manipulate the shape constructed on the authoring tool.

Ask students to construct the same shape on their tools. Continue asking questions to stimulate their exploration of the tool.

Possible Guiding Questions

Ask them to complete their constructed shape to 3D shapes which have the following dimensions:

a) 2 x 2 x 1 (Length x Breadth/Width x Height)
b) 3 x 3 x 3

Then, ask them to remove cubes so that they have 3D shapes which have the following dimensions:

a) 3 x 2 x 2
b) 2 x 2 x 2
c) 2 x 1 x 2

Move to construction examples after the discovery of GeoGebra.

Finally, it was observed that half of the participants spent less time than expected to complete designed activities. These students said that they got bored while waiting for their friends to construct their shapes. It was realized that the variability of the completion time was underestimated. Taking this into consideration, the researcher prepared extra questions for early finishers so that they continue working on these while waiting for their friends if needed.

To conclude, Study 2. 3D Shapes After-school Lessons was the microcycle of Study 3. Lesson plans were designed and tested with an initial sample of 8 students, aged 12-14. In the later cycle, the intention was to move to a whole class and so the number of students was increased to a class of students around 30. The researcher collaborated with a teacher to adapt and adopt the lessons to use in her own classes. The aim was to see her experiences and possible outcomes for the students and how students react to the RETA-based lessons when they were delivered by their teacher.

7. STUDY 3: A TEACHER'S EXPERIENCES OF TEACHING WITH THE RETA-BASED LESSONS

The fundamental goal of this study was to see whether the RETA-based lessons are engaging and effective and to actively search for chances to improve them when they were observed not to be. After making the necessary design changes reported at the end of the previous chapter, the researcher worked with a mathematics teacher who had not previously been involved as she adapted and adopted the lesson plans for her own classrooms to explore how the RETA principles worked in practice and what needs to be improved. In order to achieve this objective, this chapter describes the second implementation of the designed lessons and tries to answer two research questions:

- What are the opportunities and challenges for a mathematics teacher when adopting the RETA-based lessons?
- What are the outcomes of her teaching with the RETA-based lessons for the students?

This is an iterative cycle to see how the RETA principles worked in the lessons and both questions are about informing that wider objective. The researcher evaluated how the RETA principles have been embedded in these lessons and how they worked by considering how they impacted upon a teacher's experiences and outcomes of her teaching for the students. Study 2 explored students' experiences of the lessons (Chapter 6). This chapter shifts the focus to a teacher and tries to answer two research questions about a teacher's experience of the RETA-based lessons and the outcomes of these lessons for the students when they were adopted by a teacher. It addresses how the planned lessons were experienced by a teacher with a specific focus on the challenge of a teacher's taking the thoughtfully designed lesson plans and how it felt to implement them in a context where she was unfamiliar with the pedagogical approach, unfamiliar with the technology and the kind of underlying pedagogical stands of constructivism and students' building their own knowledge, those probably were in contradict to how she would do it. It also explored how students were helped to learn about 3D shapes using approaches that neither they nor the teacher had encountered previously.

Moreover, the teacher needed to make certain design choices during the implementation, which affected how the RETA-based lessons were implemented in this cycle. She appropriated and adopted these principles in her enacted design based on her understanding of the model. Firstly, the lesson plans suggested students' work in pairs; however, the teacher needed to decide how the pairs were composed (or not at all). In this cycle, she chose that all students should work individually for a certain time before working in pairs. When the teacher realized that students struggled to construct some of the later shapes she first decreased the time for the individual work and asked them to work in pairs, then finally stopped giving time for an individual study. All gender combinations (two females, two males, a female and a male) were observed during the pair-work of the lessons. Moreover, students were intended to work in the same pairs in all tasks. They worked with their desk mates during group activities in all lessons unless their deskmate was absent. In such cases, the teacher asked students to change their places to work with another student or asked the student to work together with the closest pair, in a group of three. Finally, the lesson plans suggested students' discussion after the real-life videos; however, the teacher needed to decide how the instructions for the discussion were given. In this cycle, the teacher gave students options and examples from her understanding of the video after watching the real-life videos and before encouraging them to discuss in their pairs. She talked about what was happening in the video and why it might happen. Students shared their ideas only after listening to teacher's explanations (if at all), and the devoted time for the discussion of the real-life examples was limited to three to five minutes per lesson whilst the original lesson plans had suggested about ten minutes for this activity.

7.1. Methods

7.1.1. Participants

Although the researcher had intended to work with a colleague who had shared some of her previous education and philosophical approaches, she was not available to do this. Kindly and happily a replacement was found but the researcher had not previously worked alongside this teacher and it was clear she was more similar to the teachers who were observed in Study 1. The teacher, Ms Aslan (pseudonym), was generally experienced but has not taught this topic previously. She was graduated from the mathematics program of a university. She worked as a mathematics tutor for a private teaching institution after her graduation. Meanwhile, she attended a post-graduate level course and got the teaching certificate in middle school mathematics education. After this, she started working in the current school in the early 2000s and she had been teaching in this school since then. She was the teacher of the observed class for the last six months. The student participants in the study were 30 seventh grade students, 16 girls and 14 boys that Ms Aslan was currently teaching.

7.1.2. Data Generation

The data were generated through observations, worksheets and interviews.

7.1.2.1. Observations

The researcher observed four 40-minute lessons prior to the main study observation to make students and the teacher accustomed to her existence in the classroom. Then, the main observation started, and the data were generated through four 40-minute lesson observations. Similar to Study 1, the researcher set at the back of the classroom with her laptop and took field notes in five-minute intervals. An observation protocol with descriptive and reflective observation notes in separate columns was used to structure the field notes during classroom observations (Creswell, 2007). In the descriptive notes, observations related to the classroom environment and students' and teachers' actions were noted. In the reflective notes, the researcher noted her comments and opinions on the actions taken. Copies of the materials used during the lessons (e.g., activity sheets and cube constructions) were also collected as additional data.

7.1.2.2. Interviews

The interview data were generated through four 20-minute semi-structured interviews prior to lessons and four 10-minute debrief discussions with the teacher after the lessons. The aims of the semi-structured interviews were to get to know the teacher and to discuss her opinions on the lesson plans and underlying pedagogical stands, and the aim of the debrief discussions was to discuss how the lessons went including strengths, problems and issues to consider in the future. Appendix F and G includes all questions from the interviews and debrief discussions together with the context in which they were asked.

The interviews were transcribed to allow the researchers to thematically analyse the data – except for one interview. It should be noted that the teacher did not want to be recorded while discussing and discovering GeoGebra, as described in Ethical Issues Section 3.3.1, item f.

7.1.2.3. Worksheets

The students were asked to complete a worksheet consisting of ten questions, as described in Section 4.1.2. It was important to see how students did (or did not) improve at the target objectives in order to evaluate whether the lessons were successful and to improve their design. The worksheets were completed before and after intervention in order to measure students' improvement (if any) at the orthogonal and isometric drawings of 3D shapes.

7.1.3. Data Analysis

7.1.3.1. Interviews and Observations

A thematic analysis of observation and interview data was carried out similar to Study 2. A deductive strategy was followed to explore how the RETA principles were experienced by the teacher. This is, the analysis looked for the experiences of the teacher, particularly focusing on the RETA (realistic, exploratory, technology-enhanced and active) principles. For example, the first one looked for how she appropriated and adapted as well as enacted the realistic principle and what are the opportunities and challenges she faced with activities involving real-life examples.

Table 7.1. Theme, codes and sub-codes of Study 3

Theme	Codes	Sub-codes
Experience of the teacher (Aspect of experience)	Realistic (Real-life examples)	Opportunity Challenge
	Exploratory (Worked examples)	Opportunity Challenge
	Technology-enhanced (GeoGebra)	Opportunity Challenge
	Active (Student-centeredness/unit cubes)	Opportunity Challenge

7.1.3.2. Worksheets

The same coding strategy with Study 1 (Section 4.1.4.2) was followed to analyse the worksheets. As a reminder, no points were given for incorrect and not attempted answers and one point was given for correct answers for each item in coding. This is, each question is scored out of four for different views: front view (one point), top view (one point), left view (one point), and right view (one point). Therefore, for each of ten questions, students are scored out of four. The completed worksheets were coded by the researcher one by one. Another expert in the field also coded 10% of the worksheets, and both researchers agreed on marking. The agreement between the raters for the coding was Kappa = 0.83 ($p<.001$), suggesting a good agreement. This value was smaller than the agreement in the previous study, and the reason for this is that there was not much variability in the codes which decreased the value of Kappa. These findings were reported in Section 7.2.2 Outcomes of the RETA-based Lessons for the Students (Students' drawing performance).

7.1.4. Ethical Issues

Study 3 was almost the same as Study 2 except for a collaboration of a teacher to adapt and deliver the lessons rather than the researcher. Amendments for this study had been submitted together with the previous ethics submission and approval had been given for both cycles. As discussed in more detail in Section 3.3 Ethical Issues, the researcher was sensitive to issues related to anonymity, privacy and data security as well as the risk of teachers feeling judged or criticized during the interviews.

7.2. Results

The results of this study presented in two sub-sections. The first of these describes the teacher's experiences of teaching with the RETA-based lesson plans. It is followed by a quantitative analysis of the outcomes of the lessons for the students.

7.2.1. A Mathematics Teacher's Experiences of the RETA-based Lessons

This section answers the research question concerning how a teacher experienced the RETA principles with separate sub-sections on each principle (realistic, exploratory, technology-enhanced and active). The aim was to look at how the planned lessons were experienced by a teacher with a specific focus on the challenge of a teacher's taking thoughtfully designed lesson plans and how it felt to implement them in addition to the opportunities for the teacher. It is mainly concerned with the specific activities described by the teacher when she was asked to talk about what she experienced in the lessons and whether and how she would do these differently. These answers were supported with the researcher's observation notes and students' constructions.

In general, the RETA-based lessons (worked because they) gave the teacher the opportunities for adopting and testing new pedagogies and technologies in teaching isometric and orthogonal drawings. Ms Aslan took the opportunities to use new technologies and pedagogies the RETA principles offered into her classroom even though some of those probably were in contradiction to how she would typically do it (hence the enacted lessons were slightly different than the designed lessons).

7.2.1.1. Realistic

As explained earlier, lesson plans suggested using real-life examples and follow-up discussions to provide concrete and real-world applications of the knowledge and skills learnt in the classroom. Ms Aslan supported the idea of integrating these real-life examples into the lessons with the belief that these examples may enhance students' awareness of the importance of the topic so that they perform better in the end. She appreciated the use of them in the lessons and found these examples relevant and useful. She said that she often gets questions from her students about whether they would need what she was teaching in real life. She added that her response to these questions on real-life use of the topic was mostly that students have to learn these if they would like to perform better in the ministry exam. She was happy that the real-life examples in the lesson plans would solve this issue for orthogonal and isometric drawings. The realistic principle in the model provided her with an opportunity to discover/find better reasons for teaching the topic and answer students' questions with realistic examples as a part of more thoughtfully designed lesson plans.

Ms Aslan's negative comments focused on the follow-up discussion after the videos. She said that she felt that she faced a choice between describing the real-life examples herself and encouraging students' discussion on them. According to her disciplinary rules, talk during the class was disrespectful and students should not be allowed to talk during the class. She also believed that middle school students were not mature enough to actively engage in fruitful discussion on the real-life importance of mathematics, regardless of whether it is a discussion on orthogonal and isometric drawings or any other topic. Hence, she struggled to bring a discussion on the real-life examples into her classrooms.

When the real-life video was showed in the second class, the clip only showed how people used the drawings in their jobs. Ms Aslan's framing of the video played an important role in the students' initial response because students tended to respond according to what they perceived from Ms Aslan's summary in addition to what appealed to their own intuition. She chose to introduce the videos herself and tell students what was happening in the video before encouraging them to discuss what was happening, if at all. In the debrief discussion after the class, she had two suggestions about the use of real-life examples. The first one was removing the discussion activities on real-life examples from the lessons "because the freedom of talking throughout the discussion might cause noise and chaos in the classroom" and replacing them with more exam practice questions. She added that

> *I think we should not focus on the initial part of the lesson. That is to say, that real-life stuff is important, of course, but we should pay attention to help them draw the shapes correctly, and maybe the most important part of the lesson is asking those questions modified from ministry exam questions.*

When she was asked what could be done differently if the desire was to still include them, she suggested moving these real-life examples to the very end of the lesson and only teacher's mentioning of them for a short time so that plenty of time could be saved to practice exam questions. She also had an assumption about students' potential learning as being limited and included this while explaining her opinions. She explained her position and suggestions with the following words

> *These are a bit more detailed than my lesson plans. I also didn't plan to spend this much time teaching [this topic]. I mean, I didn't teach it before but I know I could finish it in an hour. I aim for the students to succeed in the exam because these students won't be mathematics professors so, as best I can, I try to help them score better in the exam and meet the requirements of the high school they want. I think this lesson could be strengthened with the multiple-*

choice exam questions or some other questions similar to ministry exam questions.

To sum up, Ms Aslan appreciated aspects of the realistic principle embedded in the lessons and found it relevant and useful to find better reasons for teaching the topic. However, she had difficulties to understand the pedagogy under the discussion activities and found it time-consuming and problematic to move toward more student-centred pedagogy through the discussion of real-life examples.

7.2.1.2. Exploratory

Ms Aslan liked the activities with worked examples which support students' exploratory of the topic through practising problems, some of which with designed mistakes. She was familiar with worked examples from her regular teaching. She found the worked examples in the lesson plans very comprehensive and beneficial for students' learning.

The interviews which were aimed at discussing the lesson plans and the teacher's perspectives on these also gave her an opportunity to rehearse her teaching before the actual lessons. It was (spontaneously) started by the end of the final interview after Ms Aslan's proposal. In the fourth (which is the final) interview prior to the lessons, she wanted to practice how to describe some of the worked examples (to the students) with me. She said, "I want to describe to you how I would draw this red shape if we were in the lesson (Figure 7.1, left)." After the discussion of this question, she passed to the next one and said "Now, I would like to explain how one could draw this question (Figure 7.1, right)." and described her methods of making drawings to me. In the lessons, it was observed that Ms Aslan found it easy to explain didactic parts and solving these problems on the board, this probably is close to how she would do it typically. She was confident and good at describing how to draw 3D shapes on a paper and explaining her own strategies to do these.

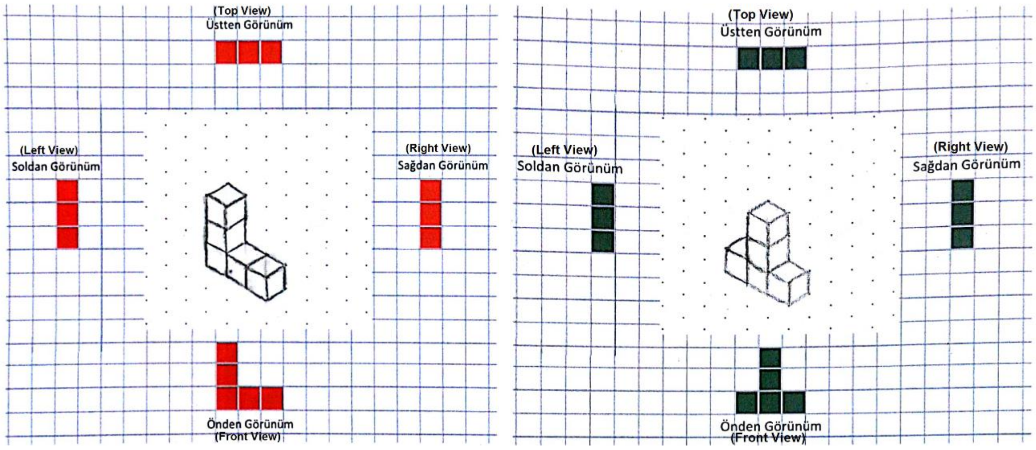


Figure 7.1. Sample worked examples and possible correct answers

Moreover, it is likely that these strategies were affected by the discussion during the interviews. Ms Aslan had opportunities to ask questions while describing the questions and her methods of making drawings to me. She asked many questions about the strategies to draw orthogonally and isometrically and particularly about the answers to the questions in the lesson plans. Some of these questions were followed by other questions where she questioned herself about an earlier question she asked. The main objective of her questions during the drawing appeared to focus on the possibility of generalizing this specific instance the discussion into a "formula" that could fit all questions. Her questions included

> *It seems correct from the right perspective but is there more than one correct answer to these questions?*

> *Do we always need to start drawing from the front view? Can I say this? Isn't it the one (to start with)?*

She found answers to all of her questions during the interviews. Most of the answers were found by herself after a short discussion on the questions. She said that these questions and the practice in the class improved her content knowledge of the orthogonal and isometric drawings which she did not realize that she lacked at the beginning. At the very first interview, she had reported that she was confident about her pedagogy and content knowledge by saying

> *Actually, until now, I didn't have any difficulty in drawing these shapes. I can visualise 3D shapes in two-dimension. I think I am good at reducing three dimensions into two dimensions. ... I don't*

think I will have any difficulty in explaining it (during the lessons). No, I don't think of any (possible difficulty).

In the last debrief discussion, on the other hand, her perceived understanding of her pedagogy and content knowledge changed and she said

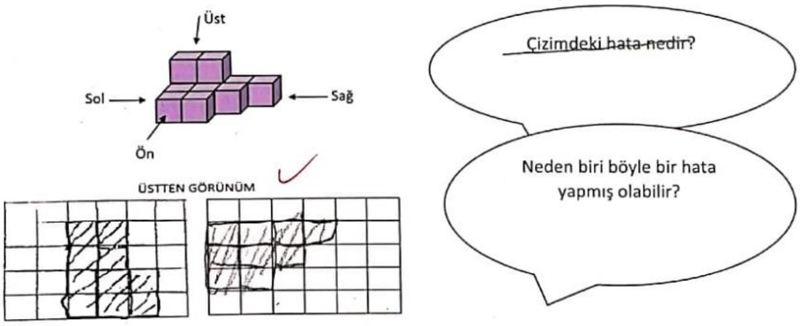
I did not think that I would have any difficulty in teaching this topic but frankly, it was not easy to teach. … I realized that I had a lack of understanding of it myself. I have solved these shortcomings during this study. For example, I did not know that there would be (more than one possible correct isometric) drawings from left and right perspectives.

Although using worked examples was familiar to Ms Aslan as a way of teaching, it is observed that she struggled found it difficult to teach the topic in the class. She explained this by mentioning her lack of understanding of the topic and lack of practice quite a few times. Above quote also can provide an explanation for her struggle challenge in relation to her understanding of the drawings, "This [orthogonal and isometric drawings] is the last topic of the school year and we barely teach it. This was my first time to properly teach it."

Similar to the discussion on real-life examples, Ms Aslan's negative comments focused on the follow-up discussion on the designed mistakes because of the same reasons described in the previous section. These reasons were found to be the teacher's perceptions about students' abilities to discuss (as being limited), her own readiness for a more student-centred pedagogy (as being challenging to leave the disciplinary rules she set for her classes) and her challenge to understand the pedagogy under the discussions. Observations also pointed toward the same conclusion that the discussions of designed mistakes in the lessons were not enacted as intended. Students struggled to describe some of their answers in the follow-up discussions and Ms Aslan struggled to guide the students during these discussions. Regarding her own challenge, she stated that

I have to say that it was challenging to answer the activity with the questions that ask what the mistake is and why it can be. In this part, it was very difficult to find the means of solutions for explaining why *it is wrong. So, I think these questions should be changed. I do not know how but you should change these questions somehow.*

She further explained the challenge by relating her response to the students’ questions in the classroom. Some of the students asked questions to her regarding how to complete the part on the possible reasons for the designed mistakes on the worksheet. As Ms Aslan was struggling to answer these questions herself, she did not give clear explanations to the students. Those who did not get the help or guidance they needed chose not to report their discussion on the worksheet although most of their drawings after the discussion were correct (see Figure 7.2 for two examples, translations are available in Section 6.2.1.2, Figure 6.3).



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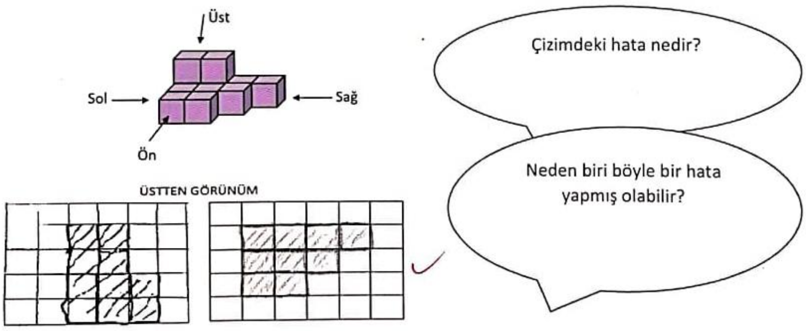


2.

Figure 7.2. Sample student worksheets to a worked example with a designed mistake

She explained this case in the debrief discussion by saying

Some children chose to leave those bubbles empty. I chose to do one of these in the class myself and ask them to write something to the remaining bubbles later. They should be able to discuss and write themselves if they have the talent for this.

To sum up, the teacher experienced much of the exploratory principle positively. She enjoyed the idea of students’ practising a number of orthogonal and isometric drawings in the lesson and found the worked examples very comprehensive and beneficial for students' learning. However, it was difficult for her to find answers to students' questions regarding the designed mistakes and to manage the classroom discussion about them.

7.2.1.3. Technology-enhanced

This section focuses on the teacher's experiences with the technology-enhanced principle of RETA, which refers to the use of dynamic geometry tools in teaching 3D shapes. In the lesson plans, it refers to the use of GeoGebra in the teaching of orthogonal and isometric drawings of 3D shapes. In general, Ms Aslan found the activities with GeoGebra empowering and useful for students, despite her being concerned about the use of the tool at the beginning.

Initially, Ms Aslan was cautious by not only asking not to be recorded during the interview on GeoGebra but also clearly stating that she had a tendency not to believe in using technology in the classrooms to support students' learning. Interviews showed that her position was mostly because of the classroom and time management issues which for her were directly related to the exam pressure/performance. She had formed presumptions about the negative effects of the technology into lesson time and flow. For example, she was thought that the use of GeoGebra makes it easy for students to be distracted during the lesson and can be time-consuming. It should also be noted that she had no previous experience in using GeoGebra. She told the researcher that she might struggle to learn how to use it and this was the main reason why she did not want to be recorded during the particular interview on this tool.

Despite her position and challenges prior to the lessons, the observations and debrief discussions after the lessons pointed toward the same conclusion, which is that she found the activities with GeoGebra effective and helpful for students. After trying GeoGebra in her classroom for a few lessons and observing her students' responses and reactions to the tool, she had no hesitation to say that she would continue using the tool in her future lessons. She came to this point step by step in each lesson and after some practice in the classroom.

For example, in the first lesson with GeoGebra, she asked her students to discover the software for themselves with only a little guidance, arguably because she was not sufficiently familiar or confident with it. Luckily, it did not take much time for students to discover the features of the software (by using the activity previously designed to help students discover them after Study 2). They helped the teacher while constructing shapes with the tool on the smartboard. She was impressed by how students engaged with the topic when GeoGebra was integrated into the lessons and surprised when she did not need to spend time on describing how to construct each and every example. She said that

> *I think the hardest part was to use GeoGebra so I felt a bit more relaxed when I saw the students were comfortable with it. I guided them. ... Students did not expect me to show them the correct constructions (in GeoGebra). They constructed the shapes themselves and asked me whether their constructions were correct. This was the most important strength of the lesson.*

Although Ms Aslan did not need to describe the way to construct the majority of the shapes in GeoGebra, she felt the need to control students' GeoGebra constructions and give feedback on them. The difficulty of the questions was increased incrementally in the lesson plans. In the debrief discussion after the second lesson, Ms Aslan explained how she felt about the difficulty of the questions with the following words

> *I showed them [the students] how to use GeoGebra but it was not wise to come to the class without the answer key. I will be bringing the answer key to the next lesson as the questions are getting harder and harder. I don't know whether I can try and draw difficult questions myself without the answer key.*

In relation to this, it was observed that she checked all students' constructions by dragging the constructed shapes in GeoGebra to compare all four views one by one (front, top, left and right) with the correct answers that she constructed during the lesson. She followed the same process for each GeoGebra construction and spent more time than intended for these questions. The time spent after the questions made the integration of the tool time-consuming despite students' spending less time to correctly answer even later/harder questions on the worksheets. Hence, it was not simply the tool making the lesson time-consuming (as she observed in the interviews) but it was related to her choices in how to integrate the tool into the classroom.

In the debrief discussion after the second lesson, she made a suggestion about the use of GeoGebra. She recommended providing the tool when the teacher feels the need, indeed more like only providing the tool to students when the teacher feels the need (this is similar to the observations made by the teachers in Study 1). This was, for her, because "the students were acclimating easily. Some of them were constructing shapes in GeoGebra and drawing the answers on the worksheet right away and correctly. However, there *would* not be GeoGebra available for them to use during the ministry exam." This quote indicated that she was aware of and listing the affordances of integrating GeoGebra into her lessons but still was worried about how they would affect the students' performance in the exam. Her concern was legitimate. However, in the lesson plans, concreteness fades away so students do not come to rely on it. The lesson plans intended to help students develop an awareness of how the views change when they manipulate the shape in GeoGebra. According to the lesson plans, first linking cubes, then GeoGebra fades away (by the end of the lessons). She found it challenging to understand the intended pedagogy of fading the concreteness away by the end of the lessons and how she could enact it.

To sum up, Ms Aslan appreciated the technology-enhanced principle embedded in the lessons and found the activities with it effective and helpful for students' learning. However, it was not easy for her to shift from a predominantly didactic and exam-focused pedagogy she had for almost twenty years to more student-centric teaching approaches supported by technology.

7.2.1.4. Active

The active principle aimed to achieve active learning environments where learners regulate the utilization of concrete manipulatives rather than observing the constructions of the teachers. In general, Ms Aslan mostly looked from the positive side and was happy with the integration of concrete manipulatives. However, as the RETA-based lessons were an innovative and different approach to teaching geometry, she struggled to shift from her didactic approach to more student-centric teaching approaches. Hence, she listed a number of difficulties in relation to the active principle and its intention to support a student-centred pedagogy by giving the control of tools and manipulatives to the students.

In the interviews prior to the lesson observations, she was quite confident in her explanations which were about the use of linking cubes and the active learning environments they (these cubes) will be used. She supported active learning environments and the use of linking cubes *by students* when teaching orthogonal and isometric drawings of 3D shapes. For example, in the interview just prior to the lessons, she expressed her opinions on the active principle with the following words:

> *They [Students] will understand better if they built the construction with the linking cubes and discover the views themselves. I don't have— there is not much to explain in this topic. It does not mean much to students if, for example, I say 'add two cubes on the top of this cube' or 'remove one from this row'. There are some topics in maths that should be taught in a teacher-centred approach but this one should definitely be in student-centred.*

She further explained in the debrief discussions that students' minds were more naturally opened up to what she was saying when they engaged with the topic in the RETA-based lessons and discovered it themselves through constructing shapes from linking cubes.

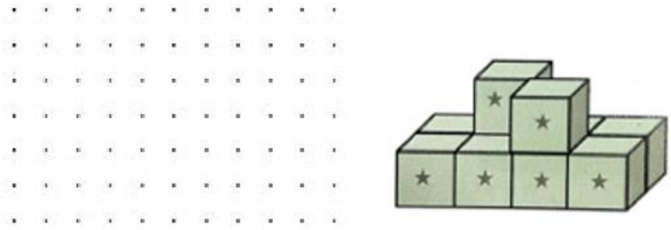
In addition to this, lessons helped her overcome her presumptions about students' performance. Most of the students drew their linking cube constructions to the dotted and isometric papers quickly and correctly. RETA-based lessons were found to be effective for all students but particularly effective for students who have not done well in school, thus narrowing the gap between high and low achievers. Ms Aslan was surprised when students who were not good at other geometry topics and/or numeracy (according to her) grasped the topic quickly in the RETA-based lessons. She appreciated that the RETA-based lessons raised her awareness of her suppositions regarding students' success. However, she found it challenging to enact more student-centric teaching approaches, and thus listed a number of difficulties in relation to the active principle.

Firstly, Ms Aslan found it hard to find the language to guide students to the correct answers, especially when they made mistakes in the construction with the linking cubes. It was observed in the lessons that she had two strategies when students made mistakes. The first of these did not include mathematical suggestions or information about the mistake. She told the pairs of students that their construction was wrong and vaguely encouraged them to put in more effort, for example, by saying they need to think more. Her second strategy was pointing to the mistake with her index finger and directing the students to where the mistake was. This strategy often included non-verbal expressions. She either showed the mistake with her finger and mimicked it was false or simply told the pair that they needed to check one of the views, for example, the right view. Regarding this difficulty, she told the following in the first debrief session:

> *It was quite hard to guide students to the correct answer when a student made a mistake (incorrect or incomplete constructions with the linking cubes), but easy to tell them how to construct the shape. I questioned myself and asked whether I am bad at expressing and teaching this topic.*

In relation to this, lesson plans suggested the teacher give reminders and ask questions for guidance during the activities. For this particular lesson, the first lesson plan recommended 'remind[ing] students that they need to face the front view to decide the required top view.' Lesson plans also included the common possible mistakes and guidance on what to do if the teacher faces any of these. An example from lesson one is the following: 'Some students might tend to draw the front view [of the fourth question] similar to the front view of the third question since the shape actually is the same and the only change is that the stars indicating the front view (see Figure 7.3). Ask them to go back to their GeoGebra constructions and manipulate their constructed shapes to indicate the front view.' The recommended explanations and questions were that the teacher not give the correct answer immediately but to give rise to thought and to encourage students to make their own decisions about mathematical knowledge. This was so that students could discover the correct constructions which lead them to the correct orthogonal and isometric drawings.

3-



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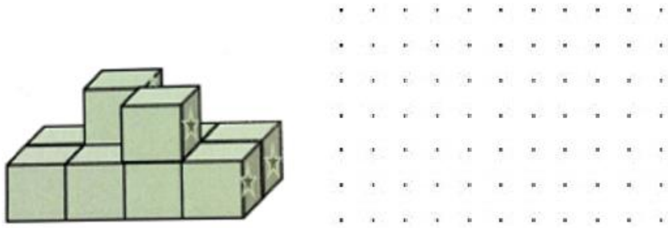


Figure 7.3. Sample examples from the first lesson plan

Secondly, it was challenging for Ms Aslan to manage the time during the RETA-based lessons with student-centred activities which activate students in class by building 3D shapes from linking cubes and/or in GeoGebra. She said she struggled to decide whether the time she gave was enough for students to construct a shape as pairs had different speeds to complete the tasks in the activities. Moreover, it was observed that she was in a hurry to get things done in the third lesson after realizing that she spent more time than intended in the second lesson to complete the designed activities. The researcher expected that classes would have different dynamics and profile of students, which made it hard to predict how things would go in different classes (e.g., how students will respond to a given activity). Thus, the allocated time for each activity on the lesson plans was only as estimate and was not intended to be rigidly followed by the teacher. Although the teacher was informed about this, she wanted to complete the activities within the suggested time and that made the lessons challenging for her.

Finally, the noise level in the class during the pair activities disturbed Ms Aslan a lot. She believed that these activities were inviting even silent students to chat and "it was inevitable to have the annoying buzz of ceaseless chatters and noise of the linking cubes on the desks". Since the beginning of the school year, she set some rules and encouraged silent working and individual study. Observations showed that she established authority and managed to get the class quiet as soon as she came into the classroom. In the pre-interviews, she talked about the classroom generally as a silent one with a couple of noisy students who immediately start chatting as soon as she set them on a task and turn her back. These certain students behaved in the lessons as she expected. They chatted about non-related topics, dropped the linking cubes and made a noise with them. During the pair activities to build shapes from linking cubes, she called their names a few times when she felt they made outright disruptive noise. She explained this case in the second debrief session by saying:

> *It was good to call the names of these children. This increased their interest in the lesson and kept the noise at an appropriate level. I especially repeated the names of those I mentioned earlier so that they stop chatting and continue to their drawing. They got low marks in the exam and were just chatting but not drawing.*

To sum up, Ms Aslan was happy with the idea of integrating concrete manipulatives into the classes. However, she was not ready for more radical changes and her beliefs about the discipline in the lessons prevented her from moving towards a student-centred pedagogy where students decide whether or not they need to use the tools and manipulatives, and how (e.g., whether to use only GeoGebra construction and/or linking cubes or both/none) and how much (i.e., time spent on using tools and manipulatives, discussion and drawing) they need to use them. The teacher had difficulties in adopting the intended pedagogical approach which gave students' the agency for their own learning.

7.2.1.5. Conclusion of the Teacher's Experiences of the Lessons

To conclude, Ms Aslan approached realistic, exploratory, technology-enhanced and active principles mostly positively despite all the challenge (e.g., difficulty in bringing a discussion on the real-life examples and worked examples, integrating GeoGebra, and managing activities with linking cubes) in shifting from a didactic and exam-focused pedagogy she had for years to more student-centric teaching approaches. Nevertheless, these experiences could be better with some design changes, which are addressed below.

7.2.2. Outcomes of the RETA-based Lessons for the Students

In an educational context, the students' learning has to be in focus; hence effort has been put into evaluating how the RETA-based lessons affected students' orthogonal and isometric drawings when they were adopted by a mathematics teacher. All four principles potentially affected students' orthogonal and isometric drawing performances and the RETA model was still found to be an effective way of teaching 3D shapes when it was used by a teacher other than the researcher. To test whether the students get better over time, one just needs their beginning and endpoints. The fact that the researcher did not was because it will be important to explore both orthogonal and isometric drawings as she needed to see if orthogonal and isometric drawings both improve, given the results of her previous study and also she now has a large enough sample to test this. Given the concern about gender, it is also important to investigate as several studies indicated that gender is a factor affecting spatial ability and geometry performance. The following section answers the research question about the outcomes of the RETA-based lessons for the students by reporting the worksheet/quantitative results of the study.

7.2.2.1. Students' Orthogonal and Isometric Drawing Performance

The results suggested that students performed better after the RETA-based lessons. Descriptive statistics appeared to show the marked improvement, and this was particularly visible on the isometric drawings. To examine the effect of question type and gender on students' performance, a mixed (2 by 2 by 2) ANOVA was performed. The design of the analysis was by gender (female and male) by time (pre-test and post-test) by question type (orthogonal and isometric) as presented in Table 7.2. The dependent variable was the students' performance, i.e., the summed scores of the orthogonal drawing questions and isometric drawing questions, each with a possible range of 0-20.

Table 7.2. Test scores at pre- and post-test by question type and gender

Question type	Orthogonal drawing (/20)				Isometric drawing (/20)			
Gender	Female (16)		Male (14)		Female (16)		Male (14)	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Pre-test	14.88	5.54	15.07	4.21	9.38	6.96	8.71	5.97
Post-test	20.00	.00	20.00	.00	18.06	2.21	17.07	3.05

Results from the ANOVA showed that there was a significant main effect of time (pre-test, post-test) on students' performance, $F(1,28)=56.17$, $p<.001$, $\eta_p^2=.667$. Moreover, a main effect of question type (orthogonal drawing, isometric drawing) was revealed,

F(1,28) =42.47, p<.001, η_p^2=.603. Irrespective of gender or time, orthogonal drawing scores and isometric drawing scores were different. The analysis revealed a significant difference in students' orthogonal drawing scores and isometric drawing scores with a medium effect size. There was no effect of gender, F(1, 28)=.121, p=.730, η_p^2=.004.

There was an interaction between time (pre-test, post-test) and question type (orthogonal drawing, isometric drawing), F(1,28)=19.16, p<.001, η_p^2=.406. Pairwise comparisons showed that students' performance statistically differed in orthogonal drawing and isometric drawing in both pre-test and post-test. Students performed significantly better in the orthogonal drawings than they did in the isometric in both pre-test (*MD*=5.92, *SE*=0.95, p<.001) and post-test (*MD*=2.43, *SE*=0.48, p<.001). Moreover, students performed significantly better in the post-test than they did in the pre-test in both orthogonal drawing (MD=5.03, SE=0.91, p<.001) and isometric drawing (*MD*=8.52, *SE*=1.06, p<.001).

However, there were no interactions between time and gender (F(1,28) =.021, p=.885, η_p^2=.001), question type and gender (F(1,28)=.519, p=.477, η_p^2=.018), and time, part and gender (F(1,28)=.007, p=.934, η_p^2<.001).

7.2.2.2. Conclusion

To conclude, the lessons based on the RETA principles provided effective instruction in this particular case (similar to the previous cycle). After RETA-based lessons, there was an observable improvement and all students reached similar levels of newly acquired knowledge.

7.3. Summary of Findings and Discussion

The chapter ends with the summary of the findings and discussion including the design changes for the next study. To sum up, the aim of this study was exploring a teacher's reflections on the lessons and investigating outcomes for the students so that lessons can be adjusted according to their needs before collaborating with many teachers in the next cycle. The researcher collaborated with a teacher and evaluated how the planned lessons were experienced by a teacher with a particular focus on the challenge of a teacher's taking existing lesson plans and how she felt about implementing them in a context. This was a particular challenge as she was unfamiliar with the pedagogical approach, unfamiliar with the technology and the kind of underlying pedagogical stance of constructivism and the necessity of students' building their own knowledge.

Section 7.2.1 examined a teacher's experiences of the RETA-based lessons in order to find possible ways to improve them. All principles of the RETA model in the RETA-based lessons were experienced positively by the teacher, who specifically looked for the opportunities to improve her teaching. Ms Aslan appreciated the use of real-life examples in the lessons and found these examples relevant and useful (*realistic*). The realistic principle in the model provided her with an opportunity to find better reasons for teaching the topic than the ministry exam. Her favourite task in the RETA-based lessons was the activities with worked examples which support students' exploratory of the topic through practising problems (*exploratory*). The interviews on the exploratory principle gave her an opportunity to rehearse her teaching before the actual lessons and improve her understanding of the topic which in her words "*she lacked*". Finally, she supported the idea of using the activities with GeoGebra (*technology-enhanced*) and concrete manipulatives (*active*) and reported that she found these effective and helpful for students' learning.

However, the RETA-based lessons were a new way of teaching geometry and were unfamiliar to her. She had experience in neither integrating GeoGebra nor using discussions on real-life examples and designed mistakes into her teaching practices, and hence struggled in adopting the RETA-based lessons. She had difficulty in leaving her habits and breaking her own rules (e.g., disciplinary rules about classroom noise) and move towards this new way that emphasises student-centeredness with technology. Moreover, the lesson plans suggested ways of moving toward more student-centred pedagogy and to give students agency for their own learning. She struggled to understand the rationale for giving students the agency for their own learning and this created some challenges for her including classroom management (e.g., managing noise during the pair activities), time management (e.g., spending less time for real-life examples or more time for GeoGebra activities), and choosing the right language for guidance.

Section 7.2.2 looked at student's outcomes when the RETA-based lessons were adapted and adopted by a mathematics teacher (other than the researcher) for her own classrooms. In other words, it reported the outcomes of the RETA-based lessons for the students regarding their orthogonal and isometric drawing performance. All students (16 girls and 14 boys) performed better than they did before. It seems reasonable to conclude that RETA-based lessons have succeeded in prompting new learning on three-dimensional shapes that leads to better orthogonal and isometric drawings. Moreover, as expected from the previous studies, students performed better on the orthogonal drawing questions than the isometric drawing questions. They still found isometric drawings much harder than orthogonal drawings and this was not affected by gender.

7.3.1. Design Changes

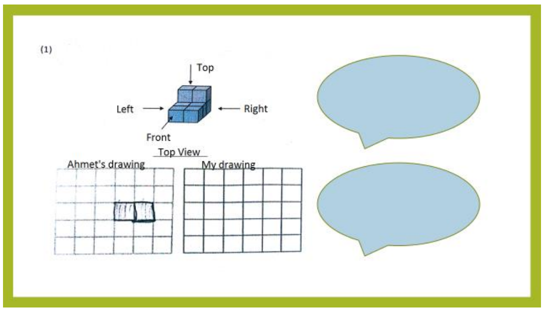
The teacher's proposals had practical implications for the design of the lessons of the next cycle, similar to those of students reported in the previous cycle.

First of all, it was found that Ms Aslan had difficulties in understanding the pedagogy under the discussion activities about real-life examples and designed mistakes hence she limited them to three to five minutes per lesson and allowed students to share their ideas only after listening to her explanations (if at all) whilst lesson plans suggested at least double the time. This did not seem to affect students' understanding of the real-life use of three-dimensional shapes that are shown in the videos; however, in the researcher's view, it is better to provide an environment for students to deeply understand the mathematics by discussing these examples with their peers instead of passively listening to their teachers. Therefore, some simplified paragraphs from articles explaining the effectiveness of discussions about real-life examples and designed mistakes were selected and translated to Turkish to be shared with the teachers prior to semi-structured interviews in the next cycle. These paragraphs were not solely limited to the benefits of discussions in the lessons. As there was the need, they included some paragraphs on student-centred learning and underlying pedagogical stands of constructivism and students' building their own knowledge.

Secondly, both students and the teacher enjoyed practising 2D representations through worked examples similar to the first application of the RETA-based lessons. However, it was still difficult for students and Ms Aslan to answer "why might somebody do this mistake?" questions – even though the questions were different than those of the worksheet questions that they completed prior to the lessons, as amended in Study 2.

There may be many reasons why it might be difficult for somebody to tell you why there was an error on the worksheet. The cognitive complexity of understanding and expressing why somebody made this mistake is unlikely to be resolved in one magic solution. To make a small step in this direction, one of the things the researcher did was personalising so that this might help them engage a bit more as students and teachers with these nominal mistakers. Hence, the wordings of the questions were changed to help them put themselves in somebody else's place. The word "somebody" in questions was replaced with common Turkish names such as Ahmet and Fatma. Moreover, lesson plans noted that it is better to use "what was s/*he* thinking" question instead of asking "what do *you* think he thought" question to push students to step into the other person's shoes a bit more.

<u>Revised worked examples for the "Activity: Finding the mistakes" in lesson 2</u>

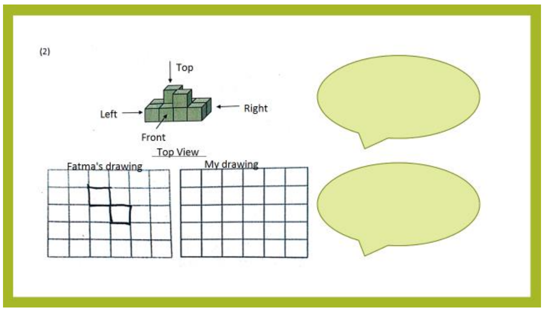


Show the slide – Find the mistake and discuss why – I. Provide one activity sheet to each student.

Say that here is Ahmet's work. Ask "what is the mistake and why is that?"

Note that it is better to use "what was *he* thinking" question instead of asking "what do *you* think he thought" question to push students to step into the other person's shoes a bit more.

Ask students to discuss why it's wrong in their groups. Invite students to share their ideas with the whole class. Ask them to draw correct representation individually.



Show the slide – Find the mistake and discuss why – II.

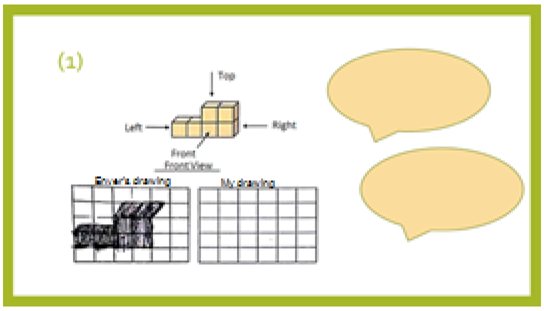
Follow the same procedure with the first question for this one as well.

Ask students to discuss why it's wrong in their groups. Invite students to share their ideas with the whole class. Ask them to draw correct representation individually.

<u>Revised worked examples for the "Activity: Finding the mistakes" in lesson 3</u>

Provide one activity sheet to each student. Activity sheets have five worked examples, specifically designed mistakes. Explain to students that now we will explore common student errors in orthogonal and isometric drawings.

(a) Show the slide – Find the mistake and discuss why – I.



1

Say that here is Enver's work. Ask them the following question. What is the mistake in Enver's first drawing and why is that?

Note that it is better to use "what was *he* thinking" question instead of asking "what do *you* think he thought" question to push students to step into the other person's shoes a bit more.

Ask students to discuss why it's wrong in their groups and to note the reason for it to their worksheets.

Invite students to share their ideas with the whole class. Ask them to draw the correct representation individually.

The following is the correct drawing for the first question.

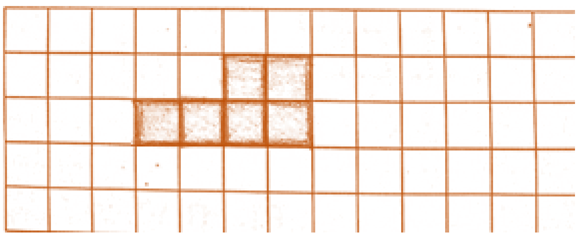


Figure 7.4. A possible correct drawing for the worked example – I

(b) Show the slide - Find the mistake and discuss why – II, III, IV, V.

Follow the same procedure with the first question for these as well.

The third change was about the teacher's discovery of GeoGebra. The researcher observed that Ms Aslan asked her students to discover the software themselves with only little guidance arguably because she was not sufficiently familiar or confident with it. Therefore, it was decided to spend extra time to support teachers' practice with GeoGebra to improve the initial preparation to teach lessons in the next cycle. In relation to the discovery of GeoGebra in the lessons, it should also be noted that students previously found GeoGebra difficult to use at the beginning of Study 2 (which is the previous cycle). Hence, the earlier lessons were intentionally included activities on discovering GeoGebra in the current cycle so that children can spend some time on the tool to structurally discover it with more confidence. This cycle showed that the activity added after the previous cycle was helpful for students' initial discovery of GeoGebra as it did not take much time for the students to discover how to construct and manipulate shapes on GeoGebra by using the activity designed to discover them after the Study 2 – despite the teachers' limited guidance.

Lastly, the teacher needed an answer key showing possible cube constructions in the lesson plans together with the correct isometric drawings. Despite the fact that she was provided with an answer key including all possible answers to the orthogonal and isometric drawing questions, she wanted to have constructions from linking cubes available and spent some time in the lessons on interpreting the isometric drawings in the answer key to construct the shapes from the linking cubes. She used this strategy rather than following the steps of the task – which asked using given orthogonal drawings to construct the shapes – as students did. Figure 7.5 shows linking cube constructions and possible correct isometric drawings to the constructions.

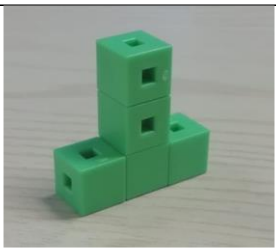
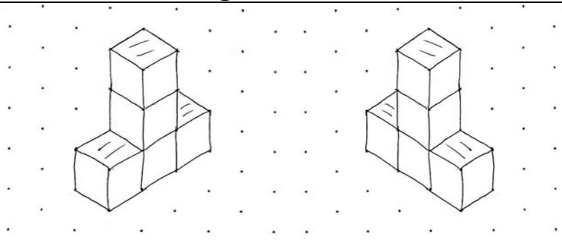
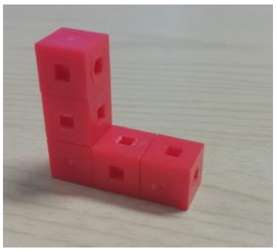
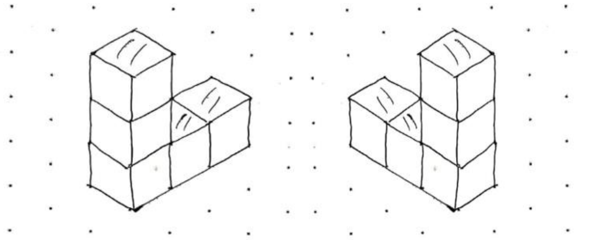
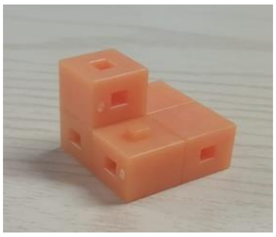
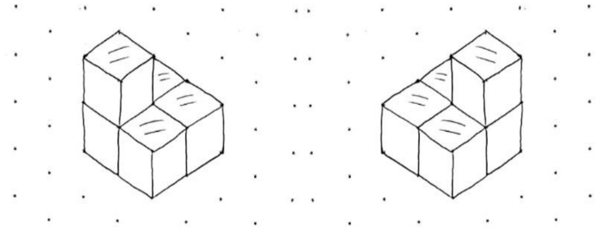
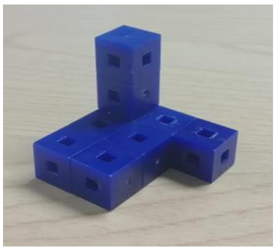
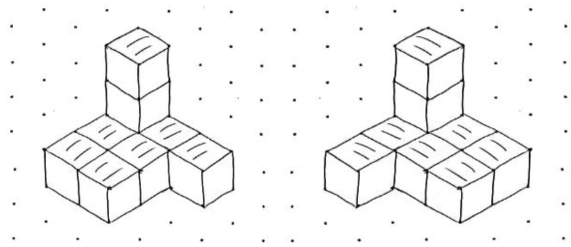
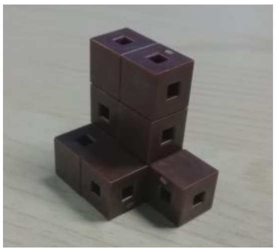
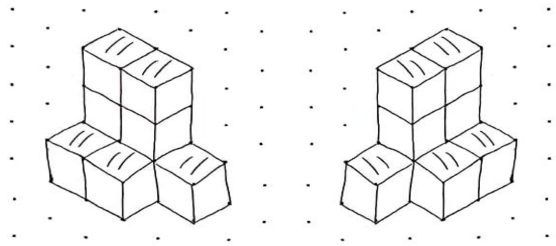
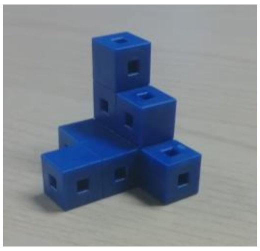
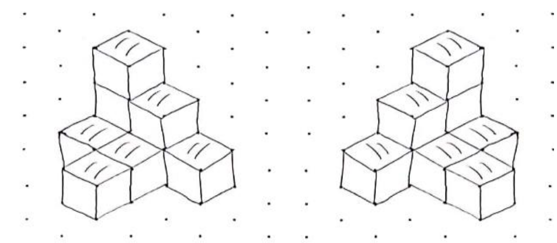
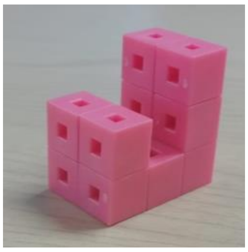
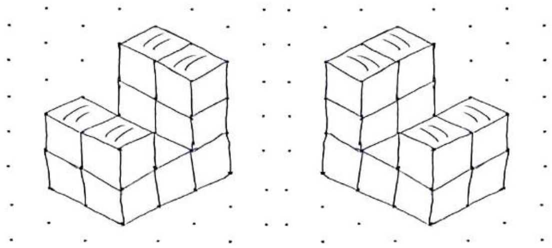
Cube construction		Isometric drawing
1-		
2-		
3-		
4-		
5-		
6-		
7-		

Figure 7.5. Sample linking cube constructions and corresponding isometric drawings

Before concluding the chapter, it is important to note that as all students answered orthogonal drawing questions correctly in the post-test; however, the difficulty of the questions they are expected to answer in the national exam was not harder than these and therefore the research continued with the same questions.

Moreover, it is important for the reader to know that Ms Aslan is a very typical Turkish teacher similar to those observed in the case study (Study 1) and thus this way of teaching was unfamiliar. However, she was able to adopt some of the practices and principles. For example, her favourite principle was exploratory with the worked examples. She was familiar with worked examples from her regular teaching and found the ones in the lesson plans very comprehensive and beneficial for students' learning. Hence, potentially, although the researcher worked with one teacher, given what is observed across the whole series of studies that it is likely that other teachers will not be dissimilar to her.

To conclude, this chapter discussed a teacher's experiences of the RETA-based lessons and outcomes of these for the students. In this second cycle of the intervention with the RETA-based lessons, the researcher collaborated with a mathematics teacher, discussed the model and shared the plans with her. The teacher adapted the lesson plans for her own classrooms and appropriated and enacted by slightly changing them. The RETA Model was still found to be an effective and engaging way of teaching orthogonal and isometric drawing when used by a teacher other than the researcher. Consequently, this research scaled up this approach to include more teachers and students and was able to report how this approach worked in mainstream contexts in the later cycle. The intention was to move to two schools and so the number of students was increased to four to six classes of students, each class around 25.

8. STUDY 4: A QUASI-EXPERIMENTAL STUDY OF THE RETA INTERVENTION

This chapter describes the third implementation of the designed lessons and tries to answer the fifth research question:

- How do learning outcomes (orthogonal and isometric drawings) differ between students who participate in the RETA-based lessons and those who study traditional lessons?
- Does gender influence this result?

This chapter describes the final quasi-experimental study of the RETA-based lessons. This study with over 200 students in two middle schools was the third and the final cycle of this design-based research. Thus, the study explores how the approach works in more typical contexts. This final study was particularly important because there was no control group in either of the previous studies, consequently, no way of knowing how much of the improvement was due to the RETA-based lessons and how much to the effects of repeated testing. It could not also address whether these lessons were better than how students traditionally learn three-dimensional geometry.

8.1. Methods

8.1.1. Participants

205 (85 intervention, 120 control) grade-7 students were recruited from two middle schools. Section 3.2.1.1 describes the sampling strategy for the thesis. Due to practical constrains, the intervention and control groups were compared using pre-existing classes in their schools. Moreover, the teachers of the classes volunteered to be in the conditions and thus were not randomly allocated. There were nine (four intervention, five control) classes in the study. However, the intervention and control groups were well distributed between two schools. Two of the intervention classes and three of the control classes were from the first school and the remaining four classes (two intervention classes and two control classes) were from the second school.

There were 42 female and 43 male students in the intervention group (Class one to four) and 65 female and 55 male students in the control group (Class five to nine).

8.1.2. Data Generation

The data was generated through worksheets. Same procedures with the previous cycle were followed. The students in both intervention and control groups were asked to complete a worksheet consisting of ten questions, as described in Section 4.1.2. The worksheets were completed before and after lessons in order to measure students' improvement (if any) at the orthogonal and isometric drawings of 3D shapes.

8.1.3. Data Analysis

The same coding strategy with the previous cycles as reported in Section 4.1.4.2 was followed to analyse the worksheets. The completed worksheets were coded by the researcher and another expert in the field also coded 10% of the worksheets, and both researchers agreed on marking. The agreement between the raters for the coding was *Kappa*=0.89 ($p<.001$), suggesting good agreement.

8.1.4. Ethical Issues

Study 4 was almost the same as Study 3 except for a collaboration with a number of teachers who volunteered to be in intervention and control groups. Hence, an amendment was required for Study 4 rather than a separate ethics submission. This was accepted on September 20th, 2018. There was not an issue raised by the ethics committee about the revised documents. As discussed in more detail in Section 3.3 Ethical Issues, the researcher was sensitive to issues related to anonymity, privacy and data security as well as the issue of teachers' volunteering to the conditions and hence students' only volunteering to be in the study but not to the conditions.

8.2. Results

The results of this study presented in two sub-sections. The first of these describes the outcomes of the lessons for students by reporting the results of a mixed measures ANOVA that examined the effect of group, gender and question type on students' performance. It is followed by a section on the results of the analysis on improvement scores which further investigates the analysis reported in the first sub-section.

8.2.1. Outcomes of the Lessons for Students

To examine the effect of group, gender, time and question type on students' performance, a mixed measures (2 by 2 by 2 by 2) ANOVA was performed. The

ANOVA was conducted with group (control, intervention) and gender (female, male) as between-group factors, and time (pre-test, post-test) and question type (orthogonal and isometric) as a within-groups factor as presented in Table 8.1. The first two factors are between groups and the second two factors are within groups. The dependent variable is the students' performance, i.e., the summed scores of the orthogonal drawing questions and isometric drawing questions, each with a possible range of 0-20.

Table 8.1. Test scores of the intervention and control group at pre- and post-test by question type and gender

	Question type	Orthogonal drawing (/20)				Isometric drawing (/20)			
Intervention	Gender	Female (42)		Male (43)		Female (42)		Male (43)	
		M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
	Pre-test	11.88	6.61	11.65	6.48	6.41	6.17	5.09	5.32
	Post-test	19.36	1.53	18.65	3.02	17.33	4.30	16.61	5.21
Control	Gender	Female (65)		Male (55)		Female (65)		Male (55)	
		M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
	Pre-test	6.89	5.66	10.44	6.88	3.95	4.14	6.15	6.07
	Post-test	8.99	5.76	12.91	6.93	4.95	4.87	8.38	6.78

Results from the ANOVA showed that there were significant main effects of time $F(1,201)=370.20$, $p<.001$, $\eta_p^2=.648$, question type $F(1,201)=158.22$, $p<.001$, $\eta_p^2=.440$ and group, $F(1,201) = 76.11$, $p<.001$, $\eta_p^2=.275$ on students' performance. These main effects were modified by further interaction as there were interactions between time and question type, $F(1,201)=16.84$, $p<.001$, $\eta_p^2=.077$, as well as time and group, $F(1,201)=156.66$, $p<.001$, $\eta_p^2=.438$ and finally a three way interaction between time, question type and group, $F(1,201)=33.28$, $p<.001$, $\eta_p^2=.142$. Given complexity interpreting a three way interaction, it was decided to simplify this by analysing difference scores (post-test -pre-test) and this is reported below in Section 8.2.2.

Additionally, there was also a main effect of gender on students' performance $F(1,201)=318.00$, $p<.05$, $\eta_p^2=.019$ which was modified by an interaction between group and gender, *F*(1,201)=10.01, *p*<.05, η_p^2=.047. Bonferroni post-hoc tests were conducted and showed that there was a significant difference between females and males in the control group (*MD* = 3.28, *SE* = 0.82, *p*<.001). However, it seems there is no difference between females and males in the intervention group (*MD* = 0.74, *SE* = 0.97, p>.05). Moreover, in the intervention group, both females and males scored higher than the control group (female *MD* = 7.55, *SE* = 0.89; male *MD* = 3.53, *SE* = 0.91, both p<.001).

No further interactions were significant and therefore for ease of reading these are omitted from this chapter but can be found in Appendix H. Moreover, because of a concern about any pre-existing differences in the classes, supplementary analyses on test scores were done for each class separately (see Appendix I).

8.2.2. Further Analysis on Students' Performance

Students' improvement between the pre-intervention test and the post-intervention test of geometry performance was calculated by subtracting their pre-intervention score from the post-intervention score. For example, the improvement scores in orthogonal drawing were calculated by subtracting students' pre-intervention orthogonal drawing scores from their post-intervention orthogonal drawing scores. Then, a mixed measures (2 by 2 by 2) ANOVA was conducted with group (control, intervention) and gender (female, male) as between-group factors, and question type (orthogonal and isometric) as a within-groups factor (see Table 8.2).

Table 8.2. Improvement scores in the intervention and control group by question type and gender

Question type	Orthogonal drawing (/20)				Isometric drawing (/20)			
Gender	Female		Male		Female		Male	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Intervention	7.48	6.40	7.00	6.28	10.93	6.05	11.51	6.23
Control	2.11	3.97	2.47	4.29	1.00	3.09	2.24	4.10

As above, there were significant effects of group (*F*(1,201)=156.68, *p*<.001, η_p^2=.438) and question type on the improvement of geometry performance (*F*(1,201)=16.84, *p*<.001, η_p^2=.077), which was modified by an interaction between question type and group, $F(1,201)=33.28$, $p<.001$, $\eta_p^2=.142$. Multiple comparisons using the Bonferroni correction showed that both orthogonal and isometric drawing improvement scores differed between the intervention and control group (orthogonal *MD*=4.96, *SE*=0.71; isometric *MD*=9.66, *SE*=0.71). There was a significant difference between orthogonal and isometric improvement scores in the intervention group (*MD*=3.99, *SE*=0.76). However, it seems there was no difference between orthogonal drawing and isometric drawing improvement scores in the control group (*MD*=0.71, *SE*=0.64). The main effect of gender was not significant, $F(1,201)=0.540$, $p=.46$, $\eta_p^2=.003$ as both female and male students improved equally. Additionally, there were no interactions between question type and gender ($F(1,201)=1.43$, $p=.23$, $\eta_p^2=.007$), gender and group ($F(1,201)=0.41$, $p=.52$, $\eta_p^2=.002$) and question type, gender and group ($F(1,201)=0.014$, $p=.91$, $\eta_p^2<.001$).

To conclude, the intervention (the RETA-based lessons) was more successful than the business as usual condition at improving students' geometry drawing scores and this was particularly noticeable for the isometric drawings (presumably because students had started with poorer scores). Male and female students benefitted equally from the intervention.

8.3. Summary of the Findings

To sum up, the purpose of this study was to see how the RETA-based lessons work in mainstream contexts. This final study was particularly important because it included a control group and so could address whether the improvement was due to the RETA-based lessons and not simply maturation or the effects of repeated testing. Thus, it could address whether these lessons were better than traditional lessons on orthogonal drawings and isometric drawings. Findings confirmed the results of previous cycles and also showed that RETA-based lessons provided more effective instruction than traditional methods.

The results showed that students got better whether they were in the intervention or control group, but students in the intervention group improved both in the orthogonal drawings and isometric drawings more than the students who attended the traditional lessons. This suggested that the intervention can improve students' geometry performance with the help of activities which are supported with realistic, exploratory, technology-enhanced and active principles more than the traditional lessons. Hence, it seems reasonable to conclude that RETA-based lessons have succeeded in prompting new learning on three-dimensional shapes. This new learning leads to better performance in orthogonal and isometric drawings than the traditional lessons. As observed, the RETA-based lessons made almost every single child taught by multiple different teachers in multiple different classes score almost 100% in orthogonal drawing and almost 90% in isometric drawing questions that are of equivalent difficulty to the national exam.

Moreover, as expected from the earlier cycles, students performed better on the orthogonal drawing questions than the isometric drawing questions in both pre-test and post-test. Similar to earlier studies, they still found isometric drawings much harder than orthogonal drawings (nonetheless, they made significant improvement). There may be many reasons for why it might be more difficult for students to make isometric drawings than orthogonal drawings. For example, students need to have more simultaneous relations in mind in order to make isometric drawings (than that of orthogonal drawings) and that is harder from what we know about human memory capacity (Ayres, 2006; Paas et al., 2003). This is, students have to have more elements (orthogonal drawings) simultaneously in mind to do an isometric drawing whereas in drawing an orthogonal drawing, students can decompose a polycube and focus on one bit (e.g., front view) at a time. Different strategies were observed to be used by students to answer orthogonal and isometric drawing questions, and these strategies did appear to be affected by initial spatial skills (Widder & Gorsky, 2013) and gender differences (Sorby, 2009). This will be further discussed in Chapter 9.

In the intervention group, students' isometric drawings showed the most improvement and the reason for that is, most likely, due to students' starting points. All students started lower on the isometric drawing (female 32%, male 25%) than in the orthogonal drawing (female 59%, male 58%). The results showed that almost all the students improved up to 100% of orthogonal drawings (female: 97%, male 93%). In isometric drawings, the mean percentage in the post-test was 87% for females and 83% for males. Although the researcher refined the lessons constantly to try and get to achieve the same degree of improvement on both isometric drawings and orthogonal drawings, this was not realised. This was partly due to the ceiling effect on the orthogonal questions. Whilst this is not ideal for inferential analysis, it does however indicate that the intervention was highly successful in relation to the national exam expectations.

Finally, both females and males benefitted from the RETA-based lessons. When the improvements were compared in Section 8.2.2 no gender difference in the improvement of the geometry performances (orthogonal and isometric drawings) was observed. This shows that both genders were equally benefitted from the RETA-based lessons. Performances of females and males did however differ in the control group who study traditional lessons with males performing better. This suggests tentative evidence to that the RETA-based lessons help narrowing the difference in the geometry performance (orthogonal and isometric drawings) between females and males.

There were however some limitations to this study. It should be noted that the mathematics teachers volunteered to be part of either intervention or control groups. Teachers were more likely to volunteer for the control group than the intervention group, therefore the study had an imbalance in the number of students in the intervention and control groups. This is potentially a limitation because it led to a smaller sample size in the intervention group and may have factored in the imbalance of pre-test scores. Moreover, the pre-existing class difference was unexpected and of course non-ideal. However, the additional analysis as seen in Appendix I revealed that all intervention groups improved more than all control groups irrespective of whether they started higher or lower. Therefore, the improved outcomes are unlikely to be due to this reason. It is impossible to rule out that teachers' self-selection may have been a factor in the difference in the intervention and control groups. As expected in any school, these two schools had different cultures and the teachers had different backgrounds and experiences of teaching, and all of these may have affected the results. Hence, further work with randomised groups is suggested.

To conclude, the findings of the present study with 205 students confirmed the results of previous cycles and showed that RETA-designed lessons provided more effective instruction than traditional methods.

9. GENERAL DISCUSSION

This chapter discusses the findings from each study in relation to the others and in the light of the existing knowledge. It discusses the findings of the thesis in three sections: current teaching practices, evaluating the RETA-based lesson plans, and the limitations and future work. It continues with the implications for educational practices and ends with the concluding remarks.

9.1. Current Teaching Practices and the RETA Principles (Research Question 1 & 2)

This section includes the discussion on what the current teaching practices are and how that combined with the literature that led to the RETA.

The first research question is:

1. **How do the seventh-grade middle school students learn 3D shapes in Turkey?**
    a. What are the students' difficulties in learning about 3D shapes?
    b. What are the students' errors in representing 3D shapes?

Study One (the case study of current teaching) was the main way this question was answered. In addition, the control group in the fourth study also provided an opportunity to look at existing practice with different teachers who it was found use a very similar pattern with the teachers in the first study.

Both studies found that the lessons about teaching 3D shapes in Turkey were exam-focused with a little interaction. There was a lack of teacher motivation to teach the topic without exam-focused instruction. Teachers (all four teachers in Study 1 and two out of four in Study 4) were observed to use past ministry exam questions and/or questions which are similar to ministry questions in their lesson plans. In doing so, teachers hoped that the students will perform better in the next exam because they believed that the integration of these questions increases students' motivation by emphasising how important the topic is to learn for exam success. These findings align with the reviewed literature which says that teachers' beliefs affect their practices and that particularly teachers of maths in Turkey who have begun to show a tendency to teach to test and avoid student-centred activities (Doruk, 2014; Karaagac & Threlfall, 2004; Saralar, 2016b). This exam-focused pedagogy might be a result of the ongoing teacher performance evaluation (at the time of the data collection) in Turkey where teachers' performance has been measured mostly by their students' outcomes from 2016 to 2018 (Konan & Yilmaz, 2017, 2018).

Moreover, consistent with the literature, the available technology was observed to be used by teachers in limited ways, mostly to show videos and tests from the ministry's Moodle page when concluding their lessons. Despite the new mathematics curriculum's emphasis on technology-integration, particularly on the use of dynamic geometry software packages in maths lessons (MoNE, 2013, 2018), none of the observed six teachers encouraged students to use a dynamic geometry software on their tablets for visualising 2D representations of 3D shapes in any of the lessons. This study supports evidence from previous observations (Bayrakdar-Çiftçi et al., 2013; Çiftci & Tatar, 2015; Tekalmaz, 2019). Similarly to them, the observed teachers reported that they believe in the effectiveness of the current maths programme and support its technology-emphasis. However, when it comes to the teaching practice, most of the time, teachers thought memorising their strategies was the only way to learn this topic and they suggested practicing with more number of questions for better and quicker performance.

The observed lessons about teaching 3D shapes were also found to be teacher-centred. It was observed that the use of tools and manipulatives were dominated by maths teachers. Very little opportunity was given for students to use concrete materials and to express themselves in these lessons. These findings confirmed the results of the earlier studies which further looked for the reasons underlying such pedagogy. These studies (Christou et al., 2006; Kali & Orion, 1996; McGee, 1979; Parzysz, 1988; Widder & Gorsky, 2013) claim that this pedagogy might be related to that the need to visualize 3D shapes from their 2D representations as it builds barriers for not only students' learning but also for teachers' teaching. As discussed in the literature review, various studies reported that teaching of 3D shapes in middle schools is reputed to be difficult among teachers and hence teachers either do not teach it by providing various excuses or use direct teaching rather than student-centred activities (e.g., Bakó, 2003).

These were found to be the main features of existing lessons. These features could have caused the difficulty in understanding 3D shapes in the observed context. The two main student difficulties in representing 3D shapes were found to be mental visualisations of these shapes and drawing the shapes in their minds on an activity sheet.

Specifically, one of the biggest reasons for students' difficulty in both orthogonal and isometric drawings of polycubes was found to be visualization (i.e., visualizing a 3D shape from its orthogonal drawings and vice versa). This finding was consistent with the reviewed studies which found and reported difficulties in many other 2D representations of various 3D shapes (Bayart et al., 2000; Duval, 1998; Fujita et al., 2017; Parzysz, 1988). Difficulty in the visualization of shapes was observed in all four cycles of this thesis. In order to help with this, the RETA-based lessons provided multiple representations of shapes, including prototypes with concrete manipulatives (unit cubes) and constructions in GeoGebra with hopes to enhance students' visualization. This suggested solution was again based on the literature which indicates that viewing real/dynamic 3D shape (e.g., a construction with plastic unit cubes) can reveal much more information than its static drawing in 2D (e.g., its orthogonal drawing) so that it can enhance learners understanding of the 3D shape (Gutierrez, 1996).

The second biggest challenge found in the study was students' difficulty in drawing the shape which they have already visualised. There were many mistakes because of the linking problems in drawings, and interviews with students confirmed that, in such cases, the visualised shape by the student was mostly correct whilst the drawing was not. Hence, in line with the literature, much erasing and incomplete answers were found (see Bishop, 2008). In addition to this, in the isometric drawings, most students drew the front views of the shapes correctly, whilst answers to the other views (views from the left, right and top) had a more number of incorrect answers. A possible reason for the difficulty in isometric drawing might be students' unfamiliarity with the oblique convention where the front view of the cube, which is a square represented with a parallelogram, is drawn and the rest of the drawing is displaced from the front (Bishop, 2008). Thus, the four key concerns about current practice were exam-focused pedagogy (not having motivation to teach and learn about 3D shapes), limited use of available classroom technologies, teacher centred instruction (passive observation of students) and difficulties with visualisation and drawings.

Integrating these concerns with reviewed literature led to the RETA principles and sample lesson plans and thus addressed the second research question:

2. What principles can inform how 2D representations of 3D shapes are best taught to grade seven students in Turkish middle schools?
    a. What are the important elements of 3D shapes lesson plans?
    b. How can specific activities be designed to teach 3D shapes?

Firstly, considering exam-focused pedagogy and the overall aim of the research, which is improving middle school students' performance on these particular drawings in the government exam, it could be seen as initially contradictory. However, this research explored how exam performance can be improved without a reliance on direct instruction and repetition of exam questions. For example, with the ***exploratory*** principle, the lessons were supported with the worked examples (which were very similar to the government exam questions) however, rather than drill and practice, these were designed to include mistakes for students to diagnose and remediate. Moreover, rather than emphasizing the importance of the topic with the exam questions, lessons were supported with the ***realistic*** principle, which refers to the intent of integrating real-life examples and contexts (pictures and videos) into the lessons. As indicated earlier in section 5.2.1, realism is important for teaching middle school geometry to make students aware of the real-life use and importance of the topics that they learnt in the classroom (Gravemeijer, 1994; Van den Heuvel-Panhuizen, 2003). With the knowledge of that all students in the sample were given tablets and all classes were provided with the smartboards by the Turkish government, the researcher also decided to take advantage of available technology and strategically integrated dynamic geometry tools in teaching three-dimensional shapes to provide multiple representations of them, with the ***technology-enhanced*** principle. Finally, as a reaction to teacher-centred instruction and teachers' dominance in using manipulatives, the ***active*** principle of the RETA emphasises learning environments where students themselves have control of the use of manipulatives instead of them watching teacher's constructions and copying drawings on the board hence active refers to the involvement of students as active participants. The RETA with the active principle is described in Turkish context in this particular way, but one can see how it relates to elements of Chi's (2009) **i**nteractive, **c**onstructive, **a**ctive and **p**assive (ICAP) framework. As described in section 5.2.4, it could be noted definition of active is more focussed as it would not include activities such as students' construction of 3D shapes from plastic unit cubes for themselves as they would be defined as constructive and looking at the cube-constructions and solutions of students and working in pairs to discuss these as they would be defined as potentially interactive.

To sum up, the first study looked at current teaching by looking at students' difficulties in learning about 3D shapes and then errors before developed RETA (realistic, exploratory, technology-enhanced and active) based lessons. Thus, these four lesson plans were evaluated and iteratively improved based on the findings of Study 2, 3 and 4.

9.2. Evaluating the RETA-based Lesson Plans (Research Question 3, 4 & 5)

In order to evaluate the RETA-based lesson plans, all three cycles of the design-based research (the second, third and fourth studies) generated data from teachers and students. Even though studies reported in the thesis each focussed on a particular research question, the researcher was able to be in the classroom in all phases and to interview both teachers and students in all cycles. In order to evaluate how successful the RETA-based lessons are, the thesis incorporated all of these views. This is to say that the researcher wanted to know (a) what students think about the RETA-based lessons, (b) how teachers find teaching with them and (c) the students' learning outcomes. The following three paragraphs discussed these three aspects by taking all three cycles of the RETA-based lessons into consideration. Moreover, although focussing on different aspects in each cycle, the researcher always actively searched for ways to improve the design when it was found to be working less well than hoped.

The third research question is:

3. How do seventh grade students experience RETA-based lessons?

Overall, the research found that the RETA-based lessons were engaging and effective. **Students** were always the main focus of the research in all phases. Hence, their experience of the lessons was important in providing them with a better environment for learning 3D shapes, an environment that fits their expectations as well as improves their performance in (particularly orthogonal and isometric) drawings of 3D shapes. Detailed analysis on students' experiences of the RETA-based lessons was reported in the second study, which is the study where the researcher herself taught to lessons but as previously said this was not the only study which students' experiences were considered. Overall, in all cycles of the RETA-based lessons, students mostly experienced the lessons positively except for general difficulties in (i) learning about GeoGebra and (ii) the discussion activities.

For the second study (first cycle), the researcher planned lessons to teach shapes with *GeoGebra* but only after coming into the classroom and seeing students' difficulty in using GeoGebra she realised that students need some time to learn how the tool works in order to use it to solve problems on the lesson activity sheets. This might seem obvious in retrospect but the researcher was not aware of this at the time of the lesson planning. Perhaps because of an unconscious assumption that students would be able to use the tool without specific step-by-step guidance as they are born into technology and support their teachers and other adults in their daily lives with unfamiliar technologies (Grabowski, 2013; Prensky, 2012). Prensky (2012) calls this new generation of students "digital natives" who have started to use technology in every aspect of their lives and emphasizes that they do not learn to use technology like their ancestors instead they are born into technology (p.67). However, it would obviously be wrong to assume all students to know every technology. In fact, in this research, students did not know everything about the particular technology which was intended to use. Hence, an activity to explore GeoGebra was added to the lesson plans after the first cycle (the second study) and this activity was observed to be helpful in all of the consequent cycles for students to explore the tool before using it to solve problems. Even when the teacher was not confident enough to detailly describe how to use GeoGebra in the second cycle (the third study), after the initial difficulty, it did not take much time for students to learn about using GeoGebra by following the step-by-step instructions in the GeoGebra-exploration activity, which was specifically designed for exploring the tool. The classes in the third cycle (the fourth study) was not dissimilar to the one described.

Secondly, *discussions* of the real-life examples and designed mistakes in the lessons always had the most resistance from both the students and the teachers. As discussed in the individual design changes sections of the studies, challenging real-life examples were removed from the RETA-based lesson plans and prompt questions were provided for teachers to help their students' discussion; and much paraphrasing and rewording of the discussion questions on the designed mistakes were done between the cycles.

Even though the discussions were iteratively improved to provide better experience to the students, some difficulty in discussion activities continued to be observed in all cycles. According to the participating teachers, students' difficulty in discussion activities was related to their maturity. For example, Ms Aslan (the teacher in the second cycle) believed that her students were not mature enough to actively engage in fruitful discussion in maths. Her argument might partially rest on the difficulty students experienced as they were not ready for the discussion as they did not know which spatial words to use and/or how to use them well (Cartmill, Pruden, Levine, & Goldin-Meadow, 2009; Newcombe, 2010). Literature suggests teaching spatial words (e.g., front, top, back, side, row, next (to), under, over and around) as early as kindergarten times (Newcombe, 2010b) and use of parent gesture in this teaching to make it more effective even before school years (Cartmill et al., 2009; Rowe & Goldin-Meadow, 2009). However, Turkish mathematics curriculum introduces spatial geometry objectives requiring 3D geometrical thinking only after the 5th grade, and teachers give very little opportunity for their students to express themselves (MoNE, 2013; Yenilmez & Yasa, 2008).

Turing now away for the students directly, the fourth research question is in thesis is:

4. What are the opportunities and challenges for a maths teacher when adopting the RETA-based lessons?

Teachers are decision-makers in the class, and their design beliefs and decisions can have a big impact on students' geometry learning (Barrantes & Blanco, 2006; Even & Tirosh, 2014; López & Nieto, 2006). As discussed in section 2.2.3.3, teachers' beliefs affect what they say and do in classrooms, and students' learning can be affected by this (Hew & Brush, 2007; Sanders, Wright, & Horn, 1997; Schoenfeld, 1998; Thompson, 1984). This is to say teachers' experiences of teaching with the RETA-based lessons were important to evaluate to see whether the RETA-based lessons work and improve them when they did not work as intended. Hence, the teachers in all cycles were interviewed prior to and after the lessons, and the second cycle (the third study) focussed specifically on the challenges and opportunities for a teacher. In general, the RETA-based lessons worked since they gave teachers the opportunities for adopting and testing new pedagogies as well as technologies in teaching 3D shapes. All teachers appreciated the new pedagogy and its outcomes. The researcher is aware that she planned lessons differently from how Turkish maths teachers typically teach the topic and she acknowledges the clashing world views between the researcher's constructivist student-centred approach and the observed teachers' teacher-centred pedagogy involving little interaction. It was not surprising to see teachers' resistance to some of the new practices (e.g., activities with GeoGebra) and principles (e.g., technology-enhanced principle) and suggestions for continuing what they have been doing for many years (direct teaching and practising more number of questions). Teachers showed negative attitude toward the use of GeoGebra and discussion activities, stressing the importance of time (with the belief that these activities are time-consuming) and the importance of the government exam at the end of the year. They also did not appear or sound confident in either using the tool or guiding the discussion activities. The reason for this might be their unfamiliarity with the suggested activities and lack of knowledge and understanding of the new pedagogy. It could be argued that similar to many countries, particularly at the secondary level, Turkish maths teachers are mostly "on the stage" and are very likely to ask predominantly managerial questions that require short responses and do not require any interaction. For the case in England, for example, Boaler (2015) says "If you walk into maths classrooms in England, particularly at the secondary school level, you would think you had been transported into the Victorian age. For the most part, teachers are still at the front of the room lecturing on methods, students are still at desks learning to calculate by hand, and the mathematics being taught is three-hundred-year-old mathematics that is not needed in the modern world" (p.16). Mathematics teaching in Turkey often perfectly matches this description.

The RETA-based lessons were a new and innovative way of teaching 3D shapes for Turkish mathematics teachers. These teachers tended to show students how something should be done and overexplain the topic when teaching it (e.g., explaining real-life videos rather done letting students discuss them or repeating themselves during the lessons such as by saying start drawing the polycubes from top, you need to start from the top, you should not forget starting from the top). Here, it is important to stress the need of teachers' having a deep knowledge of technology and constructivism, which then can be transferred from teachers to their students (Balgalmış, 2013; Balgalmış et al., 2014; Ocak & Çimenci-Ateş, 2015). For example, in the second cycle (the third study), Ms Aslan was a very typical Turkish teacher similar to those observed in the case study (the first study), and she had no hesitation in saying she has not used any technology in her lessons as yet and did not know how to use GeoGebra (as well as that she does not believe in the use of technology to support students' learning). And yet, she was able to adopt some of the other principles, and she particularly liked the worked examples and designed mistakes, which are parts of the exploratory principle. In the third cycle (the fourth study), two teachers -who taught the RETA plans to four classes- were again similar to Ms Aslan in their pedagogy and beliefs. Hence, although the researcher introduced a new and innovative way of teaching orthogonal and isometric drawings of 3D shapes, this could be adopted by teachers who did not (initially) share her beliefs. Moreover, even though this thesis included work with three teachers who adopted the RETA-based lessons for their classes, as mentioned previously, given what has observed across the whole series of studies that it is likely that many other teachers will not be dissimilar to ones included in this thesis.

Finally, the fifth and final research question addressed is:

5. What are the outcomes of the RETA-based lessons for these students?
    a. How do learning outcomes (orthogonal and isometric drawings) differ between students who participate in the RETA-based lessons and those who study traditional lessons?
    b. Does gender influence this?

Last but not least, **outcomes** of the RETA-based lessons for the students are important to discuss. Studies two, three and four all generated outcomes data: the one the researcher herself taught, the one the teacher taught and the one the researcher observed two teachers taught in four different classes. In all three cycles, the outcomes of the RETA-based lessons were not dissimilar even though different teachers taught the lessons. The descriptive statistics in the second study (the one the researcher taught in a voluntary after-school class) showed that the lessons help students improve their 2D drawings of 3D shapes. This particular case is also important as students who volunteered for extra maths lessons were mostly from the higher end of the distribution, as might be expected. The third study (the one the researcher observed a teacher's teaching to a class of 30 students) showed statistical evidence to that the RETA-based lessons help students in these drawings, and the fourth study (experiment with nine classes, four of them being in the intervention classes) compared and further confirmed that the improvement in the students' performance through the RETA-based lessons was statistically greater than of the traditional lessons. Scores of the RETA intervention classes improved almost up to 100% in orthogonal drawings and almost 90% in isometric drawings that are equivalent to ministry exam. In traditional lessons, the final performance was about 60% in orthogonal drawing and 45% in isometric drawings. Positively, traditional lessons are still somewhat effective (the students' final performance in orthogonal drawing: Study 1 60%, Study 4 45%; in isometric drawing Study 1 26%, Study 4 33%). However, the RETA-based lessons were found to be a better way of teaching 2D drawings of 3D shapes (final performance in orthogonal drawing: Study 2 99%, Study 3 100%, Study 4 95%; in isometric drawing Study 2 85%, Study 3 90%, Study 4 85%).

Analysis of the worksheets also showed that the RETA-based lessons helped improve performance irrespective of the gender of the student. That is to say males and females benefitted equally from these lessons. As reviewed in section 2.1.4, there is much debate about systematic gender differences in spatial thinking. The results of the study supports those of other studies which have utilised disciplinary spatial training to improve disciplinary spatial academic achievement in maths and maths-based courses (e.g., Miller & Halpern, 2013; Sorby, 2009; Sorby et al., 2013) for both females and males. These studies reported an important increase in the students' maths performance (measured by a disciplinary test) after the training regardless of the gender. When looking at performance in the control group who studied traditional lessons males outperformed females in both orthogonal (female 45%, male 65%) and isometric drawings (female 25%, male 42%). However, in the intervention group (studied the RETA-based lessons) a performance difference between males and females was not observed in orthogonal (female 97%, male 93%) and isometric drawings (female 87%, male 82%). Thus, this suggests that even when there are pre-existing gender differences in performance, the RETA-based lessons will minimize the gap between females and males in orthogonal and isometric drawings. Thus, this also further support the argument that differences in male and female performance on spatial tests are not due fixed biological differences which cannot be influenced by experience.

The RETA-based lessons also seemed to facilitate children's comprehension, motivation and interest when working with them in a learner-centred environment. As reported in sections 6.2.2.2 and 6.2.2.3, realistic and active principles of the RETA model had an observable effect on students' understanding of 3D shapes and their real-life applications. Moreover, the combination of exploration through and reasoning with designed mistakes and multiple types of representations (e.g., concrete unit cubes and constructions in GeoGebra) convinced doubting students to make the correct orthogonal and isometric drawings. This may have helped the students to make orthogonal and isometric drawings seem more meaningful, similar to how the isometric representations helped to children in Kunimune, Fujita and Jones's (2010) study.

Another finding that emerged consistently was that students found it harder to make isometric drawings than the orthogonal drawings. Particularly, the outcomes of the first study showed that after the regular teaching, students got only 25% in the isometric questions and approximately 50% in the orthogonal questions. The literature review and discussion sections in individual study chapters first referenced this issue referring to human memory capacity and one's need for more simultaneous relations in mind to make isometric drawings than orthogonal drawings (e.g., see section 8.3) (Ayres, 2006; Paas et al., 2003). That is to say, students have to have more elements (orthogonal drawings) simultaneously in mind in order to construct an isometric drawing whereas, in case of orthogonal drawing, students can decompose a polycube and focus on one view (e.g., front view) at a time. Moreover, one needs to interpret orthogonal views in order to construct isometric drawings corresponding to them but isometric drawings can be internalized the same way as the structure itself without further interpretation (Metzler & Shepard, 1974). As Cooper and Sweller (1989) noted "Orthogonal views of 3D objects must be interpreted before the structure they represent can be visualized. To this extent, they differ from *isometric* drawings which … are internalized in the same way as the structure itself" (p.203). They argue that the need to interpret orthogonal views to draw the structure they represent makes it harder to use orthogonal views to construct isometric drawings than using isometric drawings to construct orthogonal views. Lastly, Halford's (1980, 2005) studies found that particular age groups have difficulties in representing 3D shapes (reported in section 2.2.3.3, working memory item). Hence, within Halford's neo-Piagetian framework, the fact that the children in this thesis who are 11-12 years old had difficulty in isometric drawings could be related to their age and to the fact that their working memory has not all developed yet. This may be another explanation of why isometric drawings are harder than orthogonal drawings.

Analysis revealed a number of errors in orthogonal and isometric drawings of 3D shapes in the post-tests after the observed lessons, and this thesis categorised these for future use. The thesis considered the error types reported on 2D representations of various 3D shapes in the reviewed studies when coding the worksheets of students for the nature of errors (e.g., Fujita et al., 2017; Pittalis & Christou, 2013) but described the errors in a much more specific way for the orthogonal and isometric drawings of polycubes. The most common errors have become a part of the RETA-based lessons and presented to the students as designed mistakes to diagnose and remediate. These are the same types of errors in section 2.2.3.1. The number of errors and error types were further investigated for the interest in gender, and no gender difference was observed. The absence of gender difference in the number of errors and error types itself was very interesting because the literature says girls and boys develop different skills in spatial thinking (mostly in favour of males) (Buckley et al., 2019; Linn & Petersen, 1985; van Garderen, 2006; Widder & Gorsky, 2013) but the difference is not observable in this study. It might be related to the number of participants as there were only about 200 students but it might also be related to that in this particular case, which is not spatial thinking but geometric spatial thinking, the classroom practise comes to predominate and hence the way you taught ends up being more important than gender.

When turning to the explanation for improvements noted in this section, as reviewed in sections 5.2.1 to 5.2.4 of the RETA principles chapter, each of the RETA principles seems to work individually within its own model and in its own setting to improve mathematics performance. Realistic principle with realistic mathematics education (Gravemeijer, 1994; Van den Heuvel-Panhuizen, 2003); exploratory with the worked examples (Kalyuga et al., 2001; Renkl, 2014, 2017) and design mistakes (Evans & Swan, 2014; Pierce et al., 2011); technology-enhanced principle with the technology-enhanced learning or computer-assisted learning (Kaput, 1992; Sosnovsky, 2014) particularly through dynamic geometry environments (Oldknow & Tetlow, 2008; Simpson et al., 2006) and by providing multiple representations (Ainsworth, 2006; Pape & Tchoshanov, 2001; Pierce et al., 2011); and finally active principle with ICAP framework (Chi, 2009; Chi et al., 2018) and concrete manipulatives (Moch, 2001; Sowell, 1989; Suydam, 1984) have all been reported to be effective in improving students' learning in mathematics. The RETA model was not something developed from scratch, it is only different in that it aimed at seeing whether the combination of these principles works in support of middle school students' geometry learning in the particular context of Turkish middle students' work with the 3D shapes. However, it is hoped that even though objectives in learning 3D shapes differ across curricula, many countries include representations of 3D shapes in one form or another (DfE, 2013a, 2013b; Hoyles et al., 2002), therefore, RETA-based lessons might also be useful for other nations than Turkish one.

9.3. Limitations and Recommendations for Future Research

This section outlines the limitations of the data collections and the RETA model. All the discussion sections above are evaluations of the lessons based on the RETA principles –the students' experiences, the teachers' experiences and then the learning outcomes. Ideally, in a DBR study, researchers also evaluate the model again in the end and either change it or if they are happy with the model then recommendations for future research, refining it and refining lessons based on it become a part of the discussion. Hence, this section discusses both the limitations of data collections and the limitations of the RETA model based on experiences gained through the RETA-based lessons. This section is important because the biggest limitation of the thesis seems that it might not be clear to one (or it is very difficult to detangle) what the thesis has evaluated. Possible answers could be the RETA model, the theories that led to the RETA model or the RETA-based lesson plans. It must be clear that what the researcher has evaluated in section 9.2 (and throughout the thesis) is the RETA-based lesson plans as implemented by different teachers, not the RETA model itself. This final section is where the thesis evaluates the RETA model and reports its limitations and notes recommendations for future research.

This final section, first, considers the limitations of data collections. The first limitation concerns the relatively small sample size of teachers for the observational phase in Study 1. Having a greater sample of teachers that have been sampled according to some criteria such as years of experience and background of teaching would have been ideal. Nonetheless, in Study 2 that the researcher herself was the teacher is not a limitation of the context of the overall thesis (but a limitation of Study 2) because a number of teachers other than me taught the lessons in the following two cycles. This experience helped her understand the teacher better after experiencing teaching with the RETA-based lessons in a real classroom as a Turkish maths teacher myself. Additionally, the observations in Study 4 (experimental study) gave a chance to observe two more teachers in addition to four teachers in Study 1. The second limitation also relates to sampling. This research looked at a particular topic from 3D geometry with these principles (orthogonal and isometric drawings of polycubes) at a particular year (year seven) in a town in Turkey. There is a much greater geometric world that RETA model could be instantiated, in other years in middle schools, other towns and countries. Hence, other opportunities for future work would be to sample from different years, choosing different topics from 3D geometry and trialling the RETA-based lessons in multiple towns, and even hopefully in multiple countries. The third limitation is a technical limitation about the geometry test that is used to measure students' performance. Because the geometry test in this thesis is based on ministry tests, it has a ceiling effect. Potentially, other studies could look for a more extended geometry test. However, also, it should be noted that the geometry test that is used in this thesis is an authentic test in the researched context.

Secondly, this final section considers whether there are improvements to the RETA principles that could be made (recommendations and any changes the researcher would make to the RETA as a result of the studies) and about where the researcher thinks potentially the RETA model could go next. The RETA (realistic, exploratory, technology-enhanced, active) teaching model –developed and evaluated in this thesis– used other mathematical models and theories as a foundation to suggest this specific model. It was developed as a response to the second research question which asked: "what principles can inform how 2D representations of 3D shapes are best taught". Clearly, there are many ways that this question could be answered, and the RETA model is only one option that could have been developed. In the literature review and when reporting the findings of the first study, the researcher outlined some of the needs, including the need for student-centred environments where students use the tools and manipulatives, the need to provide reasons to teachers to motivate them to teach 3D shapes, the need to provide realistic contexts to engage students with the mathematical content and the need to provide a tool which can help them visualise the shape and facilitate their drawings. Hence, the RETA was developed to meet these particular needs of middle school students in this particular Turkish context.

The RETA-based lessons are developed and trialled in order to see whether the RETA model works in this particular context. Overall, the findings are clear that students' performance on the test was enhanced by RETA-based lessons. But this is not to conclude that RETA model and the RETA-based lessons are perfect and thus one questions how the RETA model and the RETA-based lessons could be improved?

First of all, talking about the model, the researcher does not claim that the RETA principles are sufficient; it is possible that other principles could be supplemented to the RETA. Evaluation of the RETA-based lesson plans in teaching orthogonal and isometric drawings of polycubes showed an example of this: it is realised that dialogic might be added as a principle. The dialogue activities were in all the RETA-based lesson plans but this was not in the principles[7]. A lot of what is proposed (e.g., the fact that the discussion after the videos has to be more structured) in the lesson plans show that it is the "hidden" dialogic part that caused most problems in a lesson. However, "Mathematical discourse has long been shown influential in supporting students' learning of mathematics (Bennett, 2010, p.79). Many mathematics researchers agree that dialogic teaching in mathematics is valuable (Bakker et al., 2015; Hofmann & Mercer, 2016; Kazak et al., 2015; Mercer & Sams, 2006; Ruthven et al., 2017; Warwick et al., 2016). While some (e.g., Kazak et al., 2015) argue that dialogic processes help conceptual development in mathematics, others found that the implementation of dialogic mathematics teaching can be challenging, not only for newly qualified teachers (Bennett, 2010) but also for more experienced ones (Wegerif & Scrimshaw, 1997). Particularly in Turkey, dialogic maths teaching is not a common teaching practice other than some trials for research purposes (e.g., Gürbüz & Aksu, 2017). Teachers of mathematics are still at the front and lecturing, and there is not much space for talk and interaction in maths lessons in Turkey (because of various reasons, one of them being the exam pressure) hence there might be resistance for dialogic teaching. In retrospect, the researcher herself queries why as a researcher who values dialogue in my lessons she did not include it directly as a principle. It is suspected that one explanation is being a Turkish mathematics teacher herself although the dialogic parts were so evident in the activities, she resisted not to have it as a principle emotionally whilst knowing intellectually it would be a challenge. As

[7] This again shows the difference between the RETA model and the RETA-based lesson plans.

reported in Study 3 (the second cycle of the RETA-based lessons in collaboration with a teacher) and also found in Study 4, the discussions embedded in real-life questions and worked-examples got the most resistance (among other activities in the lessons) from the teachers and students in Turkey. In line with these findings, Broza and Kolikant's (2015) study found that it is not only teachers who resist, students also often resist dialogue in mathematics classrooms. Similarly, Bennett (2010) argued that "it's hard getting kids to talk about math." (p.79).

This again brings us to necessary and sufficient argument: Do all principles need to be in the model? Do they all have the same emphasis? Are there extra principles? The researcher argues that all principles do not necessarily be in a lesson plan to make it work better, lesson developers could use them in their overall teaching as exemplified in the RETA-based lesson plans in section 5.3. She only believes that in the specific context that she is teaching in these principles are necessary. However, in England or any other context, there can be different ways to teach 3D shapes and it should be okay. Even in Turkey, students might not need all of the principles to be embedded into a lesson. This is to say, for example, if a teacher says to the researcher that s/he wants to develop the RETA-based geometry lessons but she cannot use a dynamic geometry tool (e.g., GeoGebra), the researcher would have adaptations in mind that they could do. Dynamic geometry tools are important for visualization, but students can still learn with 3D actual physical shapes and make these drawings. On the other hand, although the RETA was developed from the literature and the observations of the researcher, and to some extent, they were treated symmetrically in the design, the principle that she considers being a pinnacle is the active principle, whilst the one which could be removed and get the most outcomes seems to be the technology-enhanced principle. In this research, students used both concrete manipulative (active principle) and a dynamic tool (technology-enhanced principle) in the RETA-based lessons, but they did not use either of them in traditional lessons. Thus, there is no evidence of whether the use of only manipulatives is as helpful as use of manipulatives together with a dynamic geometry tool in this thesis, so the researcher has not got a definitive answer. Future work could thus examine whether the RETA-based lessons would gather similar findings in the absence of technology-enhanced principle.

Furthermore, there could be some refinements in RETA-based lessons on orthogonal and isometric drawings of 3D shapes. These lessons were developed as a whole, and the activities in the lessons were refined after each cycle when/if they were found not effective or not engaging. Design changes sections at the end of the design cycles (Sections 6.3.1 and 7.3.1) reported the refinements after each cycle. After the experimental study, a broader perspective was available for the researcher as multiple teachers used the RETA-based lessons. Many questions come to mind about the lessons including ‘do we need these lessons?’, ‘if they are taught well’, ‘can they be taught quickly if the researcher would be in a school which did not have access to GeoGebra?’, ‘would it be that the rest of the sequence was not enough?’ and many others. The researcher believes that all principles are important to teach these lessons but there might be other/amended lesson plan activities around some of them. First, the exploratory principle which suggests the use of worked examples (some of which with designed mistakes) seems to work in the lessons. However, despite the amendments from cycle one to two and two to three, students and teachers still found it hard to discuss possible reasons for designed mistakes on the worksheets. Hence, these examples could be supported with different prompts for discussion in a future study. Secondly, some of the teachers (in the third and fourth study) were worried about how the existence of a dynamic tool in the lessons would affect students' performance in the ministry exam, where students are not allowed to use any digital tool. Stull and colleagues' (2012) study found that this worry is legitimate and that students might become dependent upon the tool. This study (not specifically on GeoGebra but on other digital tools in educational contexts) showed that people who used 3D models in learning did not do as well in the exam as people who, for example, just learnt to gesture it because they became to rely on the 3D and they came to depend upon the tool (Stull et al., 2012). This thesis aimed at overcoming this issue through the pedagogy of fading concreteness away. In the RETA-based lessons, first physical concrete manipulatives -unit cubes- then the dynamic tool –GeoGebra- fades away by the end of the lessons. However, it was challenging for the teachers to (probably) understand and enact this pedagogy. Discussion of the lessons with “fading concreteness” was not enough for the teachers to enact the lessons with it. Therefore, another opportunity for future research is developing professional development sessions for teaching with this pedagogy.

Finally, it is worth discussing ‘what is it that children are learning?’. Orthogonal and isometric drawing performance of students was measured by a domain-specific geometry test adapted from the ministry exam questions, described in section 4.1.2. This test was used in all four studies of the thesis. The thesis presents evidence that students learnt orthogonal and isometric drawing of geometrical shapes (geometric spatial thinking). However, the research did not collect any evidence to whether children are developing spatial skills more generally. Still, we know from the studies of Hegarty and colleagues (2009) and Stieff and Raje (2008) that at the start of intervention students are often only able to draw on domain-general spatial skills while answering test questions concerning spatial reasoning in chemistry. Their studies further found that by the end of the intervention, students now not only developed disciplinary spatial skills but they may also have improved their domain-general spatial skills. This could be mapped onto the geometry test in this thesis, which is actually training children to do orthogonal and isometric drawings in geometry. Thus, it is possible that these students may improve other spatial skills. For example, they may have become better at three-dimensional mental rotation, but it is unlikely that this intervention would affect vividness of mental imagery. Hence, in the future, it would be interesting to test how specific or general are the outcomes of RETA-based interventions. Future work could look at some of the psychometric tests as pre-test and post-test, predicting potentially one could see understanding spatial relationships in geometry improve one of these generic test results. In retrospect, one of the key improvements could be implemented in the last study of this thesis is to include a mental rotation test as a pre- and post-test to see if the specific activities of the RETA-based lessons also led to a generalized improvement in this spatial test.

9.4. Implication for Educational Practices

The findings of the thesis provide valuable information for teachers, program developers and policymakers about teaching and learning of 3D shapes in Turkey, and hopefully beyond.

Firstly, such research in real classroom settings might help not only students but also teachers gain self-awareness about their own practices. All of the participated teachers reflected on their teaching practices; some of these teachers implemented the RETA-based lessons and described their experiences with them during the debrief discussions. This reflection process might help them realize their own beliefs about teaching 3D shapes and might lead them to question their current practices (Balgalmış, 2013; Balgalmış et al., 2014; Saralar et al., 2018). This could possibly result in gaining valuable insights into a less common practice in Turkey of a more student-centred and constructivist approach in teaching mathematics.

Moreover, the thesis provided the RETA model for teaching 3D shapes which could be used by program developers. The model has been published in conference proceedings of European and British mathematics education conferences. The thesis has not only showed a model but also provided sample lesson plans for guidance. These plans were iteratively improved in each cycle and hopefully are a better help for the program developers to develop other lessons based on the RETA principles. These plans could also set an example for the practitioners so that they can amend these lesson plans according to their and their students' needs. This is to say, maths teachers can adopt and implement the RETA-based lesson plans in their lessons on orthogonal and isometric drawings. However, the notion of whether we should be sharing lesson plans is problematized in the literature. This thesis created opportunities for teachers to adapt and implement the RETA-based lesson plans where possible (in studies 3 and 4). However, because of the requirements of a doctoral thesis, there was only one primary lesson plan developer and these let to detailed lesson plans for the teachers. One might believe that sharing lesson plans is deskillling teachers because it encourages the recipe (the lesson plans) that the Turkish ministry sends out, and teachers' job is not to create their own lessons but merely follow what other researchers have done. Hence, in some ways, the more detailed lesson plans are the worst that is because teachers might feel that they have no space to put their own ideas in. For example, Lieberman (2009) argues that sharing lesson plans with teachers to use might decrease the opportunity of teachers learning from each other hence Lieberman claimed that lesson study which set collaborative roles to a group of teachers to develop lesson plans is more helpful. Similarly, Penuel et al. (2007) suggested another collaborative approach where teachers and researchers work together to (co-)design innovative tools for lesson activities. Whilst being aware that provided lesson plans is problematized, the RETA-based lesson plans (that are effective and engaging and work better than traditional lesson plans in respect of learning outcomes) might set good examples for the practitioners and that could form the basis of others later.

Finally, it might be advised to the policymakers to suggest teachers learning about and benefitting from the findings of the current research. Policymakers in this context are experts in the Ministry of Turkish National Education, where the researcher will work for after completing her PhD. The researcher is lucky enough to return to a situation/work where she can help other teachers implement the designed lessons. The researcher, together with a team of experts from the ministry, can give some pre-service workshops to introduce the RETA model and explain what is aimed at while designing it and whether and how lessons should be supported with these principles. These could be followed up and taken further with in-service training sessions after trainee teachers' start to their jobs. In summary, it would be important to emphasize that both trainee teachers and teachers need some additional training in order to better integrate this new student-centred innovative model to their classrooms.

9.5. Concluding Remarks

To conclude, this thesis has contributed to the research on teaching and learning of 3D shapes. It has developed a model and provided sample lesson plans. They were used by multiple different teachers in multiple different classes and were found to improve students' understanding of 3D shapes, more than the traditional lessons did. It is hoped therefore that in the future this model and the lesson plans that were inspired by it will be helpful for teachers' teaching 3D geometry in Turkey, and maybe beyond.

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APPENDICES

Appendix A. Sample Information Sheets

Information Sheet for School Headteachers
Designing Technology-based Interventions to Help Students Learn 3D Shapes

Who is conducting the study?
My name is Ipek Saralar, and I am a PhD student at the University of Nottingham, in the Learning Sciences Research Institute, School of Education. I am a qualified middle school mathematics teacher. I have worked, volunteered and trained in numerous middle schools, so understand the practices and protocols of working within the school environment and with middle school students.

What is the project about?
My research investigates how middle school students learn three dimensional shapes in mathematics with the help of activities around a tool created through a mathematics software: GeoGebra which they have already been using in their classes but they will use it in a different way, and how lessons can be designed best to enhance their learning in this topic.

What are the aims of the research?
The research seeks to investigate a theory and practice of 3D geometry education in middle schools to explore students' progress and find ways to improve their achievement. This study is specifically designed to enhance and promote students' learning of three-dimensional geometry and support them in overcoming the misconceptions they may have through technology-based interventions.

What are you being asked to do?
I need your permission to get access to school in order to implement four to six after school (N.B. These were amended in the second cycle to say 'in class') sessions (40 to 60 minutes each) to teach 3D shapes to the seventh grade students. After also asking consents of parents and students, I will teach three-dimensional shapes to students in these sessions. A worksheet will be completed by students before the first and after the last teaching sessions. During the sessions, students will be given the opportunity to explore three-dimensional shapes such as houses and household goods by searching for them on the school Internet (protected from the risks they may encounter) using their tablets or in the school library, and they may be given optional homework to explore some points of the topic after the teaching sessions. Classes will be designed to make students active learners. There will be a student centred environment where, they will be working on the topic (e.g. through an educational program, which allows them to do various actions) and completing activities independently. They will also have a chance to ask questions to their friends or to me. I will act as a guide during the teaching sessions, answer students' questions and give clues to them when necessary. Then, 6 students (N.B. This were amended in the second cycle to say 'a small selection of students') will be invited to a one-to-one interview about their experiences in the designed sessions, which will take approximately thirty minutes. This will help me to understand their experiences in the class so that I can improve my design.

Will your school's name and students' identity in this study be kept confidential?
The data will be treated confidentially, and only the researcher and the researcher's supervisor will have access to the raw data. All information collected while carrying out the study will be stored at a location which is password protected and strictly confidential. Any resulting digital or textual data will also be stored electronically on a password protected hard drive for a period of seven years after any research is published before being destroyed. The proposed research project has been approved by the University of Nottingham Ethics committee in compliance with the University's Code of Research Conduct and Research Ethics and follows the ethical protocols outlined by the British Educational Research Association. This means school's name, and students' identity will be anonymised from the outset before digital storage and all materials relating to the school and students will be confidential and only available to my supervisor and me.

Does your school have to take part?
Your participation is entirely voluntary. It is important for you to understand that your school does not have to participate in the project at all. You also have the right to withdraw from the study at any point and this will not affect your status as a head teacher in any manner now or in the future.

Additionally, the results of the project will be provided to the school. Once I have analysed the data I will use these for my PhD thesis and maybe for research publication such as a journal, book chapter or conference. I hope that the findings will help us find ways to improve the teaching of 3D shapes to the seventh-grade middle school students, better design the lessons to enhance students' learning in 3D shapes in mathematics.

Contact details
Researcher: *Ipek SARALAR, PhD Candidate* *Ipek.Saralar@nottingham.ac.uk*
Supervisor: *Professor Shaaron Ainsworth* *Shaaron.Ainsworth@nottingham.ac.uk*
Supervisor: *Professor Geoffrey Wake* *Geoffrey.Wake@nottingham.ac.uk*

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Information Sheet for School Mathematics Teachers
Designing Technology-based Interventions to Help Students Learn 3D Shapes

Who is conducting the study?
My name is Ipek Saralar, and I am a PhD student at the University of Nottingham, in the Learning Sciences Research Institute, School of Education. I am a qualified middle school mathematics teacher. I have worked, volunteered and trained in numerous middle schools, so understand the practices and protocols of working within the school environment and with middle school students.

What is the project about?
My research investigates how middle school students learn three dimensional shapes in mathematics with the help of activities around a tool created through a mathematics software: GeoGebra which they have already been using in their classes but they will use it in a different way, and how a lessons can be designed best to enhance their learning in this topic.

What are the aims of the research?
The research seeks to investigate a theory and practice of 3D geometry education in middle schools to explore students' progress and find ways to improve their achievement. This study is specifically designed to enhance and promote students' learning of three-dimensional geometry and support them in overcoming the misconceptions they may have through technology-based interventions.

What are you being asked to do?
I need your permission to get access to your classes in order to implement four to six after school (N.B. These were amended in the second cycle to say 'in class') sessions (40 to 60 minutes each) to teach 3D shapes to the seventh grade students. After also asking consents of parents and students, I will teach three-dimensional shapes to students in these sessions. A worksheet will be completed by students before the first and after the last teaching sessions. During the sessions, students will be given the opportunity to explore different representations of three-dimensional shapes by searching for them on the school Internet (protected from the risks they may encounter) using their tablets or in the school library, and they may be given optional homework to explore some points of the topic after the teaching sessions. Classes will be designed to make students active learners. There will be a student centred environment where, they will be working on the topic (e.g. through an educational program, which allows them to do various actions) and completing activities independently. They will also have a chance to ask questions to their friends or to me. I will act as a guide during the teaching sessions, answer students' questions and give clues to them when necessary. Then, 6 students (N.B. This were amended in the second cycle to say 'a small selection of students') will be invited to a one-to-one interview about their experiences in the designed sessions, which will take approximately thirty minutes. This will help me to understand their experiences in the class so that I can improve my design.

Will your school's name and students' identity in this study be kept confidential?
The data will be treated confidentially, and only the researcher and the researcher's supervisor will have access to the raw data. All information collected while carrying out the study will be stored at a location which is password protected and strictly confidential. Any resulting digital or textual data will also be stored electronically on a password protected hard drive for a period of seven years after any research is published before being destroyed. The proposed research project has been approved by the University of Nottingham Ethics committee in compliance with the University's Code of Research Conduct and Research Ethics and follows the ethical protocols outlined by the British Educational Research Association. This means school's name, and students' identity will be anonymised from the outset before digital storage and all materials relating to the school and students will be confidential and only available to my supervisor and me.

If the school head teacher approves the research, do I have to take part as a mathematics teacher?
Your participation is entirely voluntary. It is important for you to understand you do not have to participate in the project at all if you do not want to even though the school head teacher approves the research. You also have the right to withdraw from the study at any point and this will not affect your status in any manner now or in the future.

Additionally, the results of the project will be provided to the school. Once I have analysed the data I will use these for my PhD thesis and maybe for research publication such as a journal, book chapter or conference. I hope that the findings will help us find ways to improve the teaching of 3D shapes constructed by unit cubes to the seventh-grade middle school students, better design the lessons to enhance students' learning in 3D shapes in mathematics.

Contact details
Researcher: *Ipek SARALAR, PhD Candidate* Ipek.Saralar@nottingham.ac.uk
Supervisor: *Professor Shaaron Ainsworth* Shaaron.Ainsworth@nottingham.ac.uk
Supervisor: *Professor Geoffrey Wake* Geoffrey.Wake@nottingham.ac.uk

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Participant Information Sheet for Parents
Designing Technology-based Interventions to Help Students Learn 3D Shapes

Who is conducting the study?
My name is Ipek Saralar, and I am a PhD student at the University of Nottingham, in the Learning Sciences Research Institute, School of Education. I am a qualified middle school mathematics teacher. I have worked, volunteered and trained in numerous middle schools, so understand the practices and protocols of working within the school environment and with middle school students.

What is the project about?
My research investigates how middle school students learn three dimensional shapes in mathematics with the help of activities around a tool created through a mathematics software: GeoGebra, which they have already been using in their classes but they will use it in a different way, and how lessons can be designed best to enhance their learning in this topic.

What are the aims of the research?
The research seeks to investigate a theory and practice of 3D geometry education in middle schools to explore students' progress and find ways to improve their achievement. This study is specifically designed to enhance and promote students' learning of three-dimensional geometry and support them in overcoming the misconceptions they may have through technology-based interventions.

What is your child being asked to do?
I planned four to six teaching sessions (40 minutes to 1-lesson-hour each) to teach 3D shapes to your child. Your child will be asked to join these after school (N.B. These were amended in the second cycle to say 'in class') sessions where they will be learning 3D shapes and complete a worksheet before the first and after the last teaching sessions. During the sessions, your child will be given the opportunity to explore three-dimensional shapes such as houses and household goods by searching for them on the school Internet (protected from the risks they may encounter) using their tablets or in the school library, and s/he may be given optional homework to explore some points of the topic after the teaching sessions. Classes will be designed to make students active learners. There will be a student centred environment where, s/he will be working on the topic (e.g. through an educational program, which allows them to do various actions) and completing activities independently. S/he will also have a chance to ask questions s/he input the problem they solved to her/his friends or to me. I will act as a guide during the teaching sessions, answer students' questions and give clues to them when necessary. At the end, your child (N.B. These were amended in the second cycle to say 'a small selection of students') will be invited to an interview about her/his experiences in the designed sessions, which will take approximately thirty minutes. This will help me to understand their experiences in the class so that I can improve my design.

Will your child's participation in this study be kept confidential?
The data will be treated confidentially, and only the researcher and the researcher's supervisor will have access to the raw data. All information collected while carrying out the study will be stored at a location which is password protected and strictly confidential. Any resulting digital or textual data will also be stored electronically on a password protected hard drive for a period of seven years after any research is published before being destroyed. The proposed research project has been approved by the University of Nottingham Ethics committee in compliance with the University's Code of Research Conduct and Research Ethics and follows the ethical protocols outlined by the British Educational Research Association. This means, school's name and your child's identity will be anonymised from the outset before digital storage and all materials relating to your child will be confidential and only available to my supervisor and me.

Does your child have to take part?
Your child's participation is entirely voluntary. It is important for you to understand that your child does not have to participate in the project at all and that you can decide to withdraw her/him from the project at any point. (N.B. These were amended in the second cycle and the following sentences were added 'The mathematics teacher will be teaching in the class time. This means your child will be in the classroom but not necessarily in the study. In other words, no data will be collected from your child for the research purposes without you formally providing your consent.')

Contact details
Researcher: *Ipek SARALAR, PhD Candidate* Ipek.Saralar@nottingham.ac.uk
Supervisor: *Professor Shaaron Ainsworth* Shaaron.Ainsworth@nottingham.ac.uk
Supervisor: *Professor Geoffrey Wake* Geoffrey.Wake@nottingham.ac.uk
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Information Sheet for Students

Designing Technology-based Interventions to Help Students Learn 3D Shapes

- **Who is conducting the study?**

My name is Ipek Saralar, and I am a PhD student at the University of Nottingham. I am planning to conduct a project in your school.

- **What is the project about?**

I am trying to enhance middle school students' learning of three dimensional shapes in mathematics, and understand how a class can be best designed to do it.

- **What are you being asked to do?**

I planned four to six teaching sessions (forty minutes to an hour each) to teach Three Dimensional Shapes. You will be asked to attend those sessions as an after school course (N.B. These was amended in the second cycle to say 'during your regular mathematics classes') where I will be teaching Three Dimensional Shapes and recording the sessions with a special microphone. During the sessions, you will explore three-dimensional shapes by searching them on the school Internet using your tablet or in the school library, and you may be given optional homework to explore some points of the topic after the teaching sessions. You will be working on the topic (e.g. through an educational program, which allows you to do various actions) and completing activities independently. You will also have a chance to ask questions to your friends or to me. I will act as a guide during the teaching sessions, answer your questions and give clues to you when necessary. You will be asked to complete a worksheet having questions on three-dimensional shapes two times (before the first and after the last teaching sessions). At the end of the last teaching session, I will ask you get into small groups so that we can have a chat about what you were working on during the sessions, and how your experience is (discussion). Finally, I will ask you a few questions one-to-one and record our conversations (interview). These questions will be on your opinions about teaching sessions.

- **What are the other things you need to know?**

You can always ask any questions to me. I will keep a copy of your worksheets so I can study them. All data (i.e. consent forms, worksheets, recordings and their written form and my observation notes) will be destroyed after seven years. If I write anything about you, I will give you a different name so that nobody will know who you are.

- **Do you have to take part?**

You can stop at any time. You do not have to give a reason and nothing will happen to you as a result.

If you are happy with this and want to take part in the project, please sign the consent form.

Contact details

Researcher: *Ipek SARALAR, PhD Candidate* Ipek.Saralar@nottingham.ac.uk
Supervisor: *Professor Shaaron Ainsworth* Shaaron.Ainsworth@nottingham.ac.uk
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Appendix B. Sample Translated Consent Forms

School principal or Headteachers

OKUL YÖNETİMİ YA DA MATEMATİK BÖLÜM BAŞKANI GÖNÜLLÜLÜK FORMU
Öğrencilerin Üç Boyutlu Şekilleri Öğrenmesi İçin Ders Tasarımı

Araştırmacının adı ve Eğitimi: İpek Saralar (ipek.saralar@nottingham.ac.uk)
2014, Haziran : ODTÜ – Ortaokul Matematik Öğretmenliği (Lisans)
2016, Nisan : ODTÜ – Ortaokul Fen ve Matematık Eğitimi (Tezli Yüksek Lisans)
2016, Aralık : Nottingham Üniversitesi – Eğitim Teknolojileri (Tezli Yüksek Lisans)
2020, Aralık (planlanan) : Nottingham Üniversitesi – Eğitim (Doktora)

I. Danışmanın adı : Prof. Dr. Shaaron Ainsworth (shaaron.ainsworth@nottingham.ac.uk)
II. Danışmanın adı : Prof. Dr. Geoffrey Wake (geoffrey.wake@nottingham.ac.uk)

- Bilgilendirme formunu okudum ve anladım.
- Araştırmanın amacını ve benim bu çalışmadaki rolünü anladım.
- Öğrencilerimi istediğim zaman bir yaptırımı olmaksızın araştırmadan çekebileceğimi anladım.
- Çalışma sırasında elde edilen veriden yayın yapılabileceğini anladım.
- Okulun, öğrencilerin, öğretmenlerim ve benim adımın yayınlarda kesinlikle kullanılmayacağını ve anonimize edieceğini anladım.
- Toplanan verinin (Gönüllülük formları, çalışma kağıtları, ses kayıtları ve gözlem notlarının) üçüncü şahıslarla paylaşılmayacağını ve sadece araştırma amacıyla kullanılacağını, 7 yıl sonra ise tamamen silineceğini anladım.
- Toplanan verinin gizli tutulacağını, veriye sadece araştırmacının ve danışmanlarının erişebileceğini anladım.
- Herhangi bir zamanda araştırmacı ve danışmanlarından detaylı bilgi alabileceğimi anladım. Ayrıca gerekli görürsem Nottingham Üniversitesi Araştırma Etikleri Koordinatörlüğüne şikayette bulunabileceğimi anladım. (educationresearchethics@nottingham.ac.uk)

İmza [redacted] .... (Okul Yönetimi ya da Matematik Bölüm Başkanı)

İsim ve Soyisim (Büyük harflerle) [redacted] ....

Tarih 17.11.2017......

Teachers

MATEMATİK ÖĞRETMENİ GÖNÜLLÜLÜK FORMU
Öğrencilerin Üç Boyutlu Şekilleri Öğrenmesi İçin Ders Tasarımı

Araştırmacının adı ve Eğitimi: İpek Saralar (ipek.saralar@nottingham.ac.uk)

2014, Haziran	: ODTÜ – Ortaokul Matematik Öğretmenliği (Lisans)
2016, Nisan	: ODTÜ – Ortaokul Fen ve Matematik Eğitimi (Tezli Yüksek Lisans)
2016, Aralık	: Nottingham Üniversitesi – Eğitim Teknolojileri (Tezli Yüksek Lisans)
2020, Aralık (planlanan)	: Nottingham Üniversitesi – Eğitim (Doktora)

I. Danışmanın adı : Prof. Dr. Shaaron Ainsworth (shaaron.ainsworth@nottingham.ac.uk)
II. Danışmanın adı : Prof. Dr. Geoffrey Wake (geoffrey.wake@nottingham.ac.uk)

- Bilgilendirme formunu okudum ve anladım.
- Araştırmanın amacını ve benim bu çalışmadaki rolünü anladım.
- Normalde benim dersimi takip eden öğrencilerin bir kısmının üç boyutlu cisimlerin çizimlerini öğrenebilmek için araştırmacının derslerine gideceğini anladım.
- Öğrencilerimi istediğim zaman bir yaptırımı olmaksızın araştırmadan çekebileceğimi anladım.
- Çalışma sırasında elde edilen veriden yayın yapılabileceğini anladım.
- Okulun, öğrencilerin ve benim adımın yayınlarda kesinlikle kullanılmayacağını ve anonimize edieceğini anladım.
- Toplanan verinin (Gönüllülük formları, çalışma kağıtları, ses kayıtları ve gözlem notlarının) üçüncü şahıslarla paylaşılmayacağını ve sadece araştırma amacıyla kullanılacağını, 7 yıl sonra ise tamamen silineceğini anladım.
- Toplanan verinin gizli tutulacağını, veriye sadece araştırmacının ve danışmanlarının erişebileceğini anladım.
- Herhangi bir zamanda araştırmacı ve danışmanlarından detaylı bilgi alabileceğimi anladım. Ayrıca gerekli görürsem Nottingham Üniversitesi Araştırma Etikleri Koordinatörlüğüne şikayette bulunabileceğimi anladım. (educationresearchethics@nottingham.ac.uk)

İmza .... (Matematik Öğretmeni)

İsim ve Soyisim (Büyük harflerle) ....

Tarih ...17. 11. 2017

Parents

VELİLER İÇİN GÖNÜLLÜLÜK FORMU
Öğrencilerin Üç Boyutlu Şekilleri Öğrenmesi için Ders Tasarımı

Araştırmacının adı ve Eğitimi: İpek Saralar (ipek.saralar@nottingham.ac.uk)
2014, Haziran : ODTÜ – Ortaokul Matematik Öğretmenliği (Lisans)
2016, Nisan : ODTÜ – Ortaokul Fen ve Matematik Eğitimi (Tezli Yüksek Lisans)
2016, Aralık : Nottingham Üniversitesi – Eğitim Teknolojileri (Tezli Yüksek Lisans)
2020, Aralık (planlanan) : Nottingham Üniversitesi – Eğitim (Doktora)

I. Danışmanın adı : Prof. Dr. Shaaron Ainsworth (shaaron.ainsworth@nottingham.ac.uk)
II. Danışmanın adı : Prof. Dr. Geoffrey Wake (geoffrey.wake@nottingham.ac.uk)

- Bilgilendirme formunu okudum ve anladım. Velisi olduğum öğrencinin çalışmaya katılmasına izin veriyorum.
- Araştırmanın amacını ve velisi olduğum öğrencinin araştırmadaki rolini anladım.
- Herhangi bir zamanda ben ya da kendisi çalışmaya devam etmek istemezse herhangi bir yaptırımı olmadan vazgeçme hakkımızın olduğunu anladım.
- Çalışma sırasında elde edilen veriden yayın yapılabileceğini anladım.
- Herhangi bir yayında velisi olduğum öğrencinin kimliğini belli edecek hiçbir bilgi verilmeyeceğini ve anonim olarak yayın yapılacağını anladım.
- Toplanan verinin (Gönüllülük formları, çalışma kağıtları, ses kayıtları ve gözlem notlarının) üçüncü şahıslarla paylaşılmayacağını ve sadece araştırma amacıyla kullanılacağını, 7 yıl sonra ise tamamen silineceğini anladım.
- Toplanan verinin gizli tutulacağını, veriye sadece araştırmacının ve danışmanlarının erişebileceğini anladım.
- Herhangi bir zamanda araştırmacı ve danışmanlarından detaylı bilgi alabileceğimi anladım. Ayrıca gerekli görürsem Nottingham Üniversitesi Araştırma Etikleri Koordinatörlüğüne şikayette bulunabileceğimi anladım. (educationresearchethics@nottingham.ac.uk)

Lütfen aşağıdaki kutulara seçiminizi gösterecek şekilde çarpı işareti koyunuz.

		EVET	HAYIR
1	Velisi olduğum öğrencinin belirtilen çalışmaya katılmasını istiyorum.	X	
2	Velisi olduğum öğrenciyle birebir görüşme yapılacağını ve görüşmenin kaydedileceğini anladım.	X	
3	Velisi olduğum öğrencinin ders sırasında gerekli görüldüğünde sesinin kaydedilmesine izin veriyorum.	X	

İmza .... (Veli)
Velinin Adı ve Soyadı (Büyük harflerle) ....
Çocuğun Adı ve Soyadı .. ..
Tarih 22/11/2017

Students

ÖĞRENCİ GÖNÜLLÜLÜK FORMU

Öğrencilerin Üç Boyutlu Şekilleri Öğrenmesi için Ders Tasarımı

Araştırmacının adı ve Eğitimi: İpek Saralar (ipek.saralar@nottingham.ac.uk)
2014, Haziran : ODTÜ – Ortaokul Matematik Öğretmenliği (Lisans)
2016, Nisan : ODTÜ – Ortaokul Fen ve Matematik Eğitimi (Tezli Yüksek Lisans)
2016, Aralık : Nottingham Üniversitesi – Eğitim Teknolojileri (Tezli Yüksek Lisans)
2020, Aralık (planlanan) : Nottingham Üniversitesi – Eğitim (Doktora)

I. Danışmanın adı : Prof. Dr. Shaaron Ainsworth (shaaron.ainsworth@nottingham.ac.uk)
II. Danışmanın adı : Prof. Dr. Geoffrey Wake (geoffrey.wake@nottingham.ac.uk)
Nottingham Üniversitesi Araştırma Etikleri Koordinatörlüğü : (educationresearchethics@nottingham.ac.uk)

- Bilgilendirme formunu okudum ve benden ne beklendiğini anladım.
- Çalışmaya katılmak istiyorum.
- Eğer çalışmaya devam etmek istemezsem, istediğim zaman vazgeçebileceğimi ve bununla ilgili bir sorun yaşamayacağımı biliyorum.
- Kendi derslerimin dışında derslere katılmam gerektiğini anladım.
- Sizin beni ve arkadaşlarımı ders sırasında gözlemleyeceğinizi anladım.
- İki kez bir çalışma kağıdı dolduracağımı anladım.
- Dersler, konuşmalar ve görüşme sırasında sesimin kaydedilmesinde sorun yok.
- Eğer benle ilgili bir şey yazarsan adımı kullanmayacağını ve kimsenin yazdığın kişinin ben olduğunu anlamayacağını anladım.
- Verinin (Gönüllülük formları, çalışma kağıtları, ses kayıtları ve gözlem notlarının) yedi yıl sonra yok edileceğini anladım.
- İstediğim zaman soru sorabileceğimi anladım.

İmza... [redacted] .... (öğrenci)
Öğrencinin adı ve Soyadı (büyük harflerle) [redacted] ..
Tarih 13. 11 2017

Appendix C. Ethical Approval for Study 1

The University of Nottingham

UNITED KINGDOM · CHINA · MALAYSIA

School Of Education
The Dearing Building
Jubilee Campus
Wollaton Road
Nottingham
NG8 1BB
Tel: +44 (0)115 951 4543
Fax: +44 (0)115 846 6600
www.nottingham.ac.uk/education

Our Ref: 2017/64

Dear Ipek Saralar
CC Prof Sharon Ainsworth

Thank you for your research ethics application for your project:

Study 1. Understanding How Students Learn 3D Geometry

Our Ethics Committee has looked at your submission and has the following comments.

- This was considered to be very comprehensive with very clear documents for children in user friendly language.

However, the Committee requests the following amendments are made:

- Remove the marginal comment on the worksheet.

Based on the above assessment, it is deemed your research is:

- **Approved**

We wish you well with your research.

KEFuller

Dr Kay Fuller
Chair of School of Education Ethics Committee

Appendix D. Ethical Approval for Study 2

University of Nottingham
UK | CHINA | MALAYSIA

School of Education
University of Nottingham
The Dearing Building
Jubilee Campus
Wollaton Road
Nottingham
NG8 1BB

educationresearchethics@nottingham.ac.uk

19/10/2017

Our Ref: *2017 / 94 (Ipek Saralar)*
CC: Prof. S Ainsworth

Dear Ipek,

Thank you for your research ethics application for your project:

Study 2. Designing Technology-based Interventions to Help Students Learn 3D Shapes

Our Ethics Committee has looked at your submission and has the following comments.

- Thank you for the opportunity to review this application. This was a very thorough and complete set of documents to help think about the proposed research. Please pass on thanks to the student and supervisors.
- This is a well prepared submission, unproblematic in my opinion.

Based on the above assessment, it is deemed your research is:

- **Approved**

We wish you well with your research.

KEFuller

Dr Kay Fuller, Chari of Ethics Committee, School of Education

+44 (0)115 9514470
educationadmin@nottingham.ac.uk
nottingham.ac.uk/education

Appendix E. Answer Key for the Worksheet

ANSWER KEY

Okul No: Cinsiyet: Doğum Tarihi: .../.../.....

ÇALIŞMA KAĞIDI

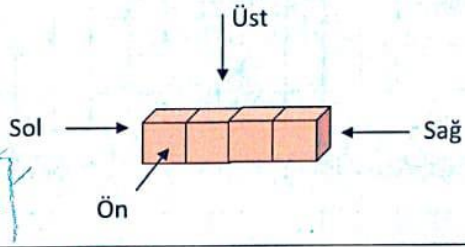
Sevgili öğrenci, iki bölümden oluşan bu çalışma kağıdı MEB YLSY görevlendirmesi ile İngiltere'de doktora yapan matematik öğretmeni İpek Saralar'ın doktora çalışmasının ikinci adımıdır. İlk bölümde, verilen üç boyutlu cisimlerin önden, üstten, soldan ve sağdan görünümlerini çizmeniz beklenmektedir. İkinci bölümde ise, önden, üstten, soldan ve sağdan görünümleri verilen birim küplerden oluşan üç boyutlu cisimleri çizmeniz beklenmektedir. Katılımınız için teşekkür ederim.

BÖLÜM 1.

Bu bölümdeki beş soruyu aşağıdaki ibareye göre yanıtlayınız:

Birim küpler kullanılarak oluşturulan üç boyutlu şeklin önden, üstten, soldan ve sağdan görünümlerini çiziniz.

1.

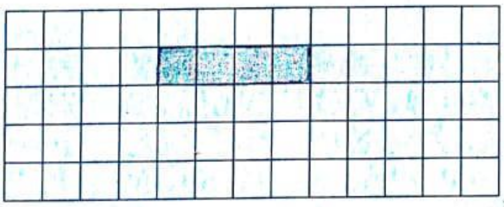


correct = 1 point
incorrect = 0 point

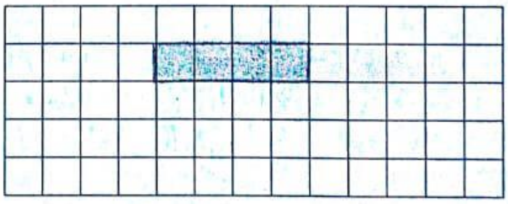
1* Drawings without dividing or filling are correct if the place and number of squares are correct.

2* If some parts of shape is divided but not others 1* applies.

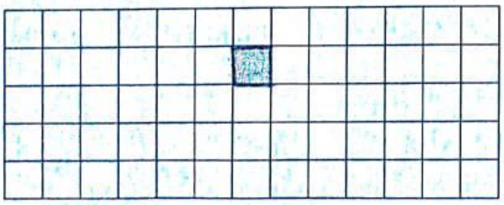
ÖNDEN GÖRÜNÜM



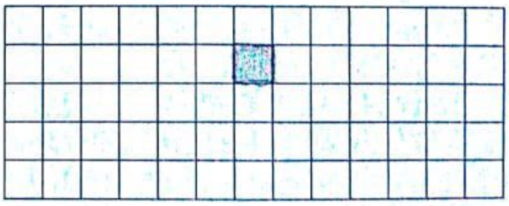
ÜSTTEN GÖRÜNÜM



SOLDAN GÖRÜNÜM



SAĞDAN GÖRÜNÜM



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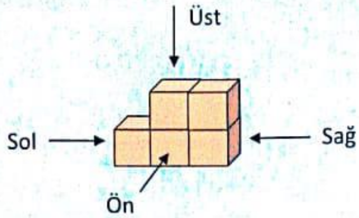
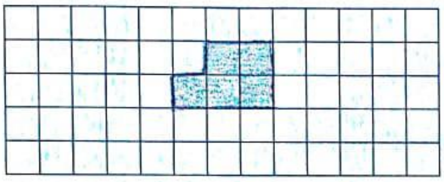


ÖNDEN GÖRÜNÜM



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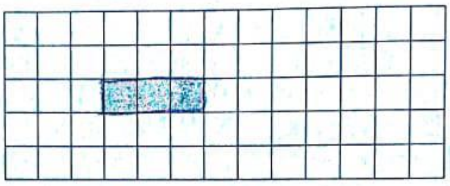
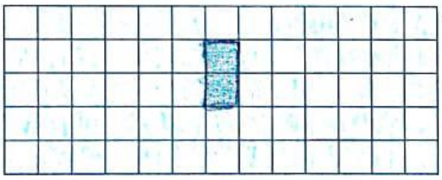
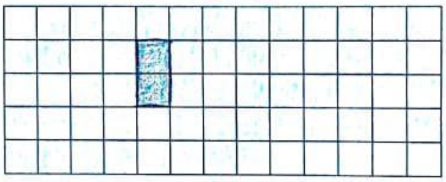


SOLDAN GÖRÜNÜM



SAĞDAN GÖRÜNÜM



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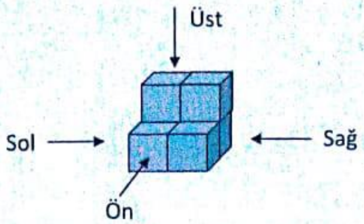

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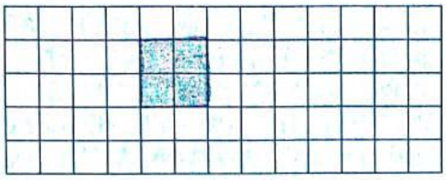

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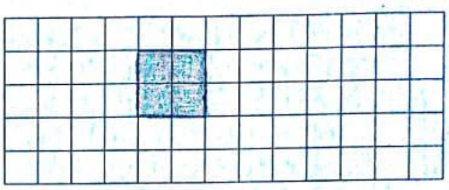

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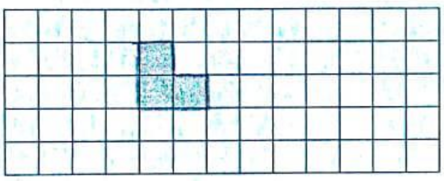

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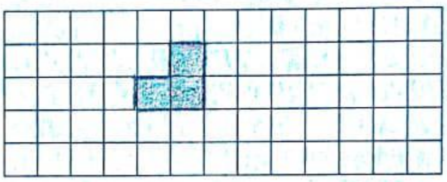

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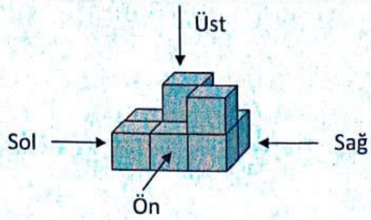

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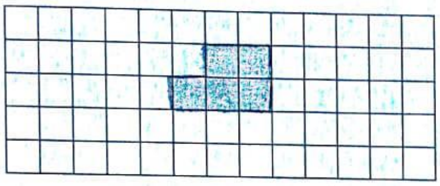

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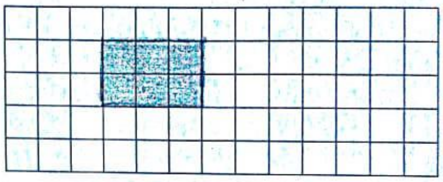
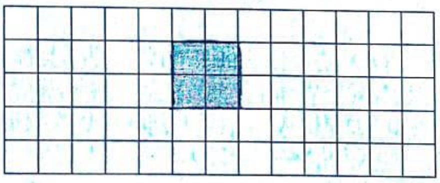


SOLDAN GÖRÜNÜM



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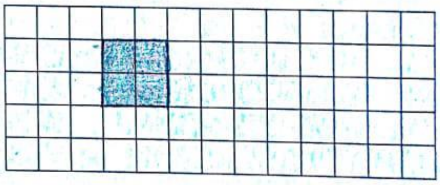

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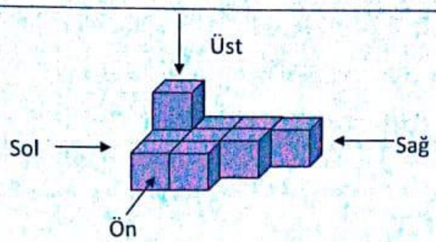

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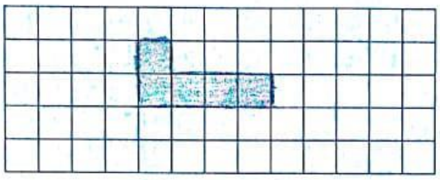

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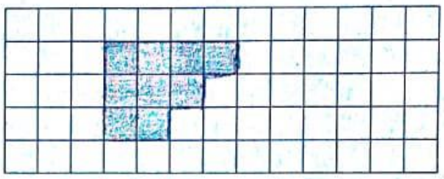
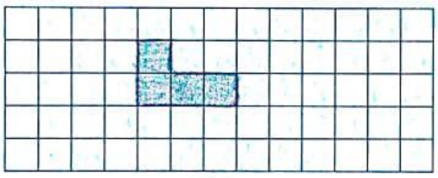


SOLDAN GÖRÜNÜM



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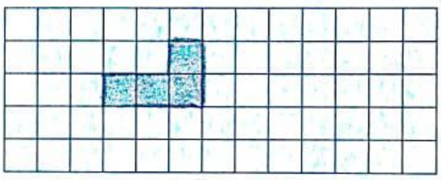


BÖLÜM 2.

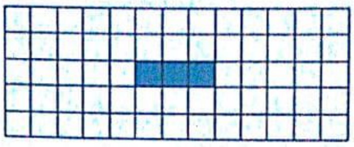
Bu bölümdeki beş soruyu aşağıdaki ibareye göre yanıtlayınız:

(1) Önden, üstten, soldan ve sağdan görünümleri verilen birim küplerden oluşan üç boyutlu cismi izometrik olarak çiziniz ve (2) şeklin önünü gösteriniz.

13*. Drawings without dividing or filling is correct if the number of cubes are correct.

ÖNDEN GÖRÜNÜM

ÜSTTEN GÖRÜNÜM



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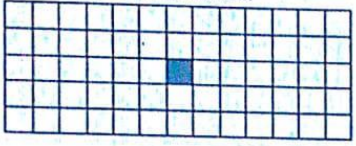

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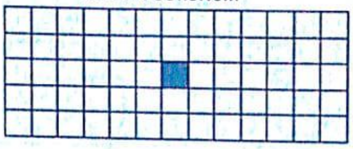
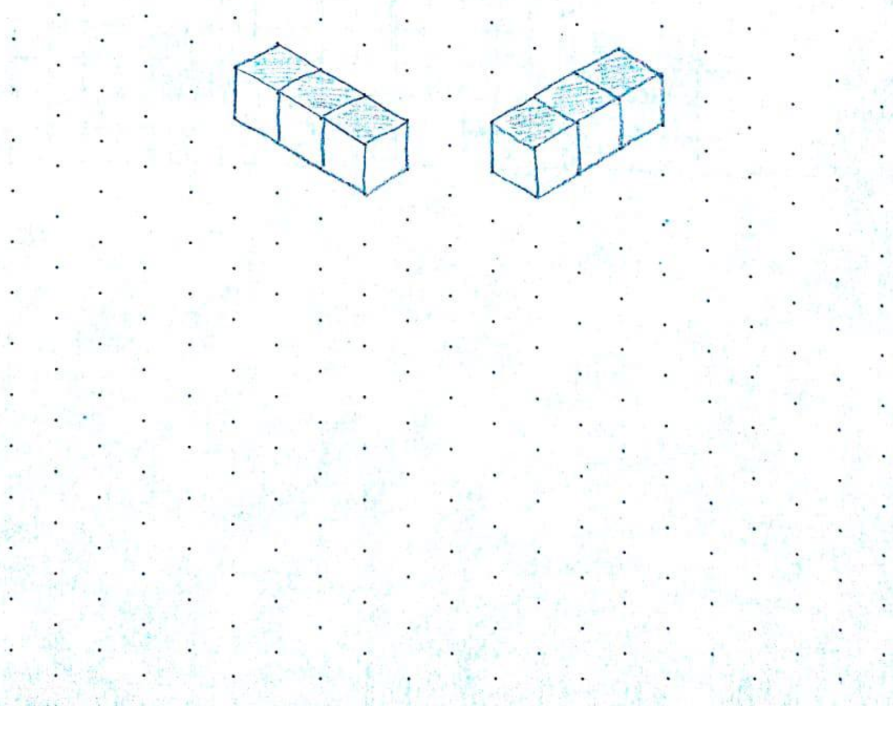
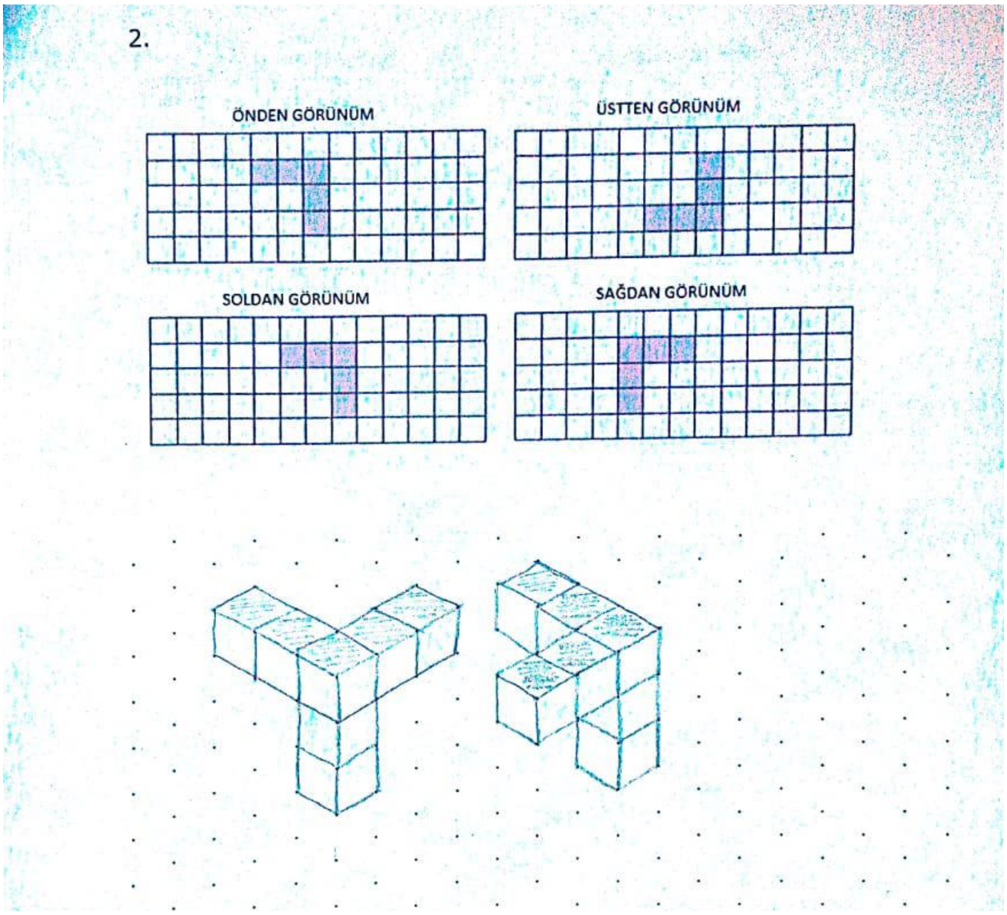
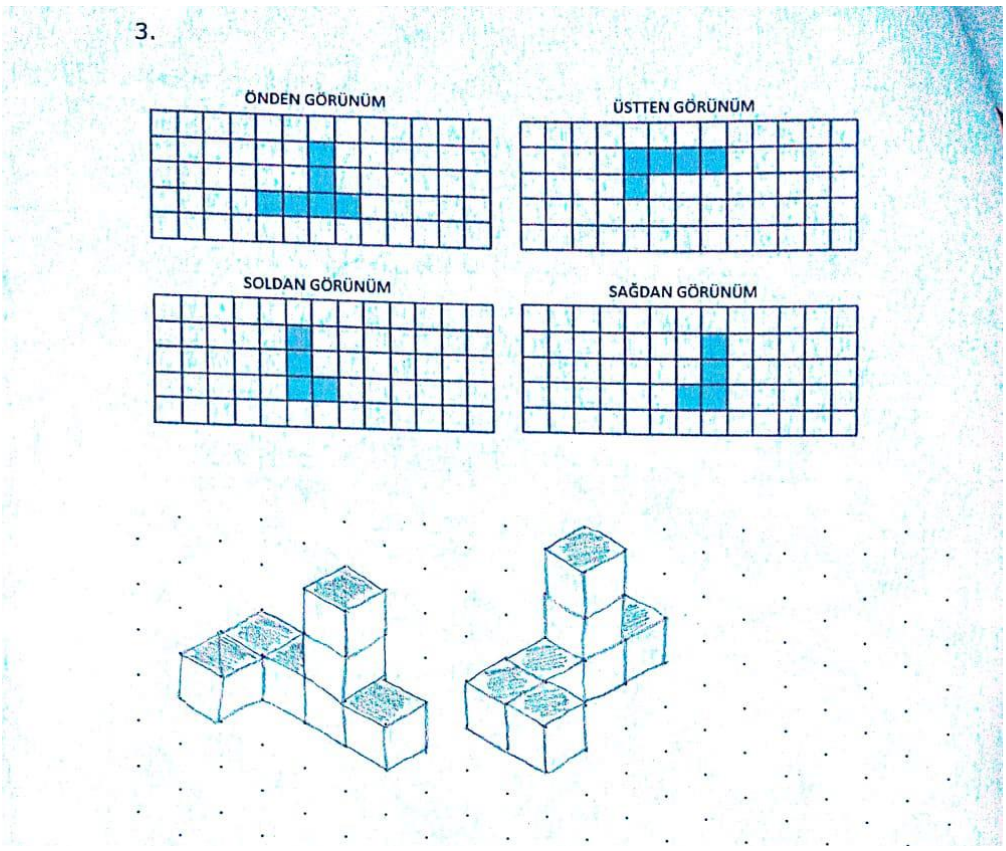





2.
ÖNDEN GÖRÜNÜM
ÜSTTEN GÖRÜNÜM
SOLDAN GÖRÜNÜM
SAĞDAN GÖRÜNÜM


3.
ÖNDEN GÖRÜNÜM
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SOLDAN GÖRÜNÜM
SAĞDAN GÖRÜNÜM

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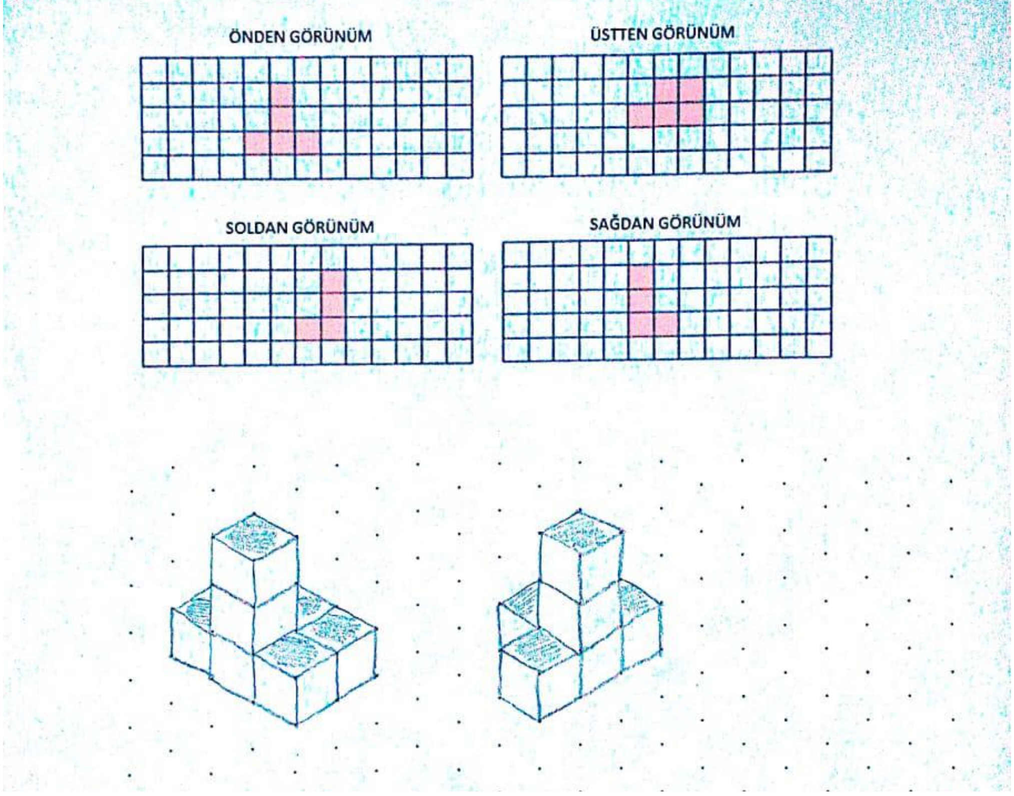
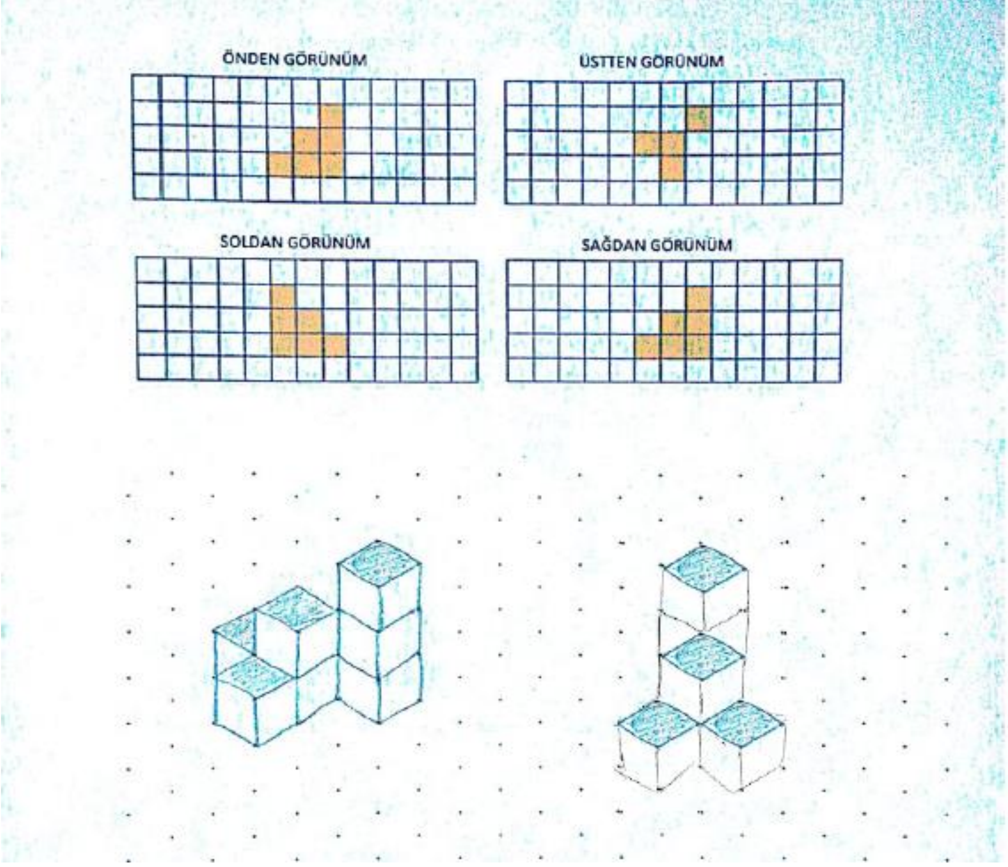

ÖNDEN GÖRÜNÜM
ÜSTTEN GÖRÜNÜM
SOLDAN GÖRÜNÜM
SAĞDAN GÖRÜNÜM

5.


ÖNDEN GÖRÜNÜM
ÜSTTEN GÖRÜNÜM
SOLDAN GÖRÜNÜM
SAĞDAN GÖRÜNÜM

Appendix F. Interview Questions: Discussion with the Teacher

Interview Protocol

This questions and notes are subject to change as there will be an informal discussion, but the general idea will be the same: getting to know the teacher and her opinions about the lesson plans. Also, I will not repeat the questions asking for the same thoughts if I have got the answer already although questions under the same title may seem a little repetitive.

Thanks for accepting to collaborate. How are you today? What beautiful weather it is, etc.

1. ***Talking about my maths journey and asking about hers***: I always feel like I was born into teaching maths. I graduated from a teacher training high school and chose to be a maths teacher. How did your teaching journey start? How did you become interested in teaching mathematics? What attracted you to teach maths?
2. ***Learning her experience as a maths teacher***: How long have you been teaching in a middle school? How long it's been in this school? How is the teaching programme in the school regarding classes? Are you teaching to this particular group of students for three years? Which field in maths do you enjoy teaching the most (algebra, numbers, geometry, probability)? Why? What about geometry (if the answer was not already geometry)?
3. ***Coming from the geometry to the 2D drawings***: I was teaching in a private middle school for a few years and I struggled a lot while teaching geometry concepts (as probably I have difficulty to understand them myself.). How do you feel about teaching geometry concepts? What about 2D drawings? What about specifically orthogonal and isometric drawings?
4. ***Learning more about her PCK***: I myself have always had difficulty to understand and teach these drawing to students. I tried to come up with solutions as a teacher but it cost me years and a PhD away from my home. Do you think it is a difficult topic to teach? Do you have any difficulty to understand it yourself? Have you experienced things similar to my experience yourself or in the classroom? If any, can you give some examples of difficulties in teaching and learning this topic from your experience? Would you talk through your strategies and methods of teaching orthogonal and isometric drawings?

Discussion on important elements of lesson plans – I think it is better to think of the important elements before discussing the lesson plans one by one and lesson flow. Now, I would like to learn what you think about the activities in the lessons as an experienced maths teacher who also knows the students in this school.

Learning her opinions on and suggestions for

1. ***The real-life examples in lesson plans***: My first solution to this was showing students my excitement of teaching the topic and trying to raise awareness for the need of the topic so that they can actually see that they are not learning just for the sake of learning or just for exams. I tried to include real-life examples into these lesson plans for the same purpose. Have you had a chance to have a look at them? (*I will open the laptop in the teachers' room beforehand and will print out the lesson plans and activity sheets.*) *There will be discussions on real-life examples in lesson plans. If the teacher watched them beforehand, I will open them just to remind but we will not watch the entire videos. If not, we will watch the videos and we will have a look at the photos together and discuss their place and importance in lessons first one by one and then in general.*
2. ***The use of concrete manipulative in the lesson plans***: My other solution was giving linking unit cubes to students to construct polycubical shapes during my lessons. While I was delivering these lesson plans a few months ago, I learnt that if I give students some time more than they need to construct the required polycubical shape, they would be happy to play and construct many other shapes and not focus on drawing instead. I also read many studies on the

positive effects of using concrete manipulatives for students and even for us, teachers. What do you think of the polycubical shape construction parts of the lessons? Why? What do you think specifically about timing (1), number (2) and sequence (3) of construction questions? Why?

3. ***The use of worked out examples or designed mistakes and discussion*:** In addition to my supervisors, during my PhD, I met with other researchers working on different topics, and they all affected me while shaping the lesson plans. They have all brought pits and pieces to the plans, and all suggestions are always welcome to have better lesson plans. For example, Mark Simmons was the one who suggested me to add more discussion on the plan and showed some examples prepared based on the English curriculum. Sheila Evans, being one of these researchers, gave me an idea of using wrong examples and asking students to correct them. I combined these ideas and designed questions based on students' common mistakes. Here are these *Spot the Mistake Questions*. (*I will open the related pages in our printed out lesson plans and activity sheets.*) What do you think about the use of these questions? What about the number (1) and difficulty (2) of these questions? Why? Are you happy with the current discussion questions?
4. ***The use of a GeoGebra tool in the lesson plans*:** My probably the best but most challenging solution was providing students with some dynamic representations so that students and also I can communicate the topic better. Have you had a chance to play with the tool I emailed? (*This question may not be asked if the answer to the question related to lesson plans is more general and includes the tool as well.*) If yes, as I am aware of the fact that she was not familiar with the tool after our previous conversation, it may continue as follows. I always feel nervous while delivering lessons with technology. This technology helped me change my teaching way but with a reasonable cost. As you may guess, there were some issues related to classroom management, bringing children's attention back to drawing on a paper again after they work with the tool etc. How do you feel about using this tool in the classroom? Shall we have a quick look together? (*It is important for the interviewer to ensure that she became capable to integrate it into her lessons, the success of the integration might be the conversation of another discussion.*) If no, I will ask to schedule another meeting for discovering the tool together by going through examples on the lesson plans.
5. ***Integration of a curriculum sub-objective (symmetry) into the lesson plans*:** It is hard for me to show the connection between two concepts and help students relate these to come up with a solution. As you already know, the hidden curriculum objective of the orthogonal drawings is 'symmetry'. I tried to integrate it into this lesson plan (*I will show the related part, and I will also open the corresponding PPT slide.*) How do you find it? Do you think it is a smooth transition? If not, how would you improve it as an experienced teacher?
6. ***Student agency*:** Although I know that most of the teaching in Turkey is teacher-centred, I tried to make the lesson plans more student-centred. I tried to give students an agency by trying to design activities meaningful and relevant to them. Those activities will be performed by them with appropriate guidance from you as their teacher. In many activities, there are more than one answers. Students' all always will be accepted as perfectly correct. Here is the answer key including all possible answers. You may have already seen these. (*They were in the email I sent you last month. I will be giving her printed copies anyway.*) What do you think about having more than one answers? What about delivering more student-centred lesson plans?
7. ***(Student profile*:** May I also learn more about students in the class that you will be teaching? How familiar are the students with the use of technology in the lessons? Is there any preparation needed to tailor the lessons for this particular class? Is there anything which might be useful to know for me about the students in this class?*)*
8. ***The lesson plans in general (flow of the plans)*:** We went through all of the activities in the lesson plans. Shall we have a look at the order of the activities within the lesson plans? I would like to hear your comments on the flow of the lessons. What difficulties do you think you are likely to encounter as a teacher while delivering these lesson plans? How would you plan to deal with them? Would you like to suggest anything I have not mentioned yet but you already knew as you know all the students and as you are more experienced than me?

Appendix G. Teacher Debriefs of the Teaching Process

Discussions after the lesson implementations *(10-15 minute breaks or lunch)*

Learning her comments on

1. ***The lesson in general and strengths***: How was the lesson in general? What went well do you think? What were the strengths of the lesson implementation?
2. ***Use of spatial geometry language***: I sometimes have difficulty expressing my thinking about spatial geometry. How easy or difficult was it for you (and your students) to express your (and their thinking) on the topic in this class?
3. ***(Un)expected problems***: We outlined the lesson flow and had discussions on it but as teachers, we always need to be flexible enough for the surprises. Were there any expected or unexpected problems in this lesson?
    a. Instant 1 - What may be the underlying reason for it? What did you do to deal with it? What was the weakness of the lesson plan? How would you improve that part of a lesson for the future iterations of the study?
    b. Instant 2 - What may be the underlying reason for it? What did you do to deal with it? What was the weakness of the lesson plan? How would you improve that part of a lesson for the future iterations of the study?
4. ***Issues to consider for the future***: What were the easiest and the most difficult parts of the lesson for the students (1) and for you (2)? What are the issues that you want to pay more attention in general for the next lesson? May I learn your thoughts on what could have been better about this lesson? If you were to change some parts, what else would you suggest? [What are the issues that you think needs to be improved for the next iteration of the study?] or [What are the issues that you would like to improve for the next iterations of this lesson?]

Discussions after lesson 2 included comparison questions (with the previous lessons) in addition to above-listed questions

Learning her comments on

1. ***Comparison with the previous lesson(s)***: How easy or difficult was it for you (and your students) to express your (and their thinking) on the topic in this class compared to your previous lesson? Is there any difference between your previous lesson and this lesson in terms of students' engagement (1), your confidence of delivering student-centred lessons (2) and integrating dynamic technologies into your lessons (3)? What are the issues that you would like to pay more attention related to the use of technology for the next lesson or that you would like to suggest me to pay more attention in future iterations of the study?

Appendix H. Outcomes of the Lessons for Students

Table 8.3. Analysis of variance for student performance

Source	df	F	η	p
Between subjects				
Group (G)	1	76.11	.52	.001
Gender (Ge)	1	318.00	.14	.048
G x Ge	1	10.01	.22	.047
Error	201	(80.08)		
Within subjects				
Question type (Q)	1	158.22	.66	.001
Q x G	1	0.02	.00	.896
Q x Ge	1	1.37	.08	.243
Q x G x Ge	1	0.09	.00	.765
Q within-group error	201	(19.91)		
Time (T)	1	370.20	.81	.001
T x G	1	156.66	.66	.001
T x Ge	1	0.540	.06	.463
T x G x Ge	1	0.41	.05	.521
T within-group error	201	(16.76)		
Q x T	1	16.84	.28	.001
Q x T x G	1	33.28	.38	.001
Q x T x Ge	1	1.43	.08	.233
Q x T x G x Ge	1	0.01	.00	.907
Q x T within-group error	201	(8.07)		

Note. Values enclosed in parentheses represent mean square errors.

Appendix I. Supplementary Analysis for each Class

This appendix reports the additional analyses on test scores for each class. As a reminder, there were four intervention (class one to four) and five control (class five to nine) classes in the study. The supplementary analyses on test scores were done for each class separately. This is, a series of paired samples t-tests were conducted to test students' performance in orthogonal and isometric drawing before and after the lessons.

As displayed in Table 8.4, in orthogonal drawings, all classes apart from class six made a significant improvement, four with large effect sizes (classes one to four), three with medium effect sizes (classes five, seven and eight) and one with a small effect size (class nine). The effect sizes of intervention classes (ranging from 0.99 to 1.21) were larger than of the control classes (ranging from 0.37 to 0.74). In the isometric drawing, seven of the classes out of nine had significant improvement (classes apart from class six and nine), six with large effect sizes (classes one to five and seven) and one with a small effect size (class eight). Similar to orthogonal drawings, the effect sizes of intervention classes (ranging from 1.47 to 2.12) were larger than of the control classes (ranging from 0.34 to 0.89).

To sum up, a series of paired-samples t-test was conducted for each class to evaluate any change between pre-test and post-test (Table 8.4). The results indicated that for all intervention classes the mean scores for post-test were significantly different than mean scores for pre-test in both orthogonal and isometric drawing, and all of them with large effect sizes. In other words, the results suggest that the performance of intervention classes improved after the RETA-based lessons with large effect sizes in both orthogonal and isometric drawing or the RETA-based lessons had a large effect on students' performance. For the control classes, the results were more complicated and varied. These results were difficult to interpret because whilst there were large effects of traditional lessons on students' isometric drawing performance in two of the classes (at two different levels), there was no effect of these lessons in two other classes.

To conclude, the results revealed that all intervention groups improved more than all control groups irrespective of whether they started higher or lower. This fact that one class starts better than another is not the factor that determines whether they improved.

Moreover, classes which started with similar levels ended at different levels after different methods of instruction. For example, class one in the intervention group and class six in the control group started with similar levels. However, the lessons (the RETA-based lessons for class one and the traditional lessons for class six) affected them differently; thus, they ended up at very different levels (for orthogonal drawing, class one: pretest 46%, posttest 94%; class six: pretest 44%, posttest 54% and for isometric drawing, class one: pretest 24%, posttest 82%; class six: pretest 24%, posttest 29%). These results do not say that the control classes did not improve at all but rather suggest that they did not improve as much and as consistently as the intervention classes.

Table 8.4. Descriptive statistics and t-test results for each class

Class	Pretest		Posttest							
	M	*SD*	*M*	*SD*	*n*	*r*	*t*	*df*	*p*	*Cohen's d*
	Orthogonal drawing									
Intervention										
1	9.13	7.78	18.80	4.11	15	.22	4.692	14	.001	1.21
2	10.60	5.40	18.43	2.56	30	.23	7.93	29	.001	1.45
3	14.46	5.67	19.50	.86	22	.59	4.539	21	.001	0.99
4	12.61	7.15	19.50	1.25	18	.24	4.204	17	.001	0.99
Control										
5	9.26	6.69	12.65	6.60	23	.76	3.536	22	.002	0.74
6	8.79	8.27	10.83	6.91	24	.75	1.812	23	.083	0.37
7	11.00	6.17	12.52	6.24	21	.94	3.269	20	.004	0.71
8	7.38	5.66	10.14	6.33	29	.78	3.668	28	.001	0.68
9	6.61	4.78	8.09	6.37	23	.87	2.203	22	.038	0.46
	Isometric drawing									
Intervention										
1	4.73	4.74	16.73	6.93	15	.38	6.895	14	.001	1.78
2	3.60	3.88	15.33	5.25	30	.30	11.628	29	.001	2.12
3	8.68	7.45	19.50	1.01	22	.14	6.871	21	.001	1.47
4	6.56	5.50	16.78	3.37	18	.44	8.632	17	.001	2.04
Control										
5	5.17	5.47	8.87	6.53	23	.78	4.282	22	.001	0.89
6	4.79	6.20	5.88	6.42	24	.88	1.679	23	.107	0.34
7	5.38	4.87	8.19	6.02	21	.82	3.754	20	.001	0.82
8	5.14	4.60	6.52	5.38	29	.83	2.462	28	.020	0.46
9	4.30	5.24	3.35	4.79	23	.85	-1.627	22	.118	0.34